PASSIVE CIRCUIT DESIGN

Heathkit
ZENITH
Educational
Systems

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INTRODUCTION

A passive component is one that cannot provide a power gain. Typical examples of passive components include resistors, capacitors, inductors, voltage and current sources, transformers, and diodes.

This course is written for persons who want to learn how to design practical circuits utilizing passive components. It is introductory in the sense that it establishes a foundation for more advanced design courses such as "Transistor Circuit Design." Similarly, this course is advanced in the sense that it assumes you already have some background in electricity and electronics.

Consequently, ideal prerequisites for this course include:

1. A basic knowledge of DC and AC electricity.
2. An exposure to, perhaps in the remote past, the concepts of elementary algebra and trigonometry.
3. A desire to learn.

Before you can design circuits, you must be able to analyze them. For this reason, the first three units provide a thorough review of the techniques and associated mathematical concepts used to design circuits.

Every circuit constructed with real circuit components always works. This does not mean that a given circuit necessarily does what you want it to, rather that the behavior of the circuit conforms to the laws of nature.

A circuit that "works" on paper may not do what you want it to once you build it with real components. Frequently, this undesirable situation arises because the properties of real components differ from those of ideal components. Ideal components can only be found in textbooks. Consequently, Unit Four is devoted exclusively to a discussion of the properties of real components. Here, various selection guides are provided to help you select an appropriate-type component for a given application.

Having emphasized the techniques of circuit analysis in the first four units, the remaining units concentrate on using these techniques to design circuits. Specifically:

1. Unit Five discusses the design of rectifier circuits, smoothing filters, and unregulated power supplies.
2. Unit Six examines how clippers, clampers, peak detectors, and other waveshaping circuits are designed.
3. Unit Seven illustrates seven general and nine measurement applications for passive circuits.

In addition, Unit Eight contains ten hands-on experiments designed to complement the text material. To perform these experiments, you will need the ET-1000 Circuit Design Trainer, or a similar trainer. A volt-ohmmeter and oscilloscope are also required for most of the experiments. All other parts, such as resistors, capacitors, inductors, and diodes, have been supplied.

In concluding, I would encourage you to design your own experiments, since I believe that experience is, without question, the best teacher. Also, don't forget to enjoy what you are doing, since learning can, and should be, fun!

Sincerely,

Vincent F. Leonard, Jr.

Associate Professor of Electrical Engineering
Jamestown Community College

COURSE OBJECTIVES

When you complete this course you should be able to:

1. Analyze resistive DC circuits.
2. Analyze RL and RC charging and discharging circuits.
3. Add, subtract, multiply, and divide complex numbers.
4. Analyze RLC circuits driven from sinusoidal sources.
5. Describe the nonideal properties of real circuit components.
6. Analyze and design half- and full-wave rectifier circuits.
7. Analyze and design choke-input, capacitor-input, and cascaded filter sections.
8. Select values for coupling and bypass capacitors.
9. Analyze and design the following diode circuits: clippers, clampers, peak detectors, and peak-to-peak detectors.
10. Select component values for passive integrators and differentiators.
11. Discuss seven general and nine measurement applications for passive circuits.

COURSE OUTLINE

UNIT 1 **DC CIRCUIT ANALYSIS**
- Introduction
- Unit Objectives
- Fundamental Concepts
  - Charge
  - Current
  - Conventions and Current Flow
  - Voltage, Resistance, and Current Flow
  - Power
  - Prefixes
- Basic Circuit Analysis
  - Kirchoff's Current Law
  - Kirchoff's Voltage Law
  - Series Circuits-Voltage Division
  - Parallel Circuits-Current Division
  - Series-Parallel Circuits
- Useful Circuit Theorems
  - Superposition Theorem
  - Thevenin's Theorem
  - Maximum Power Transfer Theorem
- Summary
- Experiment 1
- Unit Examination

UNIT 2 **MATHEMATICS FOR CIRCUIT DESIGN**
- Introduction
- Unit Objectives
- RC Switching Circuits
  - Switching Basics
  - Capacitance
  - RC Switching Circuits
- RL Switching Circuits
  - Inductance
  - RL Switching Circuits
- Logarithms
  - Basic Concepts
  - Applications for Natural Logs
  - Applications for Common Logs — Decibels
  - Reference Levels — dBm

Rectangular Complex Numbers
  Real, Imaginary, and Complex Numbers
  Operations with Rectangular Numbers
Polar Complex Numbers
  Basic Trigonometry
  Polar Numbers
  Multiplication and Division of Polar Numbers
  Rectangular — Polar Conversions
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Unit Objectives
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  Time Varying Signals
  The Sine Wave
  Frequency Domain Notation — Phasors
Reactance and Impedance
  Capacitive Reactance
  Inductive Reactance
  AC Power
Selected AC Topics
  Series Resonance
  Parallel Resonance
  Fourier's Theorem
Summary
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Unit Examination

UNIT 4 REAL CIRCUIT COMPONENTS
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  Resistor Basics
  Definitions
  Fixed Composition Resistors
  Fixed Wire-Wound Resistors
  Film Resistors
  Variable Resistors

Capacitors
  Capacitor Basics
  Definitions
  Capacitor Types
  Capacitor Applications
  Capacitor Selection Guide
Inductors
  Inductor Basics
  Core Materials
  Mutual Inductance
  Types of Coils
  Coil Resistance
Summary
Experiment 4
Unit Examination

UNIT 5 **UNREGULATED POWER SUPPLIES**

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Unit Objectives
Transformers and Diodes
  The Ideal Transformer
  Real Iron-Core Transformers
  The Ideal Diode
  Semiconductor Diodes
Rectifier Circuits
  Half-Wave Rectifier
  Output Polarity
  Full-Wave Rectifier
  Comparison of Rectifiers
Smoothing Filters
  Definitions
  Capacitor Input Filter
  Capacitor Input Filter Analysis and Design Equations
  Initial Surge Current
  Choke-Input Filter
  Choke-Input Filter Analysis and Design Equations
  Cascaded Filter Sections
Summary
Experiment 5
Experiment 6
Experiment 7
Unit Examination

UNIT 6 ADDITIONAL PASSIVE CIRCUITS

- Introduction
- Unit Objectives
- Diode Clippers
  - Negative Clippers
  - Positive Clippers
  - Biased Clippers
  - Comparison of Series- and Shunt-Type Clippers
  - Clipper Combinations
  - Designing Clipper Circuits
  - Adjustable Clipping Levels
- Diode Clampers and Multipliers
  - The Negative Clamper
  - The Positive Clamper
  - Response to Amplitude Changes
  - Shunt Diode Resistor
  - Biased Clampers
  - Designing Clamper Circuits
  - Peak Detectors
  - Designing Peak Detectors
  - Peak-to-Peak Detectors
  - Voltage Multipliers
- The Amazing RC Circuit
  - Case I — Output Taken Across Resistor
  - High-Pass Filter
  - Bode Plots
  - Coupling Circuit
  - 0 to 90° Phase Shifter
  - Differentiator
  - Case II — Output Taken Across Capacitor
  - Low-Pass Filter
  - Bypass Circuit
  - 0 to 90° Phase Shifter
  - Integrator
  - Source and Load Resistance
  - Summary
  - Experiment 8
  - Experiment 9
  - Unit Examination

UNIT 7 **SELECTED APPLICATIONS**
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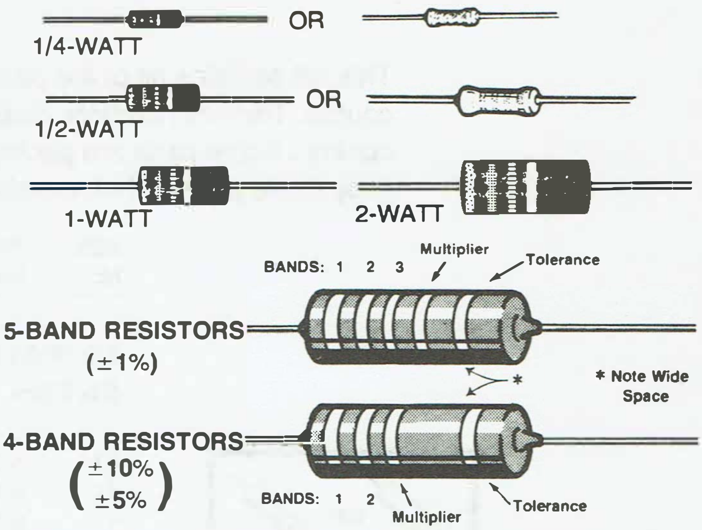
- Unit Objectives
- General Applications
  - Reversed Polarity Protection
  - Elementary Heating Control
  - Diode Arc Suppression
  - Wave Shapers
  - DC Restoration
  - Catching Diodes
  - De-emphasis Circuit
- Measurement Applications
  - Meter Protection
  - Increasing Current Ranges
  - DC Ammeter to DC Voltmeter Conversion
  - Scale Expanders
  - Peak-Type AC Voltmeter
  - Peak-to-Peak Type AC Voltmeter
  - Waveform Converters
  - Rate Meter
  - Diode Checker
- Summary
- Experiment 10
- Unit Examination
- Final Examination

UNIT 8 **EXPERIMENTS**
- Introduction
- Experiment 1
- Experiment 2
- Experiment 3
- Experiment 4
- Experiment 5
- Experiment 6
- Experiment 7
- Experiment 8
- Experiment 9
- Experiment 10

PARTS

Resistors are identified in Parts Lists and steps by their resistance value in Ω (ohms), kΩ (kilohms), or MΩ (megohms). They are usually identified by a color code and four or five color bands, where each color represents a number. These colors (except for the last band, which indicates a resistor's "tolerance") will be given in the steps in their proper order. Therefore, the following color code is given for information only. NOTE: Occasionally, a "precision" or "power" resistor may have the value stamped on it.



Band 1 1st Digit

Color	Digit
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

Band 2 2nd Digit

Color	Digit
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

Band 3 (if used) 3rd Digit

Color	Digit
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

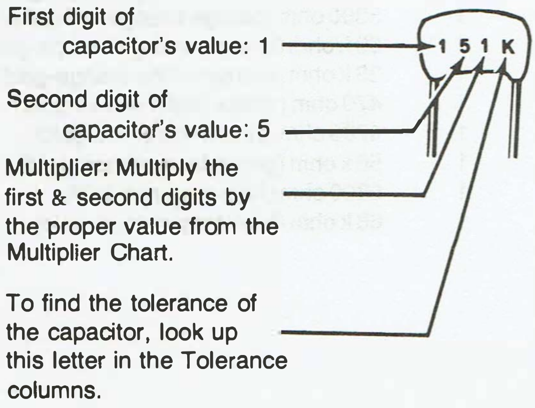
Multiplier

Color	Multiplier
Black	1
Brown	10
Red	100
Orange	1,000
Yellow	10,000
Green	100,000
Blue	1,000,000
Silver	0.01
Gold	0.1

Resistance Tolerance

Color	Tolerance
Silver	±10%
Gold	± 5%
Brown	± 1%

Capacitors will be called out by their capacitance value in µF (microfarads) or pF (picofarads) and type: ceramic, Mylar*, electrolytic, etc. Some capacitors may have their value printed in the following manner:


First digit of capacitor's value: 1
1 5 1 K
Second digit of capacitor's value: 5
Multiplier: Multiply the first & second digits by the proper value from the Multiplier Chart.
To find the tolerance of the capacitor, look up this letter in the Tolerance columns.

EXAMPLES:

151K = 15 × 10 = 150 pF
759 = 75 ×0.1 = 7.5 pF

NOTE: The letter "R" may be used at times to signify a decimal point: as in: 2R2 = 2.2 (pF or µF).

MULTIPLIER		TOLERANCE OF CAPACITOR		
FOR THE NUMBER:	MULTIPLY BY:	10 pF OR LESS	LETTER	OVER 10 pF
0	1	±0.1 pF	B	
1	10	±0.25 pF	C	
2	100	±0.5 pF	D	
3	1000	±1.0 pF	F	±1%
4	10,000	±2.0 pF	G	±2%
5	100,000		H	±3%
			J	±5%
8	0.01		K	±10%
9	0.1		M	±20%

---

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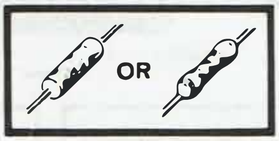
PARTS LIST

This list contains all of the parts for the experiments that you will perform with this course. The key numbers corespond to the numbers on the parts illustrated in the column. Some parts are packaged in envelopes. Except for this initial parts check, keep these parts in their envelopes until they are called for in an experiment.

1/2-WATT RESISTORS
5% Film



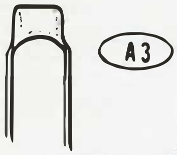
A1

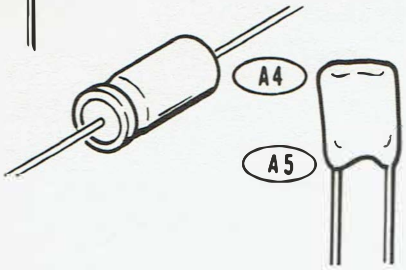
KEY No.	PART No.	QTY	DESCRIPTION
A1	6-101	1	100 ohm (brown-black-brown-gold)
A1	6-102	2	1000 ohm (brown-black-red-gold)
A1	6-103	2	10 k ohm (brown-black-orange-gold)
A1	6-104	1	100 k ohm (brown-black-yellow-gold)
A1	6-105	1	1 M ohm (brown-black-green-gold)
A1	6-222	1	2200 ohm (red-red-red-gold)
A1	6-333	1	33 k ohm (orange-orange-orange-gold)
A1	6-471	1	470 ohm (yellow-violet-brown-gold)
A1	6-472	2	4700 ohm (yellow-violet-red-gold)
A1	6-474	1	470 k ohm (yellow-violet-yellow-gold)
A1	6-681	1	680 ohm (blue-gray-brown-gold)

1/4-WATT RESISTORS
5% Film

A2

KEY No.	PART No.	QTY	DESCRIPTION
A2	6-101-12	1	100 ohm (brown-black-brown-gold)
A2	6-102-12	1	1000 ohm (brown-black-red-gold)
A2	6-103-12	3	10 k ohm (brown-black-orange-gold)
A2	6-104-12	1	100 k ohm (brown-black-yellow-gold)
A2	6-106-12	1	10 M ohm (brown-black-blue-gold)
A2	6-153-12	1	15 k ohm (brown-green-orange-gold)
A2	6-221-12	1	220 ohm (red-red-brown-gold)
A2	6-222-12	1	2200 ohm (red-red-red-gold)
A2	6-223-12	1	22 k ohm (red-red-orange-gold)
A2	6-331-12	1	330 ohm (orange-orange-brown-gold)
A2	6-332-12	1	3300 ohm (orange-orange-red-gold)
A2	6-333-12	1	33 k ohm (orange-orange-orange-gold)
A2	6-393-12	1	39 k ohm (orange-white-orange-gold)
A2	6-471-12	1	470 ohm (yellow-violet-brown-gold)
A2	6-472-12	1	4700 ohm (yellow-violet-red-gold)
A2	6-563-12	1	56 k ohm (green-blue-orange-gold)
A2	6-682-12	1	6800 ohm (blue-gray-red-gold)
A2	6-683-12	1	68 k ohm (blue-gray-orange-gold)



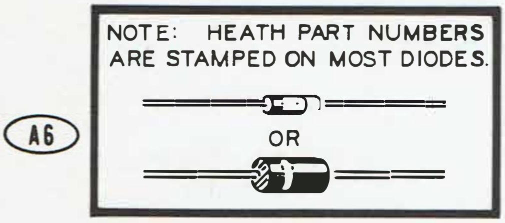


KEY No.	PART No.	QTY	DESCRIPTION

CAPACITORS

KEY No.	PART No.	QTY	DESCRIPTION
A3	21-192	1	.1 μF, 50 V ceramic (104)
A3	21-717	1	.01 μF, 50 V ceramic (103)
A3	21-784	1	.001 μF, 50 V ceramic
A4	25-868	1	47 μF, 50 V electrolytic
A4	25-870	1	100 μF, 50 V electrolytic
A4	25-881	2	10 μF, 63 V electrolytic
A5	27-79	1	.039 μF, 50 V Mylar*

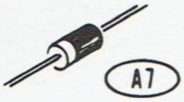
NOTE: Ceramic capacitors supplied with this course may use a new marking convention. For example, 103 = 0.01 μF, where 10 equals the capacitance in picofarads and 3 equals a powers of ten multiplier. (10×10^{-12} times $1 \times 10^{3} = 10 \times 10^{-9} = 0.01$ μF)

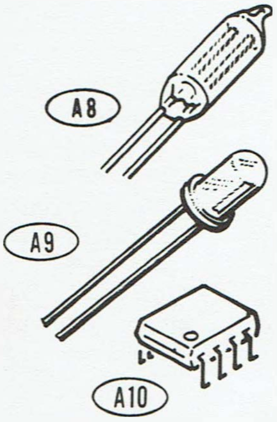


NOTE: HEATH PART NUMBERS ARE STAMPED ON MOST DIODES.

DIODES & RECTIFIERS

KEY No.	PART No.	QTY	DESCRIPTION
A6	56-84	1	1N4148
A7	57-65	4	1N4002





LEDs & INDICATORS

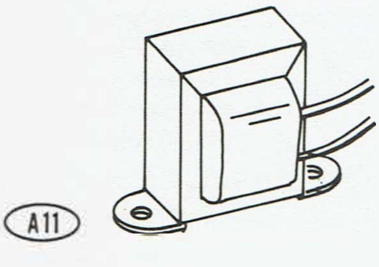
KEY No.	PART No.	QTY	DESCRIPTION
A8	412-15	1	Neon lamp
A9	412-640	2	Light Emitting Diode (LED)

INTEGRATED CIRCUITS

KEY No.	PART No.	QTY	DESCRIPTION
A10	442-22	1	741 op amp
A10	442-53	1	555 timer

AF CHOKE

KEY No.	PART No.	QTY	DESCRIPTION
A11	46-3	1	4.5 H AF choke



MISCELLANEOUS

KEY No.	PART No.	QTY	DESCRIPTION
	344-59	13′	#22 white solid wire
	266-962	1	5-compartment container

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Unit 1

DC CIRCUIT ANALYSIS

CONTENTS

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INTRODUCTION

Electronic components such as resistors, capacitors, and inductors are connected together to form electronic circuits. When you analyze a circuit your task is to determine the current through, the voltage across, and/or the power dissipated by one or more of the circuit components.

Circuit design is the opposite of circuit analysis. When you design a circuit, your task is to select appropriate components, and decide how they must be connected together so that the desired responses are produced. As you will see, the methods used to analyze circuits are very important **tools** for designing circuits.

Because of the relationship that exists between circuit analysis and circuit design, we will begin our study of circuit design by **reviewing** the techniques of circuit analysis. Once you have mastered these techniques, it will be relatively easy to "reverse the process" and begin designing circuits.

UNIT OBJECTIVES

When you have completed this unit, you should be able to:

1. Define coulomb, current, voltage, resistance, and power.
2. Specify the symbol for, and scientific notation equivalent of, standard electronic prefixes.
3. State Ohm's law.
4. Define node, loop, and branch.
5. State Kirchhoff's current and voltage laws.
6. Use Kirchhoff's laws to write and solve basic nodal and loop equations.
7. Determine the equivalent resistance of series, parallel, and series-parallel circuits.
8. State the principles of voltage and current division.
9. Apply the voltage and current division principles to series, parallel, and series-parallel circuits.
10. Analyze a simple circuit by using the superposition theorem.
11. Analyze a simple circuit by using Thevenin's theorem.
12. Convert a Thevenin equivalent circuit to a Norton equivalent circuit and vice versa.
13. Experimentally determine the Thevenin voltage and resistance of a DC circuit.
14. State the maximum power transfer theorem.

UNIT ACTIVITY GUIDE

☐ Read section on "Fundamental Concepts."

☐ Answer Self-Test Review questions 1-10.

☐ Read section on "Basic Circuit Analysis."

☐ Answer Self-Test Review questions 11-26.

☐ Read section on "Useful Circuit Theorems."

☐ Answer Self-Test Review questions 27-43.

☐ Perform Experiment 1 in Unit 8.

☐ Study Summary.

☐ Complete Experiment 1, Unit 8.

☐ Complete Unit Examination.

☐ Check Examination Answers.

FUNDAMENTAL CONCEPTS

The integrity of a building can be no better than the quality of the foundation upon which it is built. In a similar way, a number of fundamental concepts form the foundation upon which the techniques of circuit analysis and design are built. For this reason, it is important that you master the basic concepts, units, and terms that form the foundation of circuit analysis and design.

Charge

Atoms are the building blocks of matter. However, even they are composed of more fundamental building blocks - electrons, protons, and neutrons. Electrons are negatively charged; protons are positively charged; and neutrons are electrically neutral. Apparently, the smallest amount of charge possible is the amount of charge of a single electron (negative) or a single proton (positive). Thus, an electron has the same amount of charge as a proton but it is of the opposite polarity. The **coulomb** (C) is the fundamental unit used to describe the amount of charge. One coulomb represents approximately 6,240,000,000,000,000,000 (over six billion billion) times the charge of a single electron (negative) or proton (positive). Stated mathematically:

$$1C = 6.24 \times 10^{18}q \qquad \text{(Eq. 1-1)}$$

where: q = the amount of charge of a single electron (negative) or proton (positive)

Very early in the history of electricity, it was noted that like charges repel and unlike charges attract. This attraction and repulsion is explained by assuming electric lines of force exist in the space surrounding an electric charge. This space is referred to as an **electric field**; since if a second charge enters this space, it will experience an electric force.

The **direction** of an electric field is defined as the direction that a small positive test charge will move when it is placed in the electric field. For this reason, electric lines of force are directed outward from a positive charge, and inward toward a negative charge as illustrated in Figure 1-1.

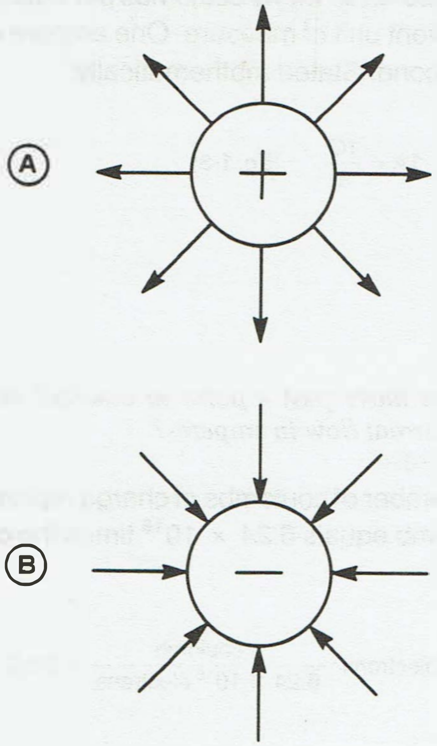


Figure 1-1

The concept of an electric field.
A Positive charge
B Negative charge

Current

Current is the **flow of electric charges**. Thus we may define the average current passing a point as:

$$I = \frac{Q}{t} \qquad \text{(Eq. 1-2)}$$

where: I = average current flow in coulombs per second (C/s)
Q = charge in coulombs (C)
t = time in seconds (s)

Normally, current flow is not stated as so many coulombs per second, since the ampere (A) is a much more convenient unit of measure. One ampere represents a current flow of one coulomb per second. Stated mathematically:

$$1\text{A} = \frac{1\text{C}}{\text{s}} \qquad \text{(Eq. 1-3)}$$

Example 1-1

15.6 × 10^{18} electrons move past a point in one-half minute. What is the average current flow in amperes?

First, you must determine the number of coulombs of charge represented by 15.6 × 10^{18} electrons. Since one coulomb equals 6.24 × 10^{18} times the charge of a single electron, you have:

$$Q = 15.6 \times 10^{18}\,\text{electrons} \cdot \frac{1\ \text{coulomb}}{6.24 \times 10^{18}\,\text{electrons}} = 2.5\text{C}$$

Next, you must convert the time required for the 2.5C of charge to move past the point from minutes to seconds — since Equation 1-2 indicates that time (t) must have units of seconds:

$$t = 0.5\,\text{min} \cdot \frac{60\text{s}}{\text{min}} = 30\text{s}$$

Finally, you can use Equation 1-2 to calculate the average current flow:

$$I = \frac{Q}{t} = \frac{2.5\text{C}}{30\text{s}} = 0.0833\ \frac{\text{C}}{\text{s}} = 0.0833\text{A}$$

Remember, **before** you substitute a particular quantity into an equation, **always** make sure that quantity has the units called for by the equation.

Conventions For Current Flow

Does current flow from plus to minus or from minus to plus? This question is an example of what logicians (people who study logic) call logically complex questions. A logically complex question is really several questions stated as if only one question was being asked. Thus, to properly answer a logically complex question, you must provide more than one answer. Recall that the definition for current is simply the flow of electric charges. This definition **does not** specify whether or not the charges that are flowing are negative charges or positive charges.

In materials like metals, the movement of electrons (which flow from minus to plus) constitutes the flow of current. In P-type semiconductor materials, the movement of positive charges, called holes, (which flow from plus to minus) constitutes the flow of current. **Thus, in a physical circuit, depending upon whether or not the movement of negative charges or positive charges constitutes the flow of current, the actual flow of the charge carriers may be from minus to plus or from plus to minus.** More importantly, as far as circuit analysis and circuit design are concerned, it makes no difference whether or not the actual flow of current is due to the movement of negative charges or positive charges, since the external effects produced are identical. Figure 1-2 illustrates this important concept. For purposes of analysis and design, we must agree to adopt one of two possible conventions: either **electron current flow**, which assumes current is due to the movement of negative charges, and flows from minus to plus; or **conventional current flow**, which assumes current is due to the movement of positive charges, and flows from plus to minus.

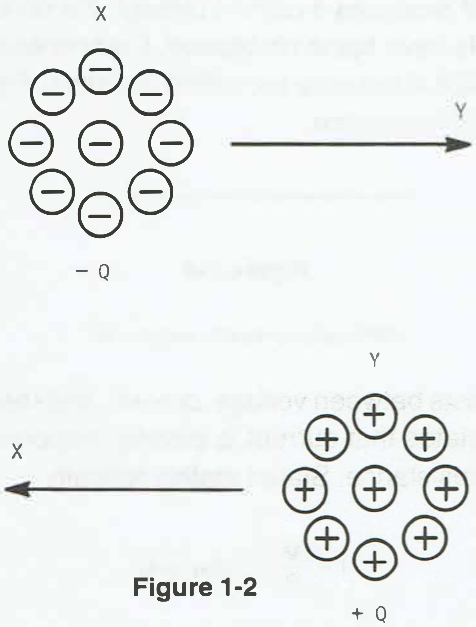


Figure 1-2

An important concept.

A given number of coulombs of negative charge (− Q) moving in one direction (X to Y) is equivalent to an equal number of coulombs of positive charge (+ Q) moving in the opposite direction (Y to X). It is assumed that the velocity and magnitude of each group of charges is the same.

Based upon a rigorous statistical analysis (a coin toss) we have decided to adopt the electron current flow convention. For purposes of analysis and design then, we will assume current flows from minus to plus.

Voltage, Resistance, and Ohm's Law

Voltage is a measure of the amount of energy that must be used in order to move electric charges through a circuit. Since current is the flow of electric charges, we can think of voltage as the **electrical pressure** that causes the current to flow. Figure 1-3 illustrates the symbol used to represent a DC voltage source. Note here that the long line is connected to the positive terminal of the voltage source, and the short line to the negative terminal of the voltage source. As you probably know, voltage is measured in units of **volts**.

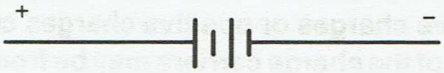


Figure 1-3

Schematic symbol for a DC voltage source. The long line indicates the positive terminal, and the short line indicates the negative terminal.

Resistance is the **opposition** to current flow. Resistance is measured in units of **ohms**. A conductor has a resistance of one ohm (abbreviated Ω), if a voltage across the conductor of one volt produces a current through the conductor of one ampere. All electrical components have some resistance. Components that have been specifically designed to exhibit resistance are called resistors. Figure 1-4 illustrates the symbol used to represent resistance.

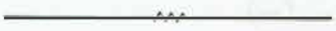


Figure 1-4

Schematic symbol for resistance.

The relationship that exists between voltage, current, and resistance is summarized by **Ohm's law**, which states that current is directly proportional to voltage and inversely proportional to resistance. Stated mathematically:

$$I = \frac{V}{R} \qquad \text{(Eq. 1-4)}$$

where: I = current in amperes (A)
V = voltage in volts (V)
R = resistance in ohms (Ω)

Alternately, we may write:

$$V = IR \quad \text{and} \quad R = \frac{V}{I}$$

It is important to realize that Ohm's law applies to an **entire circuit** as well as to **any part** of that circuit. The following examples illustrate this concept.

Example 1-2

Determine each unknown in Figures 1-5A, 1-5B, and 1-5C respectively.

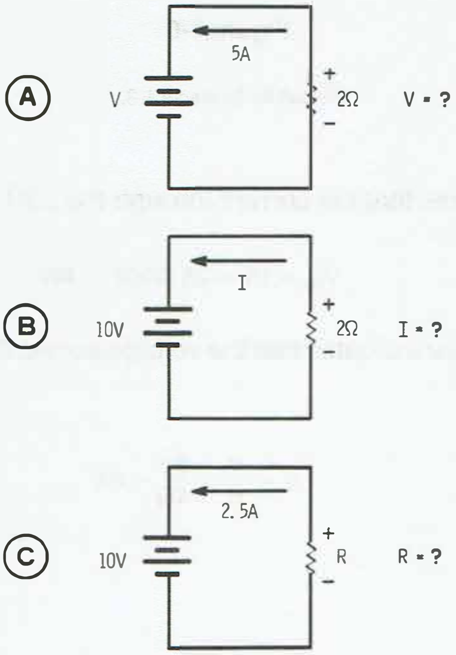


Figure 1-5

Circuits for Example 1-2.
A V = ?
B I = ?
C R = ?

In each case, the voltage source is connected directly across the resistance. Thus, you can use Ohm's law to find each unknown.

$$V = IR$$
$$V = 5A\,(2\Omega) = 10V \qquad \text{(Figure 1-5A)}$$

$$I = \frac{V}{R} = \frac{10V}{2\Omega} = 5A \qquad \text{(Figure 1-5B)}$$
$$R = \frac{V}{I} = \frac{10V}{2.5A} = 4\Omega \qquad \text{(Figure 1-5C)}$$

Example 1-3

Determine the voltage across the 20Ω resistor, and the current through the 4Ω resistor for the circuit shown in Figure 1-6.

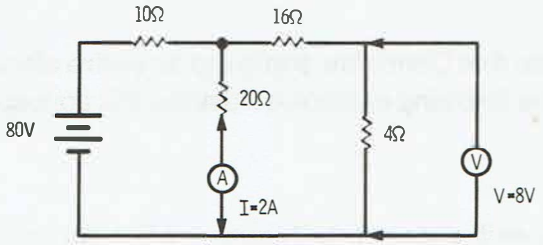


Figure 1-6

Circuit for Example 1-3.

The ammeter indicates that the current through the 20Ω resistor is 2A. Therefore:

$$V_{20} = IR = 2A\,(20\Omega) = 40V$$

Similarly, the voltmeter indicates that the voltage across the 4Ω resistor is 8V. Therefore:

$$I_4 = \frac{V}{R} = \frac{8V}{4\Omega} = 2A$$

Power

Recall that work must be done in order to move electric charges through a circuit. The electrical energy expended, or work (W) required to move a charge (Q) from one point to another, is given by:

$$W = QV \qquad \text{(Eq. 1-5)}$$

where: W = work required in joules (J)
Q = charge in coulombs (Q)
V = voltage in volts (V)

From Ohm's law, we know that $V = IR$. Similarly from our definition of current, we know that $Q = It$. Thus, substituting IR for V, and It for Q into Equation 1-5 yields:

$$W = I^2Rt \qquad \text{(Eq. 1-6)}$$

where: W = work required in joules (J)
I = current in amperes (A)
t = time in seconds (s)

Power is the **rate of doing work**. In a resistive DC circuit, the electrical power dissipated is equivalent to the rate at which electrical energy is converted into heat energy. This rate is easily computed by dividing work by time. Thus:

$$P = \frac{W}{t} = \frac{I^2Rt}{t} = I^2R \qquad \text{(Eq. 1-7)}$$

Since work has units of joules, and time has units of seconds, power must have units of joules per second. Normally, electrical power is measured in units of **watts**. One watt is defined as one joule per second. Stated mathematically:

$$1W = \frac{1J}{s} \qquad \text{(Eq. 1-8)}$$

You can derive two variations of the power formula as follows:

Substituting $\frac{V}{R}$ for I into Equation 1-7 yields:

$$P = I^2R = \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R} \qquad \text{(Eq. 1-9)}$$

Similarly substituting $\frac{V}{I}$ for R into Equation 1-7 yields:

$$P = I^2R = \frac{I^2V}{I} = IV \qquad \text{(Eq. 1-10)}$$

Summarizing our results, we have:

$$P = I^2R = \frac{V^2}{R} = IV \qquad \text{(Eq. 1-11)}$$

where: P = power in watts (W)
I = current in amperes (A)
R = resistance in ohms (Ω)
V = voltage in volts (V)

Example 1-4

Determine the power dissipated by the 6Ω resistor, the resistor labeled R, and the 16Ω resistor in Figure 1-7.

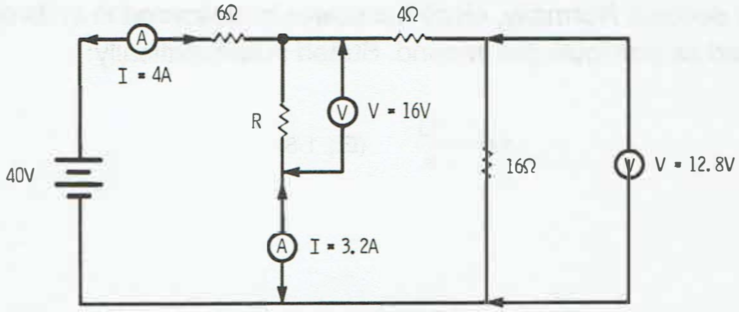


Figure 1-7

Circuit for Example 1-4.

$$P = I^2R = (4A)^2\,(6\Omega) = 96W \qquad \text{(6Ω resistor)}$$

$$P = IV = 3.2A\,(16\Omega) = 51.2W \qquad \text{(resistor R)}$$

$$P = \frac{V^2}{R} = \frac{(12.8V)^2}{16\Omega} = 10.24W \qquad \text{(16Ω resistor)}$$

Prefixes

Often units such as the ampere, volt, and ohm are too large or too small to be useful in describing the current, voltage, or resistance in a circuit. For this reason, standard prefixes are used in conjunction with scientific notation to describe very large or very small quantities. Table 1-1 lists the standard prefixes used in electronics, along with their arithmetic and scientific notation equivalents. The following examples illustrate the use of these prefixes in calculating basic electrical quantities.

Prefix	Arithmetic Equivalent	Scientific Notation Equivalent
tera (T)	1 000 000 000 000	10^{12}
giga (G)	1 000 000 000	10^{9}
mega (M)	1 000 000	10^{6}
kilo (k)	1 000	10^{3}
milli (m)	0.001	10^{-3}
micro (μ)	0.000 001	10^{-6}
nano (n)	0.000 000 001	10^{-9}
pico (p)	0.000 000 000 001	10^{-12}

Table 1-1

Standard Electronic Prefixes

Example 1-5

Calculate each unknown in Figures 1-8A, 1-8B, and 1-8C respectively.

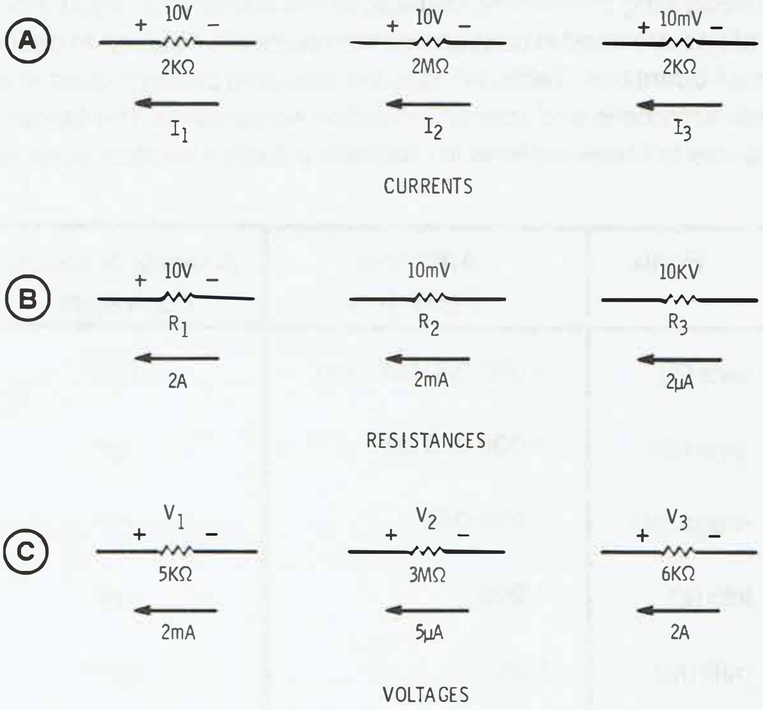


Figure 1-8

Unknowns for Example 1-5.
A Currents
B Resistances
C Voltages

$$I_1 = \frac{10\text{V}}{2\text{k}\Omega} = \frac{10\text{V}}{2 \times 10^3\Omega} = 5 \times 10^{-3}\text{A} = 5\text{mA} \qquad \text{(Figure 1-8A)}$$

$$I_2 = \frac{10\text{V}}{2\text{M}\Omega} = \frac{10\text{V}}{2 \times 10^6\Omega} = 5 \times 10^{-6}\text{A} = 5\mu\text{A}$$

$$I_3 = \frac{10\text{mV}}{2\text{k}\Omega} = \frac{10 \times 10^{-3}\text{V}}{2 \times 10^3\Omega} = 5 \times 10^{-6}\text{A} = 5\mu\text{A}$$

$$R_1 = \frac{10\text{V}}{2\text{A}} = 5\Omega \qquad \text{(Figure 1-8B)}$$

$$R_2 = \frac{10\text{mV}}{2\text{mA}} = \frac{10 \times 10^{-3}\text{V}}{2 \times 10^{-3}\text{A}} = 5\Omega$$

$$R_3 = \frac{10\text{kV}}{2\mu\text{A}} = \frac{10 \times 10^3\text{V}}{2 \times 10^{-6}\text{A}} = 5 \times 10^9\Omega = 5\text{G}\Omega$$

$$V_1 = 2\text{mA}\,(5\text{k}\Omega) = 2 \times 10^{-3}\text{A}\,(5 \times 10^3\Omega) = 10\text{V} \qquad \text{(Figure 1-8C)}$$

$$V_2 = 5\mu\text{A}\,(3\text{M}\Omega) = 5 \times 10^{-6}\text{A}\,(3 \times 10^6\Omega) = 15\text{V}$$

$$V_3 = 2\text{A}\,(6\text{k}\Omega) = 2\text{A}\,(6 \times 10^3\Omega) = 12 \times 10^3\text{V} = 12\text{kV}$$

Self-Test Review

1. Electric lines of force are directed ________________ toward a negative charge.
2. The flow of electric charges is called ________________ .
3. Electron flow assumes current flows from ____________ to ____________ .
4. ________________ can be thought of as electrical pressure.
5. An ________________ is one coulomb per second.
6. A ________________ is one joule per second.
7. If V = 8V and I = 2mA, the resistance is ____________ ohms, which is equivalent to _____ kΩ.
8. The current through a 10kΩ resistor is 5mA. The resistor is dissipating ____________ mW.
9. If 5μA of current flows through a 2MΩ resistor, the voltage across the resistor is _______ V.
10. ________________ is the rate of doing work.

Answers

1. inward
2. current
3. minus to plus
4. Voltage
5. ampere
6. watt
7. 4000Ω, 4kΩ
8. 250
9. 10
10. Power

BASIC CIRCUIT ANALYSIS

In this section, you will increase your technical vocabulary by learning the definitions of words such as node, branch, and loop. In addition, you will learn how to apply Kirchhoff's current and voltage laws so you can determine the values of the branch and loop currents. The ability to write and solve basic loop and nodal equations is essential for the analysis and design of practical electronic circuits.

Kirchhoff's Current Law

A **node** is a point where two or more **components** are connected together. A **branch** is a single path between two nodes. Figure 1-9 illustrates a circuit containing three nodes and four branches. In Figure 1-9, you should note the following:

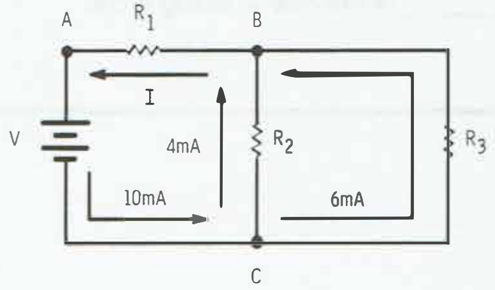


Figure 1-9

A three-node, four-branch circuit.

1. The three nodes are labeled A, B, and C.
2. There is one branch between nodes A and B. Similarly, there is one branch between nodes A and C.
3. There are two branches between nodes B and C
4. Each component in the circuit is connected between a pair of nodes.

Kirchhoff's current law is very useful in determining branch currents. This law is stated as follows:

> *The algebraic sum of the branch currents entering and leaving a node must equal zero.*

This simply means that more current cannot leave a node than enters the node. The term "algebraic sum" refers to the **direction** (into the node, or out of the node) of each branch current. An equation that describes the branch currents of a node is called a **nodal equation**. The following **rules** are used to write nodal equations for basic circuits:

1. If current **enters** a node, it is considered **negative** and will, therefore, be prefixed by a minus sign.
2. If current **leaves** a node, it is considered **positive** and will, therefore, be prefixed by a positive sign.
3. If the direction of the unknown branch current is not given, we will **initially assume** that it enters the node.
4. Once the value of the unknown current is determined, we will interpret its direction as follows:
   - (a) A positive answer means our initial assumption concerning the direction of the unknown current was correct.
   - (b) A negative answer means our initial assumption concerning the direction of the unknown current was incorrect.

The following examples illustrate the use of these rules.

Example 1-6

Write a nodal equation for node C in Figure 1-9.

$$-10\text{mA} + 4\text{mA} + 6\text{mA} = 0$$
$$-10\text{mA} + 10\text{mA} = 0$$
$$0 = 0 \qquad \text{(checks)}$$

Example 1-7

Write a nodal equation for node B in Figure 1-9. Solve for the current I.

$$I - 4\text{mA} - 6\text{mA} = 0$$
$$I - 10\text{mA} = 0$$
$$I = 10\text{mA}$$

Since we obtained a positive answer for I, the direction assumed for I (away from the node) in Figure 1-9 is correct.

Example 1-8

Determine the unknown branch current I_x in Figure 1-10.

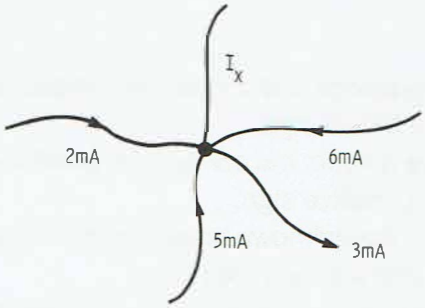


Figure 1-10

A node with four branches for Example 1-8.

Since the direction for I_x is not given in Figure 1-10, we will **initially** assume I_x enters the node.

$$
\begin{aligned}
&-2\text{mA} - I_x - 6\text{mA} + 3\text{mA} - 5\text{mA} = 0\\
&-13\text{mA} - I_x + 3\text{mA} = 0\\
&-10\text{mA} - I_x = 0\\
&I_x = -10\text{mA}
\end{aligned}
$$

Since I_x is negative, our initial assumption that I_x enters the node is incorrect. Thus $I_x = 10\text{mA}$ and is **leaving** the node.

Kirchhoff's Voltage Law

A **loop** is a closed path in a circuit. Kirchhoff's voltage law is often used to determine voltages between nodes, loop currents, etc. Kirchhoff's voltage law states:

The algebraic sum of the voltages around each closed loop in a circuit must equal zero.

The term "algebraic sum" refers to the **polarity** of each voltage in the loop. The polarity of the voltage across a resistor (and most other components) depends upon the **direction** of the current through the resistor (or component). Thus, current enters the negative terminal and leaves the positive terminal as illustrated in Figure 1-11. Voltage sources, however, are an exception to this general rule.

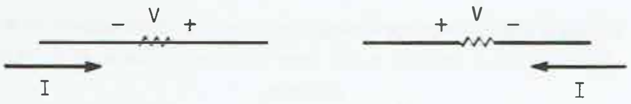


Figure 1-11

The polarity of the voltage across a resistor is determined by the **direction** of the current through the resistor.

Recall that the polarity of a DC voltage source is indicated by the symbol used to represent the source; the long line indicates the positive terminal and the short line indicates the negative terminal. Thus, the polarity of the voltage across a DC voltage source **does not** depend upon the direction of the current through the source. This concept is illustrated in Figure 1-12.

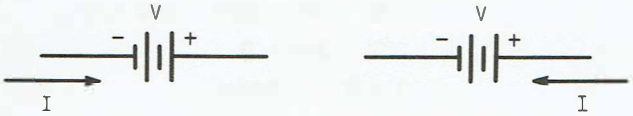
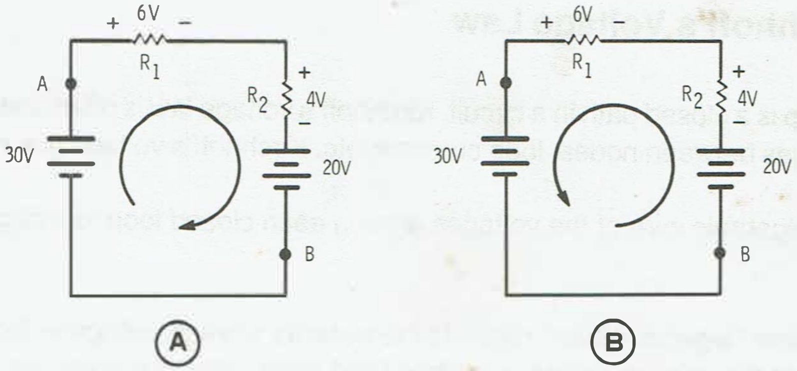


Figure 1-12

The polarity of the voltage across a DC voltage source is **independent** of the direction of the current through the source.

An equation that describes the voltages in a loop is called a **loop equation**. To write a loop equation when all loop voltages are known, you must decide two things — your starting point, and the direction (clockwise, or counterclockwise) you wish to travel around the loop.



CLOCKWISE PATH

COUNTERCLOCKWISE PATH

Figure 1-13

A loop in which all voltages are given. To write a loop equation for this circuit, select a starting point; then go around the loop in either direction.

A. Clockwise path.
B. Counterclockwise path.

To illustrate the process, consider the circuit in Figure 1-13A. In this case, start at point A, and go clockwise around the loop. As you go around the loop, write down each voltage you encounter along the way and, using Kirchhoff's voltage law, equate the result to zero. **The sign** (plus or minus) **of each voltage that you write down is determined by the sign you first encounter as you pass through each component**. Thus, for the circuit in Figure 1-13A, you have:

$$
\begin{aligned}
&+6V + 4V + 20V - 30V = 0\\
&30V - 30V = 0\\
&0 = 0 \qquad \text{(checks)}
\end{aligned}
$$

As a second example, consider Figure 1-13 B, where we start at point B, and go counterclockwise around the loop. In this case, the resulting loop equation is:

$$
\begin{aligned}
&-20V - 4V - 6V + 30V = 0\\
&-30V + 30V = 0\\
&0 = 0 \qquad \text{(checks)}
\end{aligned}
$$

Example 1-9

Calculate the voltage V_{AB} in Figure 1-13A and the voltage V_{BA} in Figure 1-13B.

V_{AB} means the voltage **from** terminal A **to** terminal B. Thus, you must start at terminal A and move through the circuit (recording any voltages encountered along the way) until you arrive at terminal B. In Figure 1-13A, if you go counterclockwise from terminal A through the 30V source to point B, the loop equation is:

$$V_{AB} = 30V$$

If you go clockwise from point A through R_1, R_2, and the 20V source, the loop equation is:

$$V_{AB} = 6V + 4V + 20V$$
$$V_{AB} = 30V$$

As you can see, the answer is the same ($V_{AB} = 30V$), regardless of the path taken in moving from point A to point B.

To calculate the voltage **from** point B **to** point A (V_{BA}) in Figure 1-13B, you must start at point B and end up at point A. If you go clockwise through the 30V source, the loop equation is:

$$V_{BA} = -30V$$

Similarly, if you go counterclockwise from point B through the 20V source, R_2, and R_1, you have:

$$V_{BA} = -20V - 4V - 6V$$
$$V_{BA} = -30V$$

Again, you obtain the same answer ($V_{BA} = -30V$), regardless of the path taken in moving from one point to another (B to A in this case). Note that $V_{AB} = -V_{BA}$. This fact should not surprise you since voltage is always measured **between** two points. Thus the statement that $V_{AB} = 30V$ means that the voltage **between** A and B **is** 30V **with** terminal A being positive with respect to terminal B. Similarly, the statement $V_{BA} = -30V$ means that the voltage **between** B and A **is** 30V **with** terminal B being negative with respect to terminal A. Thus, both statements mean the same thing! This concept is illustrated in Figure 1-14.

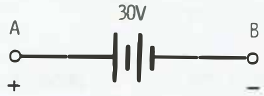


Figure 1-14

The voltage between A and B is 30V. Terminal A is positive with respect to terminal B, and terminal B is negative with respect to terminal A. Both statements mean the same thing. The equivalent mathematical statements are $V_{AB} = 30V$ and $V_{BA} = -30V$ respectively.

Kirchhoff's voltage law is very useful for determining the value of a loop current. Once the loop current is known, you can use Ohm's law to determine the values of the loop voltages. The following procedure is used to solve these problems:

1. If the direction of the loop current is not provided, **assume** one (clockwise or counterclockwise).
2. Assign polarities to the components in the loop that are **consistent** with the direction you initially assumed for the loop current.
3. Select a starting point and write a loop equation-by going around the loop in the direction you have assumed current to flow.
4. Solve the loop equation for the loop current.

The following example illustrates the procedure just outlined.

Example 1-10

Calculate the loop current in Figure 1-15A.

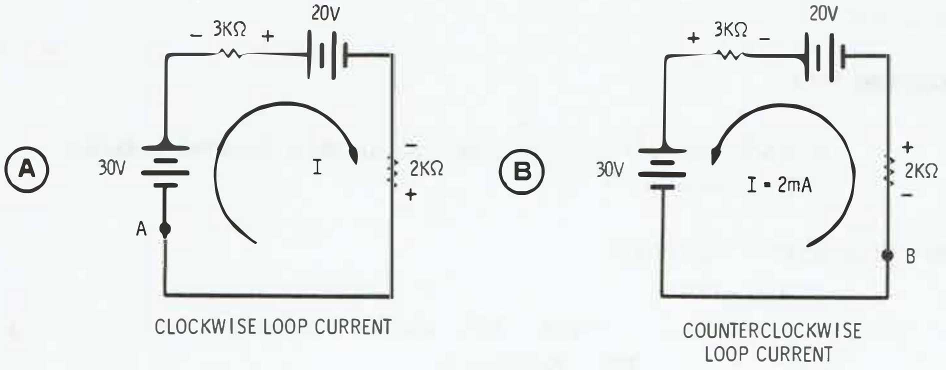


Figure 1-15

Kirchhoff's voltage law can be used to solve for the loop current I. The sign (plus or minus) of I obtained by solving the loop equation indicates whether or not the initial assumption concerning the direction of I (clockwise or counterclockwise) was correct. A negative answer indicates the initial assumption was incorrect.

A. Clockwise loop current
B. Counterclockwise loop current

To begin, we will assume a clockwise loop current. This **initial** assumption **dictates** the polarities of the voltages across the 3kΩ and 2kΩ resistors (current enters the negative terminal and leaves the positive terminal) as shown in Figure 1-15A. Naturally, the polarities of the 30V and 20V sources are indicated by the symbols for each source (long line = positive, short line = negative). Next we select a starting point which, in this example, is point A. As we go around the loop, we will use Ohm's law (V = IR) to describe the voltage across each resistor. Thus, starting at point A and going around the loop in the direction we have assumed current to flow (clockwise), we have:

$$-30\text{V} - 3\text{k}\Omega\,(\text{I}) + 20\text{V} - 2\text{k}\Omega\,(\text{I}) = 0$$

$$-10\text{V} - 5\text{k}\Omega\,(\text{I}) = 0$$

$$\text{I} = -\frac{10\text{V}}{5\text{k}\Omega} = -2\text{mA}$$

The **negative sign** for the loop current I tells us that the initial assumption concerning the direction of I was incorrect. Thus, the loop current **is** 2mA, but it flows in a counterclockwise direction as shown in Figure 1-15B. The polarities of the voltages across the resistors in Figure 1-15B are the **actual** polarities you would measure in the circuit.

Example 1-11

> *Rework Example 1-10 by initially assuming a counterclockwise loop current.*

Starting at point B, you have:

$$-2\text{k}\Omega\,(\text{I}) - 20\text{V} - 3\text{k}\Omega\,(\text{I}) + 30\text{V} = 0$$

$$10\text{V} - 5\text{k}\Omega\,(\text{I}) = 0$$

$$\text{I} = \frac{10\text{V}}{5\text{k}\Omega} = 2\text{mA}$$

In this case, the **positive sign** for the loop current I indicates that the initial assumption concerning the direction of I (counterclockwise) was correct.

Series Circuits - Voltage Division

When the **same current** flows through two or more (two terminal) components, as illustrated in Figure 1-16A, the components are connected in **series**. Starting at point A and going counterclockwise around the loop in Figure 1-16A, we obtain the following loop equation:

$$V_T - V_n \ldots - V_2 - V_1 = 0$$
$$V_T - I_T R_n \ldots - I_T R_2 - I_T R_1 = 0$$
$$I_T (R_1 + R_2 + \ldots R_n) = V_T$$

or

$$I_T = \frac{V_T}{R_1 + R_2 + \ldots R_n} = \frac{V_T}{R_T}$$

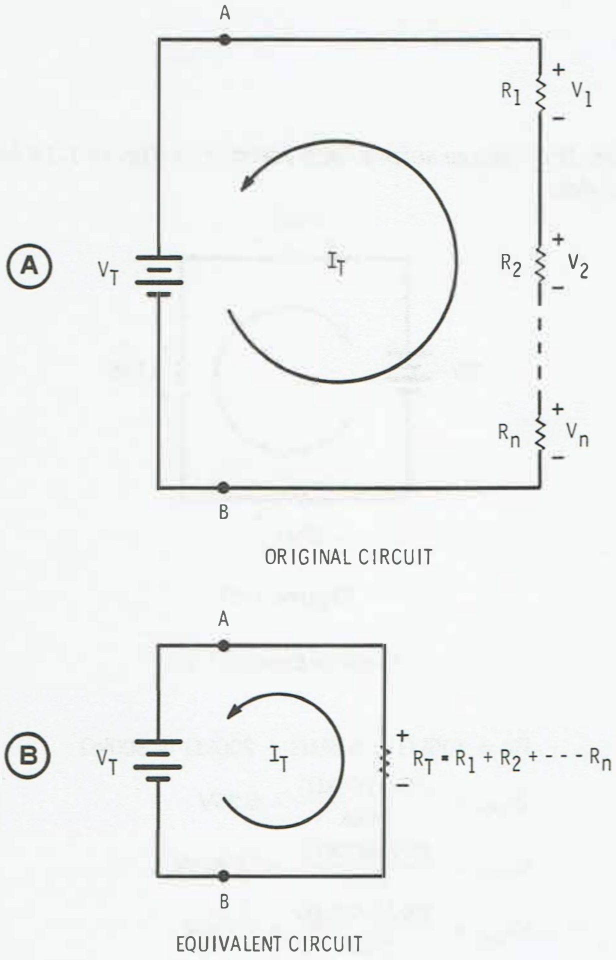


Figure 1-16

The series circuit.

A. Original circuit.

B. Equivalent circuit.

The equivalent resistance (R_T) of a series circuit, as viewed from the source equals the sum of the individual resistances. Since there is only one path for current flow, the total current (I_T) supplied by the source (V_T) to the circuit **is** the current flowing through **each** resistance. From Ohm's law, we can compute the voltage across any resistor (R_n) in the series circuit. Thus,

$$V_{R_n} = I_T R_n$$

$$V_{R_n} = \frac{V_T R_n}{R_T} \qquad \text{(Eq. 1-12)}$$

Equation 1-12 is important because it summarizes the principle of **voltage division**. This principle lets you quickly calculate the voltage across any resistor (R_n) in a "series string" of resistors as illustrated by the following example.

Example 1-12

Calculate the voltage across each resistor in Figure 1-17 by voltage division.

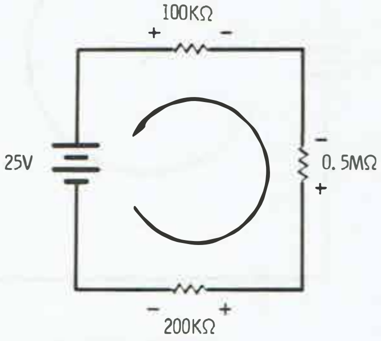


Figure 1-17

Circuit for Example 1-12.

$$R_T = 100\text{k}\Omega + 0.5\text{M}\Omega + 200\text{k}\Omega = 800\text{k}\Omega$$

$$V_{200k} = \frac{25\text{V}\,(200\text{k}\Omega)}{800\text{k}} = 6.25\text{V}$$

$$V_{0.5M} = \frac{25\text{V}\,(500\text{k}\Omega)}{800\text{k}} = 15.625\text{V}$$

$$V_{100k} = \frac{25\text{V}\,(100\text{k}\Omega)}{800\text{k}} = 3.125\text{V}$$

Note that the sum of the voltages across the resistors equals the applied voltage (25V).

To review some of the ideas discussed earlier, let's calculate the remaining responses. The current through each resistor is I_T. Thus,

$$I_{R_1} = I_{R_2} = I_{R_3} = \frac{V_T}{R_T} = \frac{25V}{800k\Omega} = 31.25\mu A$$

The power dissipated by each resistor is simply VI. Thus,

$$P_{200k} = 6.25V\ (31.25\mu A) = 195.31\mu W$$
$$P_{0.5M} = 15.625V\ (31.25\mu A) = 488.28\mu W$$
$$P_{100k} = 3.125V\ (31.25\mu A) = 97.66\mu W$$

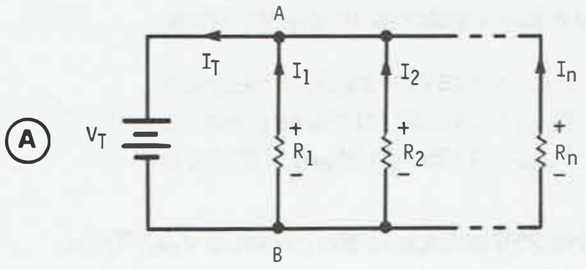
The power supplied by the 25V source to the circuit is V_TI_T. Thus,

$$P_T = V_TI_T = 25V\ (31.25\mu A) = 781.25\mu W$$

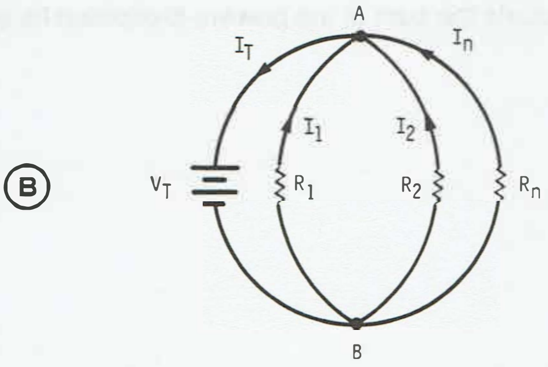
Note that the total power equals the sum of the powers dissipated by each resistor.

Parallel Circuits - Current Division

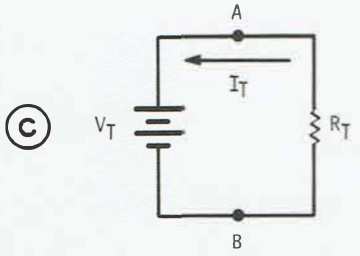
When the **same voltage** is across two or more (two terminal) components as illustrated in Figure 1-18A, the components are connected in **parallel**. Even though it may appear that the circuit in Figure 1-18A contains four nodes, only two nodes are present, as illustrated in Figure 1-18B. Remember, when components are connected in parallel, **each component** is connected between the **same pair** of nodes. Applying Kirchhoff's current law to node A, we have:



ORIGINAL CIRCUIT



EACH COMPONENT IS CONNECTED BETWEEN THE SAME TWO NODES



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}$$

EQUIVALENT CIRCUIT

Figure 1-18

The parallel circuit.

A. Original Circuit.
B. Each component is connected between **the same two nodes**.
C. Equivalent circuit.

$$I_T - I_1 - I_2 \ldots - I_n = 0$$

$$I_T - \frac{V_T}{R_1} - \frac{V_T}{R_2} \ldots - \frac{V_T}{R_n} = 0$$

$$V_T\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n}\right) = I_T$$

From Ohm's law we know that $I_T = \frac{V_T}{R_T}$. Thus, the expression

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n}\right)$$

must equal $\frac{1}{R_T}$. In a parallel circuit, then:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n} \qquad \text{(Eq. 1-13)}$$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n}}$$

For the special case of **two parallel resistors**, Equation 1-13 reduces to:

$$R_T = \frac{R_1 R_2}{R_1 + R_2} \qquad \text{(Eq. 1-14)}$$

Similarly, when **equal resistors** are connected in parallel, the equivalent resistance is:

$$R_T = \frac{R}{n} \qquad \text{(Eq. 1-15)}$$

where n is the number of equal parallel resistors.

Example 1-13

Determine — the current through, the voltage across, and the power dissipated by — each resistor in Figure 1-19.

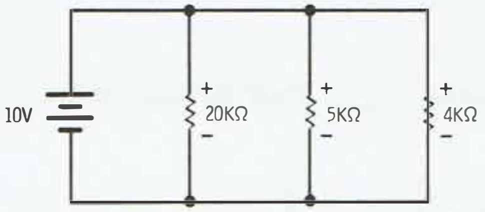


Figure 1-19

Circuit for Example 1-13.

Since the same voltage (10V) is across each resistor, you can determine the current through each resistor as follows:

$$I_{20k} = \frac{10V}{20k\Omega} = 0.5mA$$

$$I_{5k} = \frac{10V}{5k\Omega} = 2mA$$

$$I_{4k} = \frac{10V}{4k\Omega} = 2.5mA$$

Via Kirchhoff's current law, the sum of the currents through each resistor must equal the total current (I_T) supplied by the 10V source to the circuit. Thus,

$$I_T = 0.5mA + 2mA + 2.5mA$$
$$I_T = 5mA$$

The power dissipated by each resistor is VI. Thus,

$$P_{20k} = 10V\,(0.5mA) = 5mW$$
$$P_{5k} = 10V\,(2mA) = 20mW$$
$$P_{4k} = 10V\,(2.5mA) = 25mW$$

The total power supplied by the source to the circuit is V_TI_T. Thus,

$$P_T = V_TI_T$$
$$P_T = 10V\,(5mA) = 50mW$$

Note that the total power (50mW) equals the sum of the powers dissipated by each resistor. The equivalent resistance, R_T, can be calculated from Equation 1-13.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{\underline{20k\Omega}} + \frac{1}{\underline{5k\Omega}} + \frac{1}{4k\Omega} = \frac{1 + 4(I) + 5(I)}{20k\Omega} = \frac{10}{\underline{20k\Omega}}$$

$$R_T = \frac{20k\Omega}{10} = 2k\Omega$$

Alternately, you can calculate R_T from Equation 1-14 and Equation 1-15 as follows:

$$20k\Omega \| 5k\Omega = \frac{20(5)}{20 + 5} = \frac{100}{25} = 4k\Omega$$

$$4k\Omega \| 4k\Omega = \frac{4k\Omega}{2} = 2k\Omega$$

Note that $\|$ indicates two resistors in parallel.

Finally, you can also calculate R_T from the ohm's law relationship $R_T = \frac{V_T}{I_T}$.

$$R_T = \frac{V_T}{I_T} = \frac{10V}{5mA} = 2k\Omega$$

Previously, we defined resistance as the opposition to current flow. The **reciprocal of resistance** is called **conductance**, and is a measure of the ability of a conductor to "conduct current flow." Thus,

$$G = \frac{1}{R} \qquad \text{(Eq. 1-16)}$$

where: R = resistance in ohms (Ω)
G = conductance in mhos ($\mho$) **or** siemens (S)

For many years, the accepted unit of conductance was the **mho** (ohm spelled backwards) - whose symbol is an inverted ohms symbol. Recently, the **siemen** (S) has become an increasingly popular unit for the measurement of conductance. Today, you will see both units used to represent conductance, although the siemen is gradually replacing the mho as the **preferred** unit of measure. By substituting G for $\frac{1}{R}$ into the equations given previously, we obtain:

$$I = GV \qquad \text{(Eq. 1-17)}$$

$$V = \frac{I}{G} \qquad \text{(Eq. 1-18)}$$

$$P = VI = \frac{I^2}{G} = V^2G \qquad \text{(Eq. 1-19)}$$

$$G_T = G_1 + G_2 + \ldots G_n \text{ (n parallel conductances)} \qquad \text{(Eq. 1-20)}$$

$$\frac{1}{G_T} = \frac{1}{G_1} + \frac{1}{G_2} + \ldots \frac{1}{G_n} \text{ (n series conductances)} \qquad \text{(Eq. 1-21)}$$

Example 1-14

Determine: (a) The conductance of a 2Ω resistor. (b) The equivalent conductance of 20Ω and 5Ω resistors connected in parallel, and in series. (c) The current through a 2S conductance that has 5V across it.

(a) $G = \frac{1}{R} = \frac{1}{2\Omega} = 0.5S$

(b) $G_{20} = \frac{1}{20\Omega} = .05S \qquad G_5 = \frac{1}{5\Omega} = 0.2S$

For the parallel connection:

$$G_T = G_1 + G_2 = .05 + 0.2 = 0.25S$$

For the series connection:

$$\frac{1}{G_T} = \frac{1}{G_1} + \frac{1}{G_2} = \frac{1}{.05S} + \frac{1}{0.2S} = 20\Omega + 5\Omega = 25\Omega$$

$$G_T = \frac{1}{25\Omega} = .04S$$

(c) $I = GV = 2S\,(5V) = 10A$

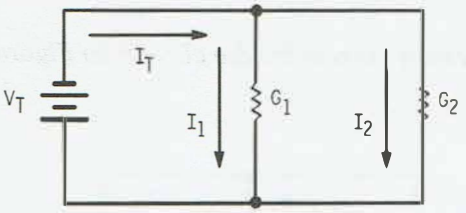


Figure 1-20

Current division.

Consider the circuit shown in Figure 1-20. The total conductance of the circuit is:

$$G_T = G_1 + G_2$$

Thus, the source voltage V_T is:

$$V_T = \frac{I_T}{G_T} = \frac{I_T}{G_1 + G_2}$$

Similarly, the current through each conductance is:

$$I_1 = V_T G_1 = I_T \frac{G_1}{G_1 + G_2}$$

$$I_2 = V_T G_2 = I_T \frac{G_2}{G_1 + G_2}$$

Since we normally work with resistance rather than conductance substitution of $\frac{1}{R_1}$ for G_1 and $\frac{1}{R_2}$ for G_2 yields:

$$I_1 = I_T \frac{R_2}{R_1 + R_2} \qquad \text{(Eq. 1-22)}$$

$$I_2 = I_T \frac{R_1}{R_1 + R_2} \qquad \text{(Eq. 1-23)}$$

Equations 1-22 and 1-23 summarize the principle of **current division**.

Example 1-15

Calculate each branch current for the circuits in Figures 1-21A and 1-21B.

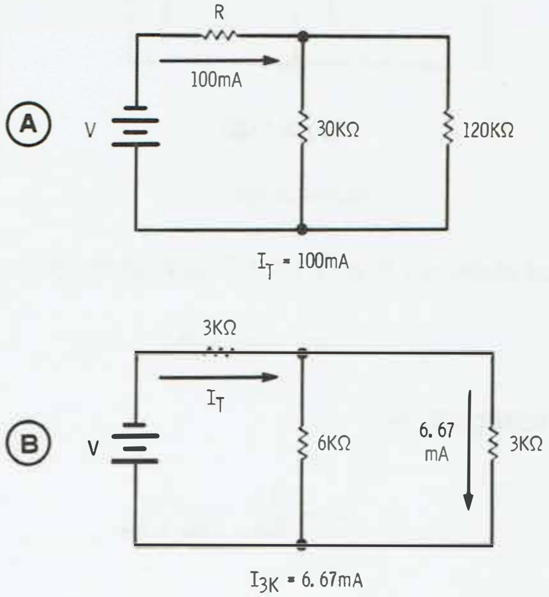


Figure 1-21

Circuits for Example 1-15.
A. $I_T = 100\text{mA}$
B. $I_{3k} = 6.67\text{mA}$

Figure 1-21A

$$I_{30k} = \frac{100\text{mA}\,(120\text{k}\Omega)}{30\text{k}\Omega + 120\text{k}\Omega} = \frac{100\text{mA}\,(120\text{k}\Omega)}{150\text{k}\Omega} = 80\text{mA}$$

$$I_{120k} = \frac{100\text{mA}\,(30\text{k}\Omega)}{30\text{k}\Omega + 120\text{k}\Omega} = \frac{100\text{mA}\,(30\text{k}\Omega)}{150\text{k}\Omega} = 20\text{mA}$$

Figure 1-21B

$$I_{3k} = 6.67\text{mA} = \frac{I_T\,(6\text{k}\Omega)}{6\text{k}\Omega + 3\text{k}\Omega} = \frac{I_T\,(6\text{k}\Omega)}{9\text{k}\Omega}$$

$$I_T = \frac{6.67\text{mA}\,(9\text{k}\Omega)}{6\text{k}\Omega} = 10\text{mA}$$

$$I_{6k} = \frac{10\text{mA}\,(3\text{k}\Omega)}{6\text{k}\Omega + 3\text{k}\Omega} = \frac{10\text{mA}\,(3\text{k}\Omega)}{9\text{k}\Omega} = 3.33\text{mA}$$

Series - Parallel Circuits

Series - parallel circuits consist of **combinations** of series- and parallel-connected components in the same circuit. By finding the equivalent resistance of the series and parallel portions of the original circuit, you can reduce it to a simpler equivalent circuit. Finally, you can "work backwards" from the equivalent circuit to the original circuit in order to determine the desired responses. The following example illustrates this process.

Example 1-16

Determine — the current through, the voltage across, and the power dissipated by — each resistor in Figure 1-22A.

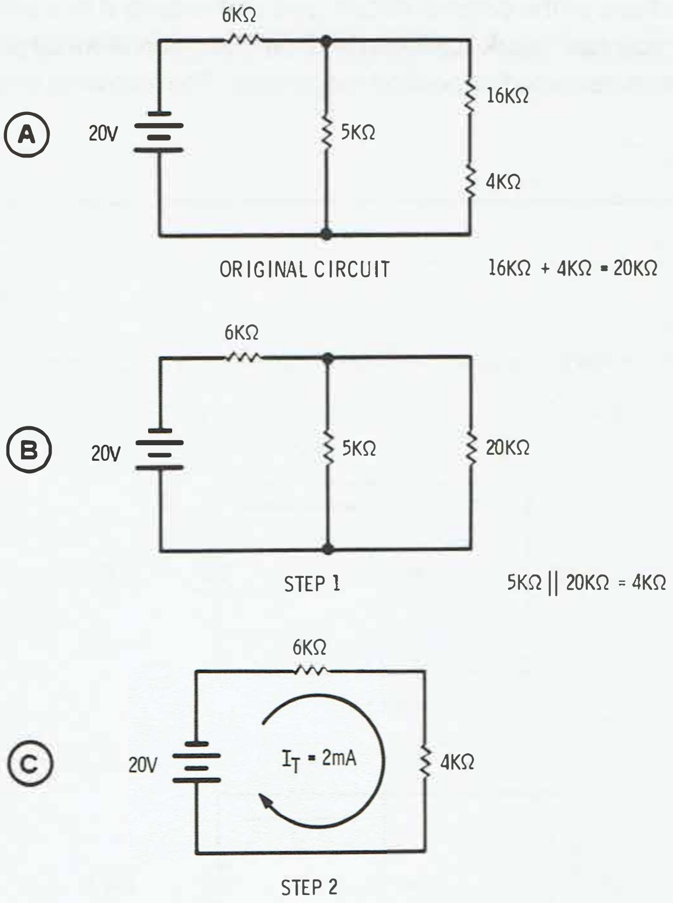


Figure 1-22

Original and simplified circuits for Example 1-16.

A. Original circuit.
B. Step 1.
C. Step 2.

To begin, note that the 16kΩ and 4kΩ resistors are connected in series. The equivalent resistance of these two resistors is:

$$R = 16k\Omega + 4k\Omega = 20k\Omega$$

This equivalent 20kΩ resistor is connected in parallel with the 5kΩ resistor in the original circuit, as shown in Figure 1-22B. In this Figure, the 5kΩ and 20kΩ resistors are connected in parallel. Thus,

$$R = 5k\Omega \| 20k\Omega = \frac{5k\Omega(20k\Omega)}{25k\Omega} = 4k\Omega$$

The equivalent 4kΩ resistor is connected in series with the 6kΩ resistor in the original circuit, as shown in Figure 1-22C. Now:

$$R_T = 6k\Omega + 4k\Omega = 10k\Omega$$

$$I_T = \frac{V_T}{R_T} = \frac{20V}{10k\Omega} = 2mA$$

Since it flows through the 6kΩ resistor:

$$I_{6k} = I_T = 2mA$$
$$V_{6k} = 2mA\,(6k\Omega) = 12V$$
$$P_{6k} = 12V\,(2mA) = 24mW$$

In Figure 1-22B, you can see that after flowing through the 6kΩ resistor, I_T divides between the 5kΩ and 20kΩ resistors. Applying the current division principle, you have:

$$I_{5k} = \frac{2mA\,(20k\Omega)}{5k\Omega + 20k\Omega} = \frac{2mA\,(20k\Omega)}{25k\Omega} = 1.6mA$$
$$I_{20k} = \frac{2mA\,(5k\Omega)}{5k\Omega + 20k\Omega} = \frac{2mA\,(5k\Omega)}{25k\Omega} = 0.4mA$$

Thus:

$$V_{5k} = 1.6mA\,(5k\Omega) = 8V$$
$$P_{5k} = 8V\,(1.6mA) = 12.8mW$$

Note that the current through the 20kΩ resistor (0.4mA) in Figure 1-22B is the same current that flows through the 16kΩ and 4kΩ resistors in Figure 1-22A. Thus,

$$V_{16k\Omega} = 0.4mA\,(16k\Omega) = 6.4V$$
$$P_{16k\Omega} = 6.4V\,(0.4mA) = 2.56mW$$

$$V_{4k\Omega} = 0.4mA\,(4k\Omega) = 1.6V$$
$$P_{4k\Omega} = 1.6V\,(0.4mA) = 0.64mW$$

The calculated responses are summarized in Table 1-2.

Component	I(mA)	V(v)	P(mW)
6kΩ	2	12	24
5kΩ	1.6	8	12.8
16kΩ	0.4	6.4	2.56
4kΩ	0.4	1.6	0.64

Table 1-2

Summary of Responses for the Circuit in Figure 1-22A.

There are several checks you can make to verify the calculated answers. First note that the sum of the powers dissipated by each resistor is:

$$P_T = 24 + 12.8 + 2.56 + 0.64 = 40\text{mW}$$

This should equal the power supplied by the source to the circuit.

$$P_T = V_T I_T = 20\text{V}\,(2\text{mA}) = 40\text{mW} \qquad \text{(checks)}$$

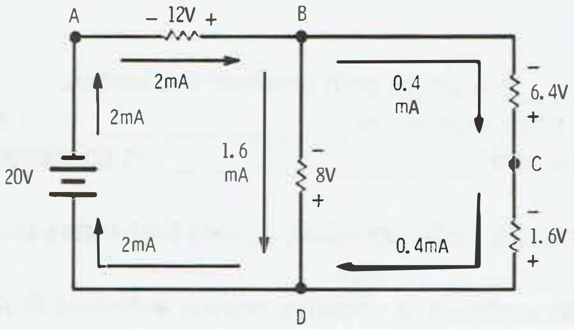


Figure 1-23

Checking the calculated answers to Example 1-16.

The circuit of Figure 1-22A is redrawn in Figure 1-23 with the calculated currents and voltages inserted. Note that each component in the circuit is connected between a pair of nodes. We can verify the answers by applying Kirchhoff's laws to the various nodes and loops. First, applying Kirchhoff's current law, we have:

Node A $-2\text{mA} + 2\text{mA} = 0$ (checks)
Node B $-2\text{mA} + 1.6\text{mA} + 0.4\text{mA} = 0$ (checks)
Node C $-0.4\text{mA} + 0.4\text{mA} = 0$ (checks)
Node D $-0.4\text{mA} - 1.6\text{mA} + 2\text{mA} = 0$ (checks)

Finally, applying Kirchhoff's voltage law, we have:

Left loop (starting at node A)
$-12\text{V} - 8\text{V} + 20\text{V} = 0$ (checks)

Right loop (starting at node B)
$-6.4\text{V} - 1.6\text{V} + 8\text{V} = 0$ (checks)

Outer loop (starting at node A)
$-12\text{V} - 6.4\text{V} - 1.6\text{V} + 20\text{V} = 0$ (checks)

Self-Test Review

11. A ________________ is a single path between two nodes.
12. You can tell if you have located all ________________ on a schematic by checking to see that each ____________________ is connected between a pair of nodes.
13. For purposes of writing a nodal equation, current that enters the node is prefixed with a ________________ sign.
14. After solving a nodal equation, a negative answer indicates that the initial direction assumed for I was ____________________ .
15. The polarity of a voltage source ____________________ (does or does not) depend upon the direction of the current flow through the source.
16. If $V_{xy} = -12V$, then V_{yx} = ______________
17. After solving a loop equation, a negative answer for I indicates the initial direction assumed for the loop current was ____________________ .
18. Two components are connected in ________________ if the current through them is the ________________ current.
19. Two components are connected in ____________________ if the voltage across them is the ________________ voltage.
20. Thirty-three resistors, each having a value of 33kΩ, connected in parallel have an equivalent resistance of _______ kΩ.
21. A conductance of 0.2S has a resistance of _______ ohms.
22. The current I_x in Figure 1-24A is _______ A.
23. The current I in Figure 1-24B is _______________ A.
24. Using voltage division, the voltge across the 4kΩ resistor in Figure 1-24C is _______ V.
25. Using current division, the current through the 5kΩ resistor in Figure 1-24D is _______ A.
26. Complete the following table for the circuit in Figure 1-24E.

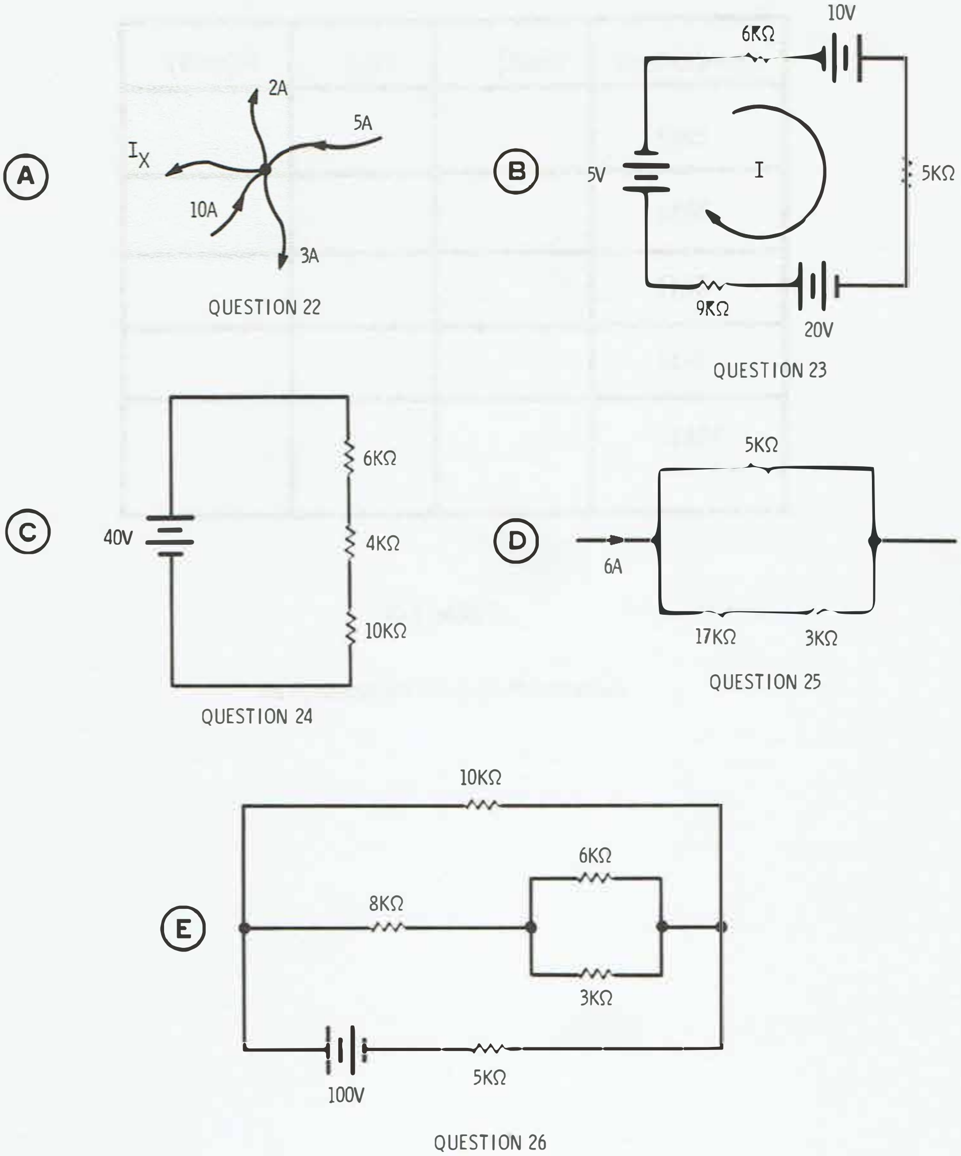


Figure 1-24

Circuits for Self-test Review.

A. Question 22.
B. Question 23.
C. Question 24.
D. Question 25.
E. Question 26.

Component	I(mA)	V(v)	P(mW)
5kΩ			
8kΩ			
6kΩ			
3kΩ			
10kΩ			

Table 1-3

Calculated Responses For Question 26

Answers

11. branch
12. nodes, component
13. minus
14. incorrect
15. does not
16. +12V
17. incorrect
18. series, same
19. parallel, same
20. 1kΩ

21. $R = \frac{1}{G} = \frac{1}{0.2S} = 5\Omega$

22. $I_x + 2A - 5A + 3A - 10A = 0$

$I_x - 10A = 0$

$I_x = 10A$

23. $5V - I(6k\Omega) - 10V - I(5k\Omega) - 20V - I(9k\Omega) = 0$

$-25V - I(20k\Omega) = 0$

$I(20k\Omega) = -25V$

$I = \frac{-25V}{20k\Omega} = -1.25mA$

Thus, I is 1.25mA, but it flows in a counterclockwise direction

24. $V_{4k} = \frac{40V\,(4k\Omega)}{20k\Omega} = 8V$

25. $I_{5k} = \frac{6A\,(20k\Omega)}{25k\Omega} = 4.8A.$

26.

Component	I(mA)	V(v)	P(mW)
5kΩ	10	50	500
8kΩ	5	40	200
6kΩ	1.67	10	16.7
3kΩ	3.33	10	33.3
10kΩ	5	50	250

Table 1-4
Answers For Question 26

The solutions to question 26 follow:

Refer to Figure 1-25.

$6k\Omega \| 3k\Omega = 6\,(3)/6 + 3 = 2k\Omega$ (Figure 1-25A)

$8k\Omega + 2k\Omega = 10k\Omega$ (Figure 1-25B)

$10k\Omega \| 10k\Omega = 10/2 = 5k\Omega$ (Figure 1-25C)

$R_T = 5k\Omega + 5k\Omega = 10k\Omega$ (Figure 1-25D)

$$I_T = V_T/R_T = 100V/10k\Omega = 10mA = I_{5k}$$

Applying the current division principle to Figure 1-25C:

$$I_{10k} = 10mA\,(10k\Omega)/20k\Omega = 5mA$$

In Figure 1-25B, 5mA also flows through the 8kΩ resistor.

Applying the current division principle to Figure 1-25A:

$$I_{6k} = 5mA\,(3k\Omega)/9k\Omega = 1.67mA$$

$$I_{3k} = 5mA\,(6k\Omega)/9k\Omega = 3.33mA$$

The various voltages are readily calculated from $V = IR$. Similarly, the powers are calculated from $P = VI$.

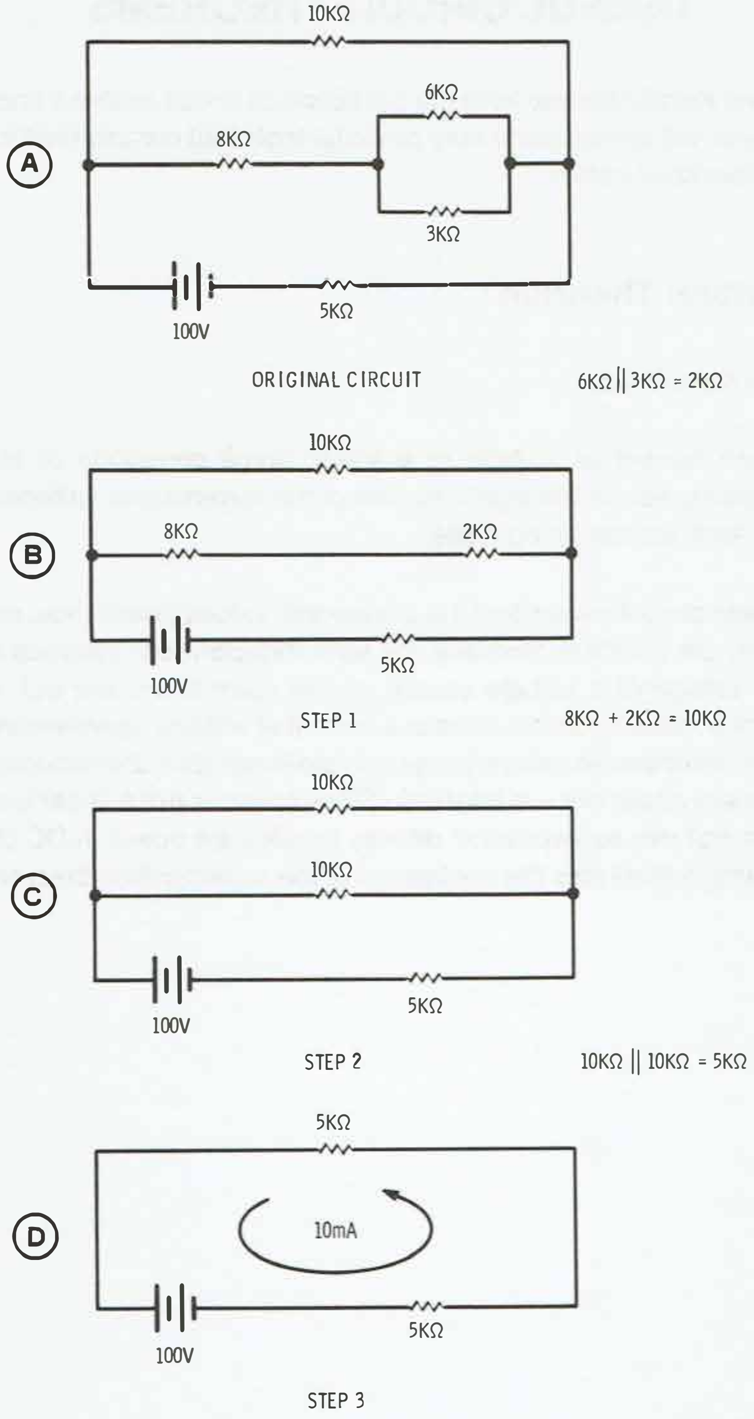


Figure 1-25

Reducing the circuit in Figure 1-24E to a single-loop equivalent circuit.

A. Original circuit.
B. Step 1.
C. Step 2.
D. Step 3.

USEFUL CIRCUIT THEOREMS

Ohm's law and Kirchhoff's law form the backbone of circuit analysis and design. In this section, you will review some very powerful tools that complement the analysis techniques discussed earlier.

Superposition Theorem

This theorem states that:

> *Each current or voltage in a linear circuit consisting of independent sources, equals the algebraic sum of the currents and voltages produced by each source acting alone.*

The term **linear circuit** means that the component values (resistance, capacitance, or inductance) are constant. Similarly, the term **independent sources** means that the terminal voltage of a voltage source, or the current into and out of a current source, do not depend upon the value of a current or voltage anywhere in the circuit. Naturally, real components can only approximate these ideal characteristics. We will have more to say about this in a later unit. Since power is **not** a linear quantity ($P = I^2R$), you **can not** use superposition directly to calculate power in DC circuits. The following example illustrates the application of the superposition theorem.

Example 1-17

Calculate the current through the 6kΩ resistor in Figure 1-26A by superposition.

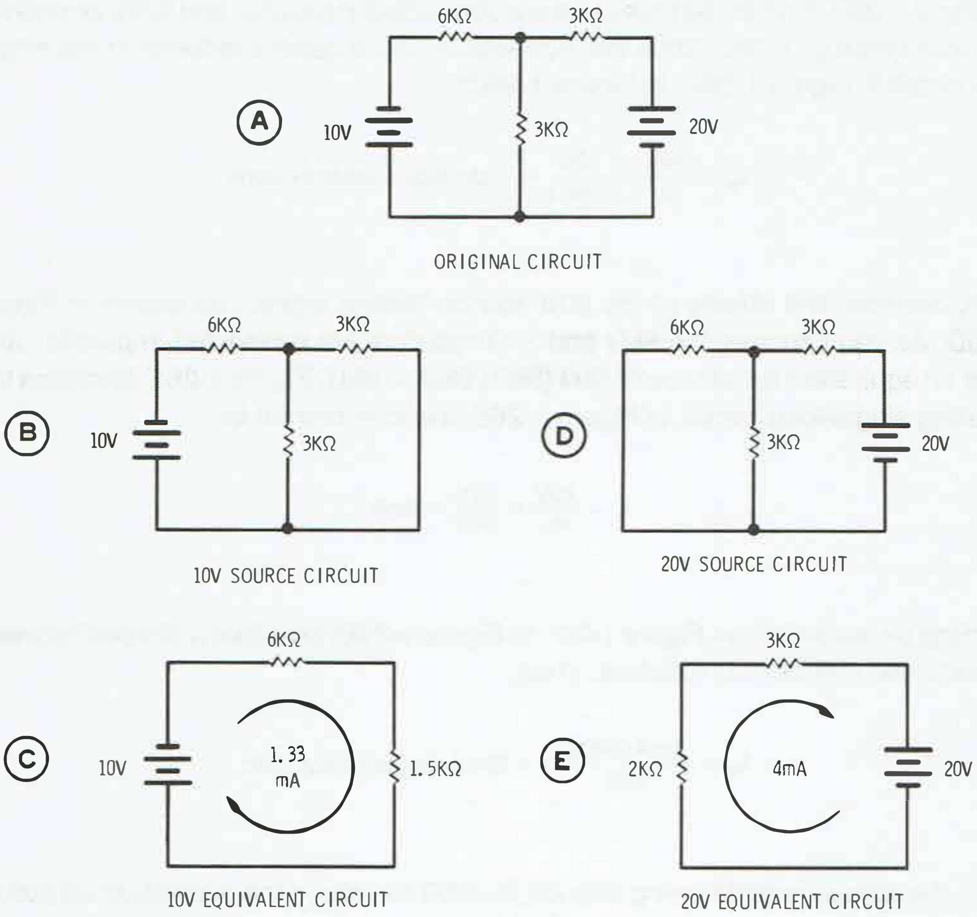


Figure 1-26

Using superposition to solve a two-source circuit.

- A. Original circuit.
- B. 10V source circuit.
- C. 10V equivalent circuit.
- D. 20V source circuit.
- E. 20V equivalent circuit.

To begin, consider the 10V source "acting alone." To do this, you must reduce all other sources to zero. This means any other voltage sources are replaced by short circuits as shown in Figure 1-26B. If the circuit contained current sources, they would be replaced by open circuits.

In Figure 1-26B, the two 3kΩ resistors are connected in parallel, and have an equivalent resistance of 1.5kΩ. Thus the 10V source circuit quickly reduces to the single loop circuit in Figure 1-26C. In Figure 1-26C:

$$I_{6k} = \frac{10V}{R_T} = \frac{10V}{7.5k\Omega} = 1.33mA \text{ directed to the right.}$$

Next, consider the effects of the 20V source "acting alone", as shown in Figure 1-26D. As you can see, the 6kΩ and 3kΩ resistors are connected in parallel, and have an equivalent resistance of 6kΩ (3kΩ)/9kΩ = 2kΩ. Figure 1-26E illustrates the resulting single-loop circuit. In Figure 1-26E, the total current is:

$$I_T = \frac{20V}{R_T} = \frac{20V}{5k\Omega} = 4mA$$

Working backwards from Figure 1-26E to Figure 1-26D, note that I_T divides between the 6kΩ and middle 3kΩ resistors. Thus,

$$I_{6k} = \frac{4mA\,(3k\Omega)}{9k\Omega} = 1.33mA \text{ directed to the right.}$$

Thus, the actual current flowing through the 6kΩ resistor in the original circuit due to **both** sources is:

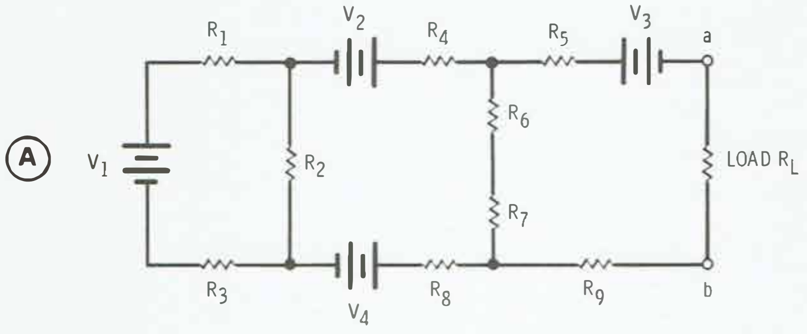
$$I_{6k} = 1.33mA + 1.33mA = 2.66mA \text{ directed to the right.}$$

Thevenin's Theorem

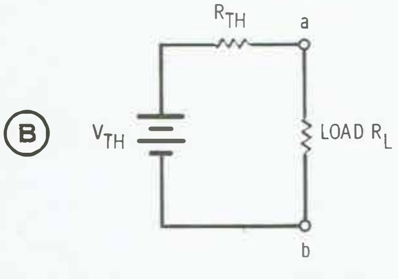
Thevenin's theorem is, without question, the single most important tool yet developed to simplify the tasks of circuit analysis and design. This is especially true if you are mainly interested in obtaining the response associated with a single component - such as a resistor. We will refer to the single component of interest as the **load**. Thevenin's theorem states that:

> *Everything in the original circuit, except the load, may be replaced by an equivalent circuit. The equivalent circuit consists of the series combination of a voltage source and resistance.*

Figure 1-27 illustrates this idea. In Figure 1-27A, we have a circuit that consists of four voltage sources, and ten resistors. We are only interested in the responses produced in the resistor that is connected between terminals a and b (R_L). Thevenin's theorem lets us replace the original circuit to the left of terminals a and b in Figure 1-27A by the Thevenin equivalent circuit shown in Figure 1-27B. The following rules are used to calculate the values of V_{TH} and R_{TH} for a given circuit.



ORIGINAL CIRCUIT



THEVENIN EQUIVALENT CIRCUIT

Figure 1-27

Thevenin's theorem
The circuit in A can be replaced by the Thevenin equivalent circuit in B. In each circuit - the current through, the voltage across, and the power dissipated by - the load (R_L in this case) is the same.

A. Original circuit.
B. Thevenin equivalent circuit.

To calculate V_{TH}:

1. Remove the load from the circuit.
2. Calculate the voltage between the terminals where the load was connected. This voltage is called the open-circuit voltage and equals V_{TH}.

To calculate R_{TH}:

1. Remove the load.
2. Reduce all sources to zero. This means voltage sources will be replaced by short circuits, and current sources by open circuits.
3. Calculate the resistance between the terminals where the load was connected. This resistance equals R_{TH}.

The following examples illustrate how the rules are applied.

Example 1-18

Calculate the current through the 5kΩ load resistor in Figure 1-28A.

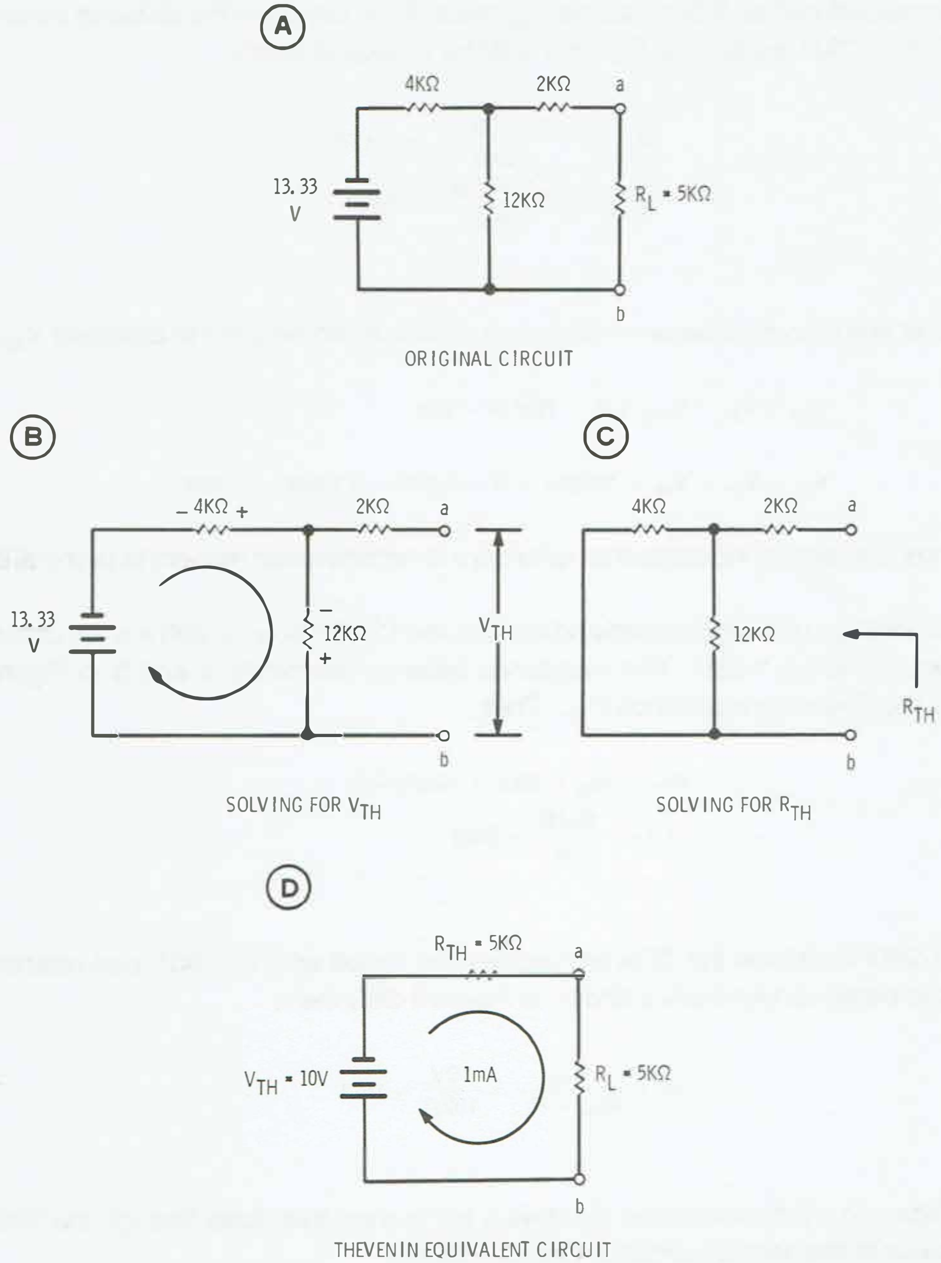


Figure 1-28

Circuits for Example 1-18.

A. Original circuit.
B. Solving for V_{TH}.
C. Solving for R_{TH}.
D. Thevenin equivalent circuit.

To calculate V_{TH}, remove the load as shown in Figure 1-28B. Once the load is removed, the 4kΩ and 12kΩ resistors are connected in series since, in Figure 1-28B, the **same** current flows through each resistor. Also, since no current flows through the 2kΩ resistor, the voltage across the 2kΩ resistor is 0. The voltage between terminals a and b in Figure 1-28B is the Thevenin voltage, V_{TH}. To review some of the concepts discussed earlier, let's calculate V_{ab} twice. First, calculate the voltages across the 4kΩ and 12kΩ resistors in Figure 1-28B by voltage division.

$$V_{4k} = \frac{13.33\,(4k\Omega)}{16k\Omega} = 3.33V$$
$$V_{12k} = \frac{13.33\,(12k\Omega)}{16k\Omega} = 10V$$

Next, write two loop equations — either one of which can be used to calculate V_{ab}.

$$V_{ab} = V_{2k} - V_{12k} = 0 - 10V = -10V$$

$$V_{ab} = V_{2k} + V_{4k} - 13.33V = 0 + 3.33V - 13.33V = -10V$$

The minus sign simply indicates that terminal a is negative with respect to terminal b.

To calculate R_{TH}, remove the load and replace the 13.33V source with a short circuit as shown in Figure 1-28C. The resistance between terminals a and b in Figure 1-28C is the Thevenin resistance R_{TH}. Thus,

$$R_{TH} = R_{ab} = 2k\Omega + 12k\Omega \| 4k\Omega$$
$$= 2 + \frac{12\,(4)}{16} = 5k\Omega$$

Figure 1-28D illustrates the Thevenin equivalent circuit with the 5kΩ load resistor connected between terminals a and b. In Figure 1-28D then:

$$I_{RL} = \frac{V_{TH}}{R_{TH} + R_L} = \frac{10V}{10k\Omega} = 1mA$$

Thus, Thevenin's theorem states that this is the current that flows through the 5kΩ load resistor in the original circuit of Figure 1-28A.

Example 1-19

The circuit in Figure 1-29A has both a voltage and a current source. Use Thevenin's theorem to calculate the power dissipated by the 4kΩ load resistor.

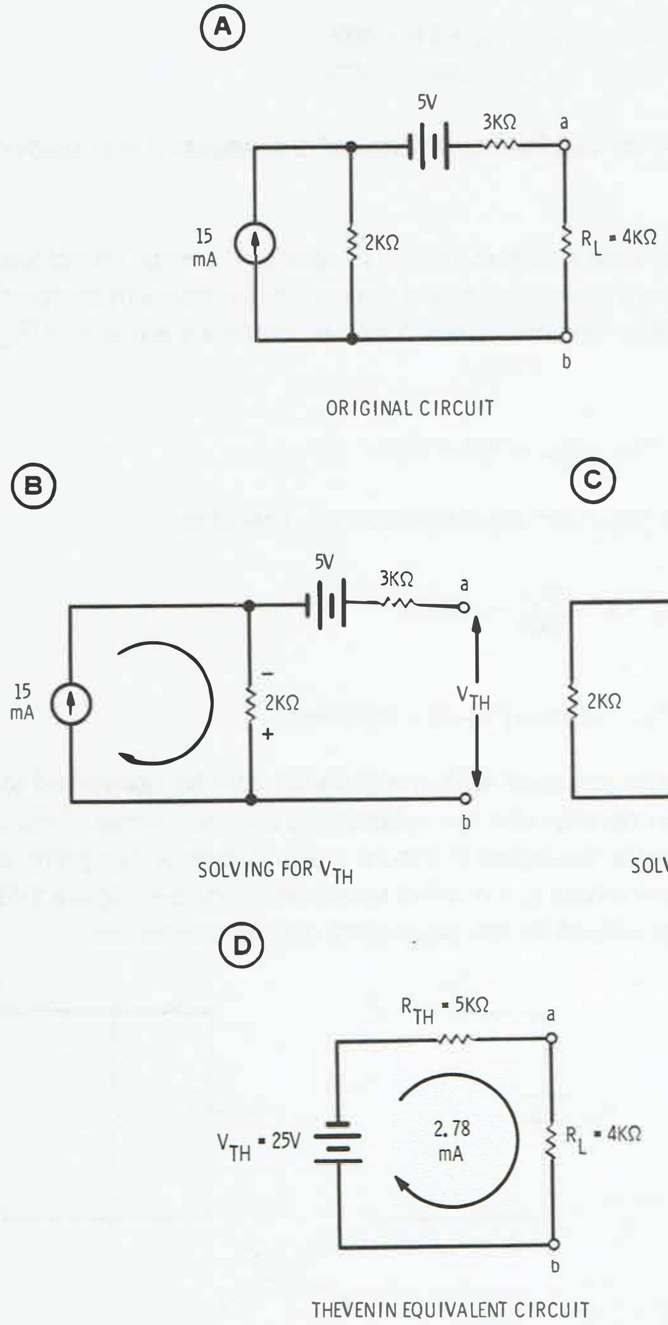


Figure 1-29

Circuits for Example 1-19.

A. Original circuit.
B. Solving for V_{TH}.
C. Solving for R_{TH}.
D. Thevenin equivalent circuit.

To calculate V_{TH}, remove the load as shown in Figure 1-29B. With the load removed, the 15mA of current from the current source flows through the 2kΩ resistor. Note that no current flows through the 3kΩ resistor in this Figure. Thus, $V_{2k} = 15\text{mA}\,(2\text{k}) = 30\text{V}$ with the polarity shown in Figure 1-29B. To find V_{TH}, write the following loop equation:

$$V_{TH} = V_{ab} = V_{3k} + 5\text{V} - 30\text{V}$$
$$V_{TH} = 0 + 5\text{V} - 30\text{V} = -25\text{V}$$

Once again, the minus sign for V_{TH} indicates terminal a is negative with respect to terminal b.

To find R_{TH}, you must remove the load and reduce all sources to zero. Therefore, replace the voltage source with a short circuit, and the current source with an open circuit as shown in Figure 1-29C. The resistance between terminals a and b in Figure 1-29C is R_{TH}.

$$R_{TH} = R_{ab} = 3\text{k}\Omega + 2\text{k}\Omega = 5\text{k}\Omega$$

Figure 1-29D illustrates the Thevenin equivalent circuit. Therefore:

$$I_{4k} = I_T = \frac{25\text{V}}{9\text{k}\Omega} = 2.78\text{mA}$$

$$P_{4k} = (2.78\text{mA})^2\,(4\text{k}\Omega) = 30.91\text{mW}$$

Incidentally, a voltage source in series with a resistance can be converted to an equivalent current source in parallel with the resistance, and vice versa. This concept of **source conversions** is illustrated in Figure 1-30. To clairify this point, consider the Thevenin equivalent circuit to the left of terminals a and b in Figure 1-29D. Referring to Figure 1-30, the values for the equivalent current source are:

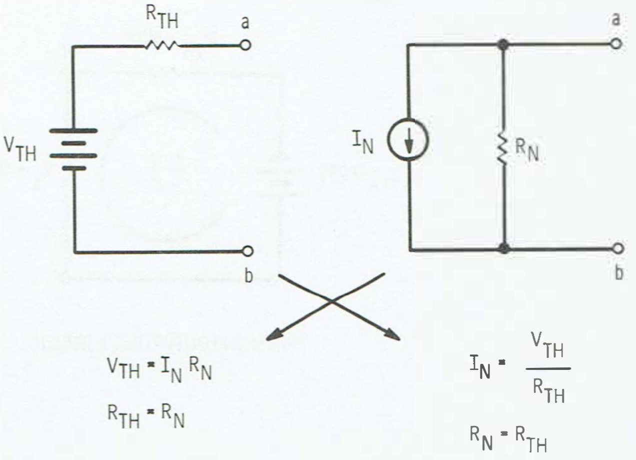


Figure 1-30

Source conversions.
A voltage source can easily be converted to an equivalent current source and vice versa, as shown above. The sources are equivalent in the sense that a load connected between terminals a and b of either source will exhibit the **same** response.

$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{25V}{5k\Omega} = 5mA$$

$$R_N = R_{TH} = 5k\Omega$$

A current source equivalent circuit is called a **Norton** equivalent circuit. The Norton equivalent circuit just developed is illustrated in Figure 1-31. Here, the load current is obtained by current division. Thus,

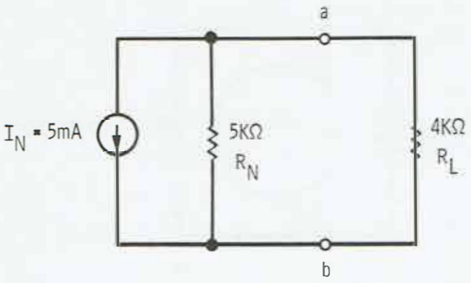


Figure 1-31

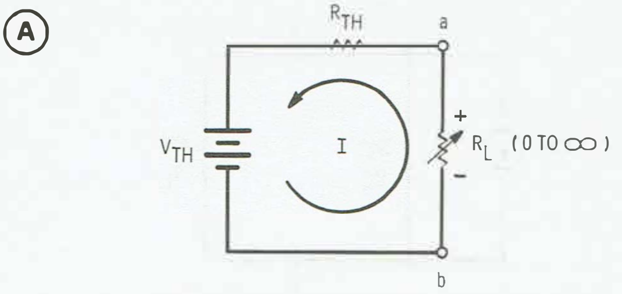
The (Norton) current source equivalent circuit of the Thevenin equivalent circuit in Figure 1-29D.

$$I_{4k} = \frac{5mA\,(5k\Omega)}{9k\Omega} = 2.78mA$$

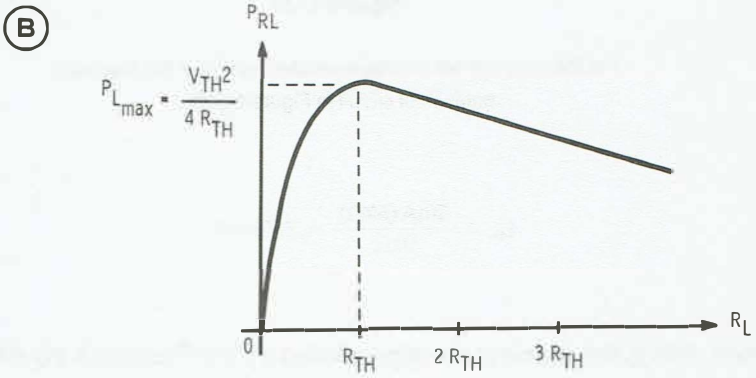
As you can see, this is the same current predicted by the Thevenin equivalent circuit in Figure 1-29D.

Maximum Power Transfer Theorem

Practical voltage and current sources always have some internal resistance. In addition, as far as the load is concerned, the rest of the circuit may be replaced by a Thevenin equivalent circuit. The circuit in Figure 1-32A can then represent either a real voltage source, or the Thevenin equivalent circuit of a more complex circuit as viewed from the load terminals (a and b). In either case, the load current is:



THEVENIN EQUIVALENT CIRCUIT WITH LOAD CONNECTED



LOAD POWER VS LOAD RESISTANCE

Figure 1-32

A circuit (or source) delivers maximum power to a load when the load resistance equals the Thevenin resistance of the circuit (or source).

A. Thevenin equivalent circuit with load connected.

B. Load power vs load resistance.

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

Similarly, the power delivered to the load is:

$$P_L = I_L^2 R_L = \frac{V_{TH}^2 R_L}{(R_{TH} + R_L)^2} \qquad \text{(Eq. 1-24)}$$

Naturally, for a given circuit (or source), different values of R_L will produce different values of load power. In any circuit however, a particular value of R_L will either be less than, equal to, or greater than the value of R_{TH}. Thus, by letting R_L equal multiples of R_{TH} ($R_L = \frac{R_{TH}}{2}$, $R_L = R_{TH}$, $R_L = 2R_{TH}$, etc) and then substituting these values into Equation 1-24, you can obtain the data necessary to plot the curve sketched in Figure 1-32B. As you can see from this chart:

> *Maximum power is transferred from a source to a load when the resistance of the load equals the resistance of the source.*

This simple fact is known as the **maximum power transfer theorem**.

Substituting R_{TH} for R_L into Equation 1-24, results in a simple formula for calculating the **maximum possible** load power:

$$P_{Lmax} = \frac{V_{TH}^2 (R_{TH})}{(R_{TH} + R_{TH})^2} = \frac{V_{TH}^2 (R_{TH})}{4\,R_{TH}^2}$$

or

$$P_{Lmax} = \frac{V_{TH}^2}{4\,R_{TH}} \qquad \text{(Eq. 1-25)}$$

Example 1-20

What value of R_L in Figure 1-33A produces maximum transfer of power from the circuit to the load? What is the maximum possible load power?

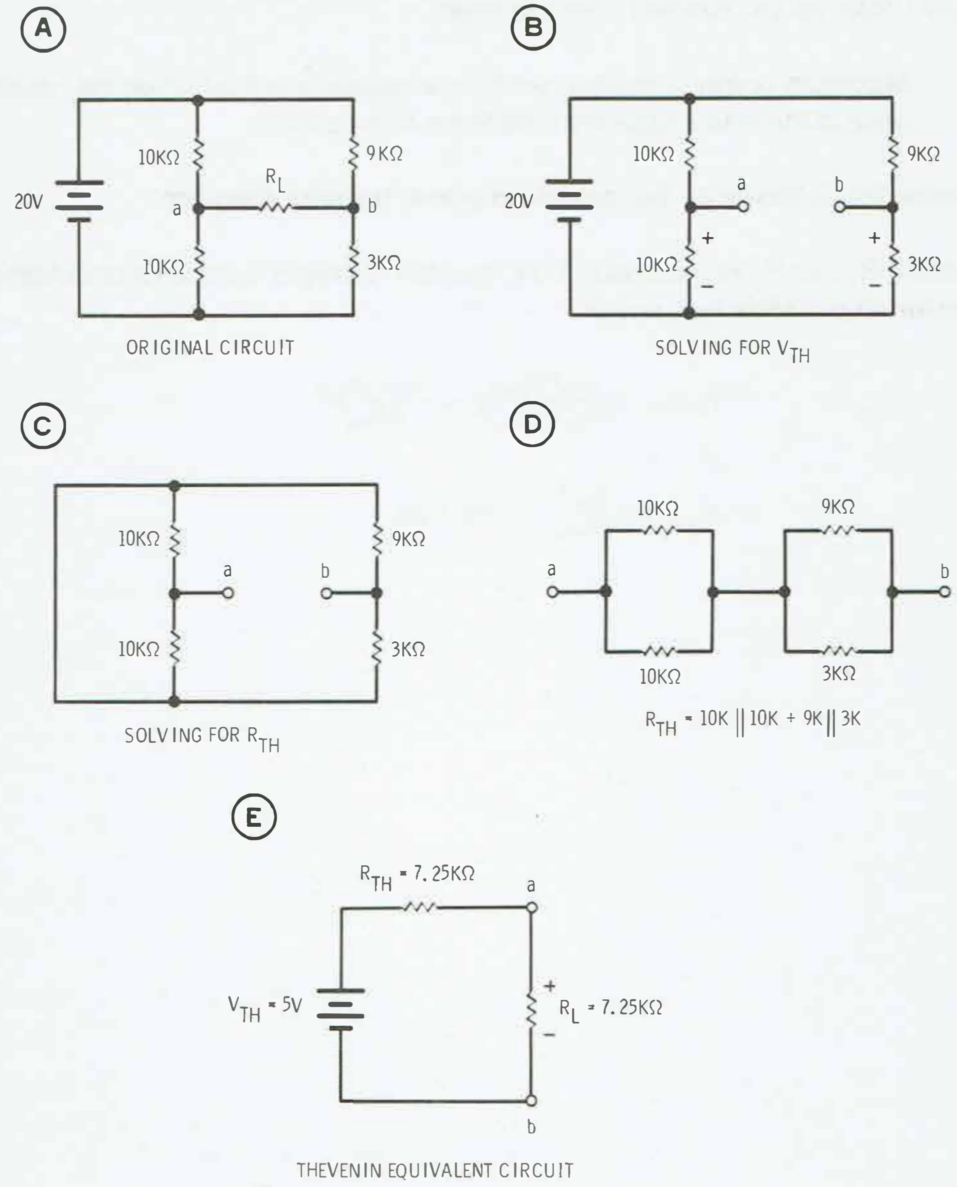


Figure 1-33

Circuits for Example 1-20.

A. Original circuit.
B. Solving for V_{TH}.
C. Solving for R_{TH}.
D. $R_{TH} = 10k \| 10k + 9k \| 3k$.
E. Thevenin equivalent circuit.

To determine V_{TH}, remove the load as shown in Figure 1-33B. Note that once the load is removed, the two 10kΩ resistors are connected in series, as are the 3kΩ and 9kΩ resistors. Applying the voltage division principle, you have:

$$V_{10k} = \frac{20V\,(10k\Omega)}{20k\Omega} = 10V$$

$$V_{3k} = \frac{20V\,(3k\Omega)}{12k\Omega} = 5V$$

Since in Figure 1-20B, $V_{ab} = V_{TH}$, you have:

$$V_{TH} = V_{10k} - V_{3k}$$

$$V_{TH} = 10V - 5V = 5V$$

To determine R_{TH}, remove the load and reduce the 20V source to zero as shown in Figure 1-33C. The circuit of Figure 1-33C is redrawn in Figure 1-33D to help you visualize the equivalent resistance between terminals a and b. Thus,

$$R_{ab} = R_{TH} = 10k\Omega \| 10k\Omega + 9k\Omega \| 3k\Omega$$

$$R_{TH} = 5k\Omega + 2.25k\Omega$$

$$R_{TH} = 7.25k\Omega$$

The maximum power transfer theorem states that when $R_L = 7.25k\Omega$, maximum power will be transferred from the circuit in Figure 1-33A to the load. To determine the amount of maximum power, you can use Equation 1-25. Thus,

$$P_{Lmax} = \frac{{V_{TH}}^2}{4\,R_{TH}} = \frac{(5V)^2}{4\,(7.25k\Omega)} = \frac{25V}{29k\Omega}$$

$$P_{Lmax} = 0.862mW = 862\mu W$$

Figure 1-33E illustrates the Thevenin equivalent circuit. Incidentally, the original circuit in Figure 1-33A is called a **Wheatstone bridge**. If you look carefully at the circuit in Figure 1-33A, you can see that the resistors are **not** connected in simple series or parallel combinations. Thus, a Wheatstone bridge **cannot** be analyzed by series-parallel techniques. This is a classic example where Thevenin's theorem simplifies an analysis that would otherwise be difficult.

Self-Test Review

27. In a ________________ circuit, component values are constant.
28. ________________________ cannot be used to calculate power directly in DC circuits.
29. When sources are reduced to zero, voltage sources are replaced by ________ circuits and current sources are replaced by _____________ circuits.
30. Another name for the Thevenin voltage is the ___________ _______________ __________________.
31. The Thevenin resistance is calculated by removing the _____________, and reducing all __________________ to zero.
32. A voltage source in series with a resistance can be converted to an equivalent __________________ source in __________________ with the resistance.
33. A ________________ circuit is a current source equivalent circuit.
34. Maximum power transfer occurs when the ______________ resistance equals the Thevenin resistance of the circuit or source.
35. The ______________________ bridge is an example of a circuit that cannot be reduced by series-parallel methods.

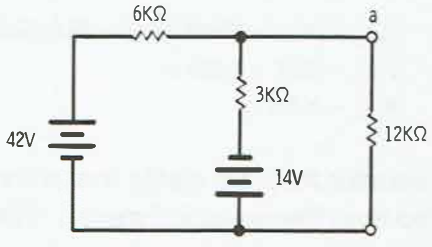


Figure 1-34

Circuit for Self-test Review questions 36-43.

36. In Figure 1-34, the current through the 12kΩ resistor due to the 42V source, is _____ mA directed ____________________.
upward/downward
37. In Figure 1-34, the current through the 12kΩ resistor due to the 14V source, is _________ mA directed ____________________.
upward/downward
38. The actual current through the 12kΩ resistor in Figure 1-34 due to both sources, is _________ mA directed ______________________.
upward/downward
39. The Thevenin voltage V_{TH} in Figure 1-34 is _________ V.
40. the Thevenin resistance R_{TH} in Figure 1-34 is _______ kΩ
41. The Norton equivalent circuit for Figure 1-34 has a _________ mA current source connected in parallel with a resistance of _______ kΩ.

42. The Norton equivalent circuit for Figure 1-34 predicts a current of _____ mA directed ______________________ through the 12 kΩ resistor.
upward/downward

43. To achieve maximum power transfer, the 12kΩ resistor in Figure 1-34 should be changed to _______ kΩ If this is done, the maximum load power will equal _________ mW

Answers

27. linear
28. Superposition
29. Short, open
30. open circuit voltage
31. load, sources
32. current, parallel
33. Norton
34. load
35. Wheatstone
36. 1, upward
37. 0.67, downward
38. 0.33, upward
39. 4.67
40. 2
41. 2.34, 2
42. 2, downward
43. 2, 2.73

The solutions to questions 36-43 follow.

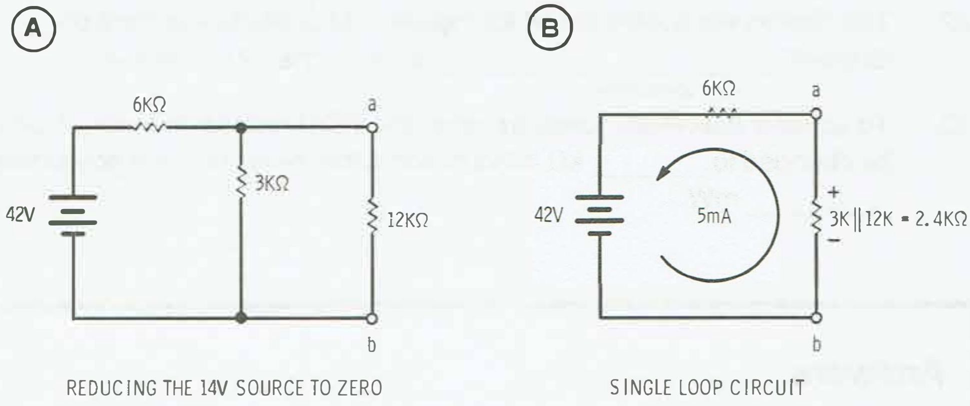


Figure 1-35

Considering the effect of the 42V source in Figure 1-34.

A. Reducing the 14V source to zero.

B. Single loop circuit.

36. Refer to Figure 1-35.

Once the 14V source is reduced to zero, the 3kΩ and 12kΩ resistors are connected in parallel as shown in Figure 1-35A. Thus,

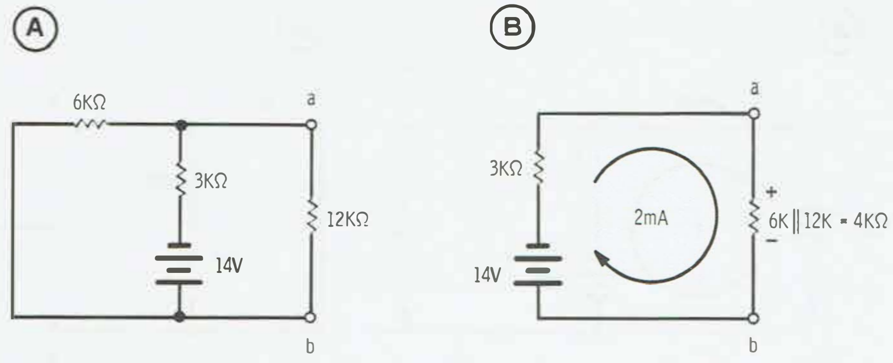
$$3\text{k}\Omega \| 12\text{k}\Omega = \frac{3\,(12)}{15} = 2.4\text{k}\Omega \text{ as shown in Figure 1-35B.}$$

The total current from the 42V source is:

$$I_T = \frac{42\text{V}}{8.4\text{k}\Omega} = 5\text{mA}$$

The total current divides between the 3kΩ and 12kΩ resistors in Figure 1-35A. Applying the current division principle, you have:

$$I_{12k} = \frac{5\text{mA}\,(3\text{k}\Omega)}{15\text{k}\Omega} = 1\text{mA directed upward}$$



REDUCING THE 42V SOURCE TO ZERO

SINGLE LOOP CIRCUIT

Figure 1-36

Considering the effect of the 14V source in Figure 1-34.

A. Reducing the 42V source to zero.

B. Single loop circuit.

37. Refer to Figure 1-36.

Reducing the 42V source to zero yields the circuit shown in Figure 1-36A. Note that the 6kΩ and 12kΩ resistors are connected between points a and b and are, therefore, connected in parallel.

$$6\text{k}\Omega \| 12\text{k}\Omega = \frac{6(12)}{18} = 4\text{k}\Omega \text{ as shown in Figure 1-36B.}$$

The total current from the 14V source is:

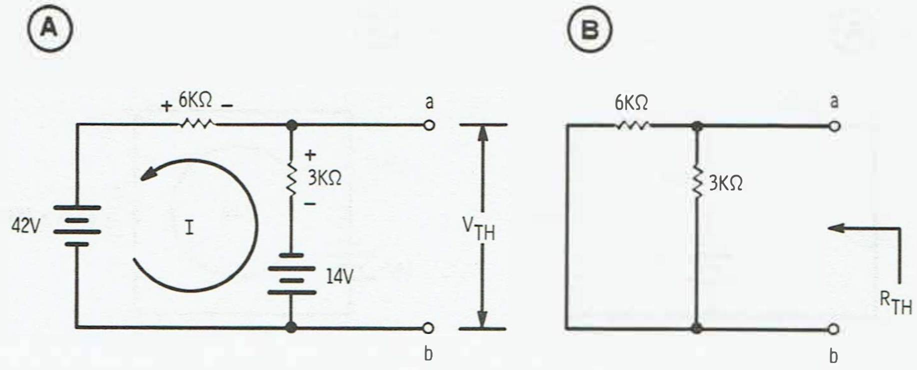
$$I_T = \frac{14\text{V}}{7\text{k}\Omega} = 2\text{mA}$$

The total current divides between the 6kΩ and 12kΩ resistors in Figure 1-36A. Using the current division principle once more yields:

$$I_{12k} = \frac{2\text{mA}\,(6\text{k}\Omega)}{18\text{k}\Omega} = 0.67\text{mA directed downward}$$

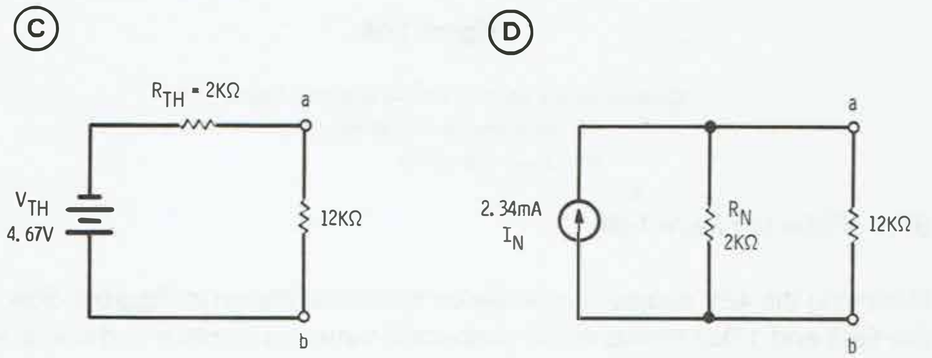
38. Via the superposition principle, you have:

$$I_{12k} = 1\text{mA}\uparrow + 0.67\text{mA}\downarrow = 0.33\text{mA directed upward}$$



SOLVING FOR V_{TH}

SOLVING FOR R_{TH}



THEVENIN EQUIVALENT

NORTON EQUIVALENT

Figure 1-37

Applying Thevenin's theorem to the circuit in Figure 1-34.

A. Solving for V_{TH}.
B. Solving for R_{TH}.
C. Thevenin equivalent.
D. Norton equivalent.

39. Refer to Figure 1-37.

To calculate V_{TH}, remove the 12kΩ load resistor as shown in Figure 1-37A. Applying Kirchhoff's voltage law, you have:

$$42V + 14V - I(3k\Omega) - I(6k\Omega) = 0$$

$$I(9k\Omega) = 56V$$

$$I = \frac{56V}{9k\Omega} = 6.22mA$$

The voltage across the 3kΩ resistor is therefore:

$$V_{3k} = 6.22\text{mA}\,(3\text{k}\Omega) = 18.67\text{V}$$

Finally: $V_{ab} = V_{TH} = V_{3k} - 14\text{V}$

$$V_{TH} = 18.67\text{V} - 14\text{V} = 4.67\text{V}$$

40. Refer to Figure 1-37B.

Removing the 12kΩ load resistor and reducing the sources to zero, you have the circuit shown in Figure 1-37B. Thus,

$$R_{ab} = R_{TH} = 6\text{k}\Omega \| 3\text{k}\Omega = 2\text{k}\Omega$$

41, 42. The Thevenin equivalent circuit is illustrated in Figure 1-37C. Converting to an equivalent current source, you have:

$$R_N = R_{TH} = 2\text{k}\Omega$$

$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{4.67\text{V}}{2\text{k}\Omega} = 2.34\text{mA}$$

Figure 1-37D illustrates the Norton equivalent circuit.

43. For maximum power transfer:

$$R_L = R_{TH} = 2\text{k}\Omega$$

$$P_{Lmax} = \frac{{V_{TH}}^2}{4\,R_{TH}} = \frac{(4.67\text{V})^2}{4\,(2\text{k}\Omega)} = \frac{21.81\text{V}}{8\text{k}\Omega} = 2.73\text{mW}$$

SUMMARY

The following points summarize the key ideas of this unit.

Charge is measured in units of coulombs. The flow of electric charges is current. In this course, we assume current is due to the movement of negative charges and, therefore, flows from minus to plus. This assumption is referred to as the electron current flow convention.

Resistance is the opposition to current flow and has units of ohms. The reciprocal of resistance is called conductance and has units of mhos, or siemens. Voltage has units of volts, and may be thought of as the electrical pressure that forces current to flow. The relationship between current, voltage, and resistance is summarized by Ohm's law, which states that:

$$I = \frac{V}{R}$$

Power is the rate of doing work. For DC circuits, the following relationships may be used to compute electrical power that has units of watts.

$$P = VI = I^2R = \frac{V^2}{R}$$

Kirchhoff's current law states that the algebraic sum of the currents entering and leaving a node must be zero. Similarly, Kirchhoff's voltage law states that the algebraic sum of the voltages around a closed loop must equal zero. The term algebraic sum refers to the direction of a current, or the polarity of a voltage. If you get a negative answer, when you are solving nodal or loop equations, your initial assumption concerning the direction of a current, or polarity of a voltage, is incorrect, so you must reverse the direction.

In a series circuit, the same current flows through each component. The equivalent resistance of a series circuit equals the sum of the resistors in the circuit. The voltage division principle lets you quickly calculate the voltage across any resistor in a series circuit from:

$$V_{Rn} = \frac{V_T (R_n)}{R_T}$$

In a parallel circuit, the voltage across each component is the same. The equivalent resistance of a parallel circuit can be determined by:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots \frac{1}{R_N}}$$

For two parallel resistors, the equivalent resistance is:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

The current division principle applies to two parallel resistors, and enables you to quickly calculate the current through either resistor. Specifically:

$$I_1 = I_T \frac{R_2}{R_1 + R_2} \text{ and } I_2 = I_T \frac{R_1}{R_1 + R_2}$$

Series-parallel circuits are combinations of series and parallel circuits. Such circuits are analyzed by reducing the given circuit to a single loop, and then working backwards.

Several circuit theorems are especially useful for analyzing multisource circuits. The superposition theorem lets you calculate a current or voltage by algebraically adding the current or voltage produced by each source acting alone. The superposition theorem cannot be used to calculate power directly, since power is not a linear quantity.

Thevenin's theorem is an extremely useful technique for analyzing circuits. This theorem allows you to picture a complex circuit (as seen from the load terminals) as the series combination of an equivalent voltage source and resistance. A Thevenin equivalent circuit can easily be converted to an equivalent current source, or Norton, equivalent circuit.

Another useful circuit theorem is the maximum power transfer theorem. This theorem states that maximum load power results when the load resistance equals the Thevenin resistance of the source (or circuit) supplying power to the load.

Complete Experiment 1, Unit 8 at this time.

UNIT EXAMINATION

The following multiple choice examination is designed to test your understanding of the material presented in this unit. Place a check beside the multiple choice answer (A, B, C, or D) that you feel is most correct. When you have completed the examination, compare your answers with the correct ones that appear after the exam.

1. A coulomb is a unit of:
    - A. Current.
    - B. Charge.
    - C. Work.
    - D. Voltage.
2. A flow of one coulomb per second is:
    - A. One volt.
    - B. One ohm.
    - C. One ampere.
    - D. One joule.
3. Voltage represents:
    - A. Opposition to current flow.
    - B. The rate of doing work.
    - C. Electrical pressure.
    - D. Charge per unit time.
4. A conductance of 0.2 siemens equals:
    - A. 5 ohms.
    - B. 5 mhos.
    - C. 0.2 mhos.
    - D. A and C.
5. $1\text{M}\Omega$ is equivalent to:
    - A. $1000\text{k}\Omega$.
    - B. $1000 \times 10^{-3}\Omega$.
    - C. 1×10^{6} mhos.
    - D. $1000 \times 10^{-6}\Omega$.
6. $V = 20\text{V}$, $R = 5\text{k}\Omega$, I is therefore:
    - A. 4A.
    - B. $4\mu\text{A}$.
    - C. $0.25\mu\text{A}$.
    - D. 4mA.
7. All parallel circuits contain only:
    - A. Two loops.
    - B. Linear components.
    - C. Two nodes.
    - D. Two resistors.

8. A single path between two nodes is called a:
   - A. Loop.
   - B. Branch.
   - C. Source.
   - D. Resistance.

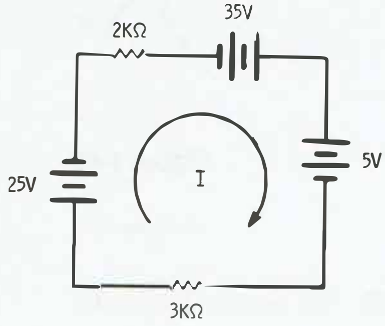


Figure 1-38

Circuit for Question 9 and 10.

9. Refer to Figure 1-38. The correct loop equation for the circuit is:
   - A. $25V + I(2k\Omega) - 35V + 5V + I(3k\Omega) = 0$
   - B. $-25V - I(2k\Omega) + 35V - 5V - I(3k\Omega) = 0$
   - C. $25V - I(2k\Omega) - 35V + 5V - I(3k\Omega) = 0$
   - D. $25V + I(2k\Omega) + 35V + 5V + I(3k\Omega) = 0$

10. Refer to Figure 1-38. The loop current I equals:
   - A. 0.
   - B. 1mA.
   - C. −1mA.
   - D. 2mA.

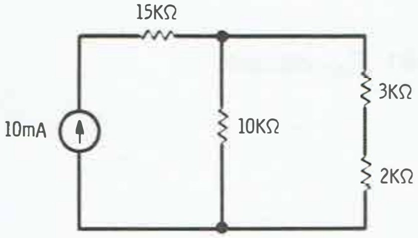


Figure 1-39

Figure 1-39 circuit for Question 11.

11. Refer to Figure 1-39. Using current division, the current through the 3kΩ resistor is:
   - A. $I = 10mA\,(10k\Omega)/15k\Omega$.
   - B. $I = 10mA\,(10k\Omega)/13k\Omega$.
   - C. $I = 10mA\,(5k\Omega)/15k\Omega$.
   - D. $I = 10mA\,(10k\Omega)/20k\Omega$.

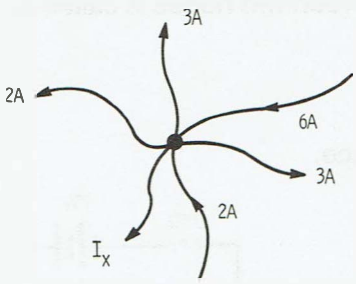


Figure 1-40

Question 12.

12. Refer to Figure 1-40. The current I_x equals:
    - A. 0.
    - B. 1A.
    - C. 2A.
    - D. 3A.

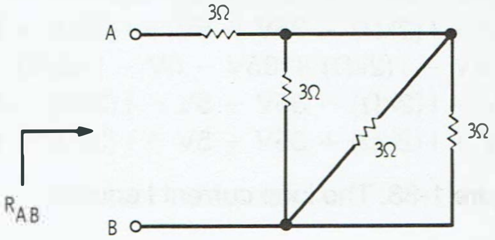


Figure 1-41

Circuit for Question 13.

13. Refer to Figure 1-41. R_{AB} equals:
    - A. 3Ω.
    - B. 4Ω.
    - C. 9Ω.
    - D. 12Ω.

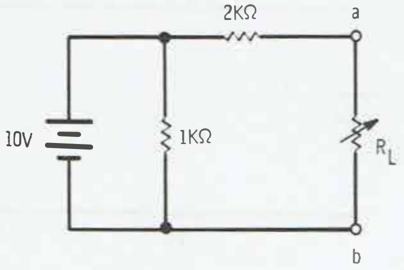


Figure 1-42

Circuit for Questions 14 to 19.

Refer to Figure 1-42 for Questions 14 through 19.

14. The Thevenin voltage equals:
    - A. 3.3V.
    - B. 6.7V.
    - C. 5V.
    - D. 10V.
15. The Thevenin resistance equals:
    - A. 2kΩ.
    - B. 3kΩ.
    - C. 1kΩ.
    - D. 0.67kΩ.
16. Maximum power transfer occurs if R_L equals:
    - A. 2kΩ.
    - B. 3kΩ.
    - C. 1kΩ.
    - D. 0.67kΩ.
17. The maxiumum possible load power is:
    - A. 12.5μW.
    - B. 12.5mW.
    - C. 12.5W.
    - D. 125W.
18. The value of the Norton current I_N is:
    - A. 5μA.
    - B. 5mA.
    - C. 5A.
    - D. 50A.
19. The value of the Norton resistance R_N is:
    - A. 2kΩ.
    - B. 3kΩ.
    - C. 1kΩ.
    - D. 0.67kΩ.

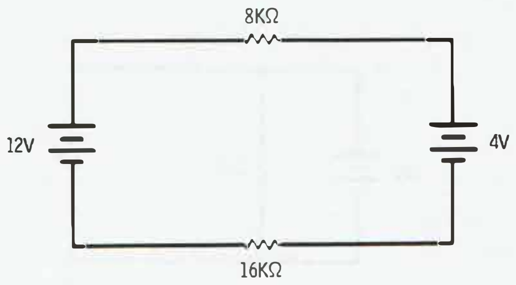


Figure 1-43

Circuit for Question 20.

20. Refer to Figure 1-43. By superposition, the voltage across the 16kΩ resistor is:
    A. 5.33V.
    B. 10.67V.
    C. 12V.
    D. 16V.

EXAMINATION ANSWERS

1. B — The unit of electrical charge is the coulomb.
2. C — One ampere is defined as one coulomb per second.
3. C — Voltage is the electrical force or pressure that causes current to flow.
4. D — Conductance is the reciprocal of resistance and vice versa. The units of conductance are mhos or siemens - with siemens being the preferred units. Thus 0.2S = 0.2℧. The equivalent resistance is:

$$R = \frac{1}{G} = \frac{1}{0.2S} = 5\Omega$$

5. A — $1M\Omega = 1 \times 10^6\Omega = 1000 \times 10^3\Omega$ or $1000k\Omega$
6. D — From Ohm's law:

$$I = \frac{V}{R} = \frac{20V}{5k\Omega} = 4mA$$

7. C — Each component in a parallel circuit is connected between the same pair of nodes.
8. B — A branch is a single path between two nodes.
9. C — Starting on the positive side of the 25V source and moving clockwise around the circuit, you have:

$$25V - I(2k\Omega) - 35V + 5V - I(3k\Omega) = 0$$

10. C —

$$25V - 35V + 5V = I(2k\Omega) + I(3k\Omega)$$
$$-5V = I(5k\Omega)$$
$$I = \frac{-5V}{5k\Omega} = -1mA$$

The negative sign indicates I actually flows in a counterclockwise direction rather than the clockwise direction shown in Figure 1-38.

11. A — The 3kΩ and 2kΩ resistors have an equivalent resistance of 5kΩ. Therefore:

$$I_{5k} = I_{3k} = 10mA\,(10k\Omega)/15k\Omega = 6.67mA$$

12. A — Starting with I_x, you can write the following nodal equation:

$$I_x + 2A + 3A - 6A + 3A - 2A = 0$$
$$I_x + 8A - 8A = 0$$
$$I_x = 0$$

13. B — The three 3Ω resistors on the right side of the circuit are connected in parallel, and thus have an equivalent resistance of 3/3 or 1Ω. This 1Ω equivalent resistor is connected in series with the remaining 3Ω resistor, giving a total resistance of 4Ω.

14. D — Once the load is removed, the current through the $2k\Omega$ resistor is 0. Thus the voltage across the $2k\Omega$ resistor is also 0. Starting at terminal a and moving around the circuit to terminal b, you have:

$$\begin{aligned} V_{ab} &= V_{TH} = V_{2k} + 10V \\ V_{TH} &= 0 + 10V \\ V_{TH} &= 10V \end{aligned}$$

15. A — Once the 10V source is reduced to zero, the $1k\Omega$ resistor is shunted by a short circuit. Thus,

$$R_{ab} = R_{TH} = 2k\Omega + 0 = 2k\Omega$$

16. A — Maximum power transfer occurs when $R_L = R_{TH}$. Since $R_{TH} = 2k\Omega$, R_L must $= 2k\Omega$

17. B — The maximum possible load power can be found from Equation 1-25. Therefore:

$$P_{Lmax} = \frac{V_{TH}{}^2}{4R_{TH}} = \frac{(10V)^2}{4\,(2k\Omega)} = 12.5mW$$

18. B — $I_N = \dfrac{V_{TH}}{R_{TH}} = \dfrac{10V}{2k\Omega} = 5mA$

19. A — $R_N = R_{TH} = 2k\Omega$

20. A — When the 4V source is reduced to zero:

$$I = \frac{12V}{24k\Omega} = 0.5mA \text{ (counterclockwise)}$$

Similarly reducing the 12V source to zero:

$$I = \frac{4V}{24k\Omega} = 0.167mA \text{ (clockwise)}$$

Thus the actual current with both sources active is:

$$I = 0.5mA - 0.167mA = 0.333mA \text{ (counterclockwise)}$$

Therefore:

$$V_{16k} = 0.333mA\,(16k\Omega) = 5.33V$$

Unit 2

MATHEMATICS FOR CIRCUIT DESIGN

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INTRODUCTION

In the previous unit we reviewed the techniques used to analyze resistive DC circuits. In this unit we introduce two additional circuit components: inductors and capacitors. We will review the DC characteristics of these components, and develop the mathematics necessary to describe how these components work in DC circuits. In addition we will discuss logarithms and the mathematics of complex numbers. Complex numbers are necessary to analyze and design AC circuits. Without a background in the language of electronics (mathematics), your ability to analyze, and design circuits will be very limited. The purpose of this unit, then, is to help you learn this language. It will significantly increase your ability to design circuits and to understand how they work.

UNIT OBJECTIVES

When you have completed this unit, you should be able to:

1. Define capacitance and inductance.
2. Calculate the energy stored in a capacitor or inductor.
3. Determine the equivalent capacitance of series, parallel, and series-parallel combinations.
4. Determine the equivalent inductance of series, parallel, and series-parallel combinations.
5. State the formula for capacitive voltage division.
6. Write an equation that describes capacitor voltage and current in single-loop RC circuits.
7. Write an equation that describes inductor voltage and current in single-loop RL circuits.
8. Convert natural logarithms to common logarithms, and vice versa.
9. Calculate the amount of time for the voltage across a capacitor to reach a particular value in a single-loop RC circuit.
10. Convert current, voltage, and/or power ratios to their dB equivalents, and vice versa.
11. Define real, imaginary, and complex numbers.
12. Add, subtract, multiply, and divide rectangular numbers.
13. Multiply and divide polar numbers.
14. Convert rectangular numbers to polar numbers, and vice versa.

UNIT ACTIVITY GUIDE

☐ Read the section on "RC Switching Circuits."

☐ Answer Self Test Review Questions 1-18.

☐ Read the section on "RL Switching Circuits."

☐ Answer Self Test Review Questions 19-34.

☐ Read the section on "Logarithms."

☐ Answer Self Test Review Questions 35-46.

☐ Read the section on "Rectangular Complex Numbers."

☐ Answer Self Test Review Questions 47-58.

☐ Read the section on "Polar Complex Numbers."

☐ Answer Self Test Review Questions 59-68.

☐ Study the Summary.

☐ Complete Experiment 2 in Unit 8.

☐ Complete the "Unit Examination."

☐ Check the "Examination Answers."

RC SWITCHING CIRCUITS

With the exception of resistors, capacitors are the most frequently used components in electronic circuits. Although capacitors are generally found in AC circuits, the characteristics they exhibit in DC circuits are also very important. This is especially true in DC circuits where abrupt voltage changes can occur. In this section, you will learn how the current through and voltage across a capacitor in a DC circuit can be described. In later units, you will see how these "mathematical descriptions" are used to design a number of useful circuits.

Switching Basics

As the words imply, **switching circuits** employ one or more switches in addition to numerous other components. The ideal switch is an example of a perfect binary (two state) device. When closed, an ideal switch has a resistance of zero ohms (short circuit), and when open its resistance is infinite (open circuit).

The currents and voltages in a switching circuit can change abruptly when a switch is "switched" from one state to the other. Figure 2-1 illustrates this concept, and introduces some relevant switching notation. Initially, the switch in Figure 2-1 A is open; and since the current (I) is zero, the output voltage is also zero as indicated in Figure 2-1 B. At some point in time, $t = 0$, the switch is closed. Thus, the current (I) instantaneously rises to the Ohms law value V/R (5mA in this case), producing an output voltage of 5V as shown in Figure 2-1 C.

The curve in Figure 2-1 D is a "picture" of the circuit action just described. It is important to realize that we are assuming, for simplicity, that the circuit in Figure 2-1 is composed of "ideal" components. The non-ideal effects of real components will be discussed in a later unit. In Figure 2-1, you should note the following:

$t = 0$ refers to the instant the switch is thrown from one position to the other.

$t = 0^-$ refers to the point in time just before $t = 0$.

$t = 0^+$ refers to the point in time just after $t = 0$.

Clearly, the value of the output voltage is 0V at $t = 0^-$ and 5V at $t = 0^+$. The value of the output voltage at $t = 0$ is not defined, because at this instant it is assumed to change instantaneously from 0V to 5V. The point to remember is simply this: before the switch is closed ($t = 0^-$), the currents and voltages have particular initial values; after the switch is closed ($t = 0^+$), the values of the currents and voltages have changed to their final values. Incidentally, the notation $\nu(t)$ (pronounced ν of t) means "voltage as a function of time". This notation is used to describe responses whose values change with time.

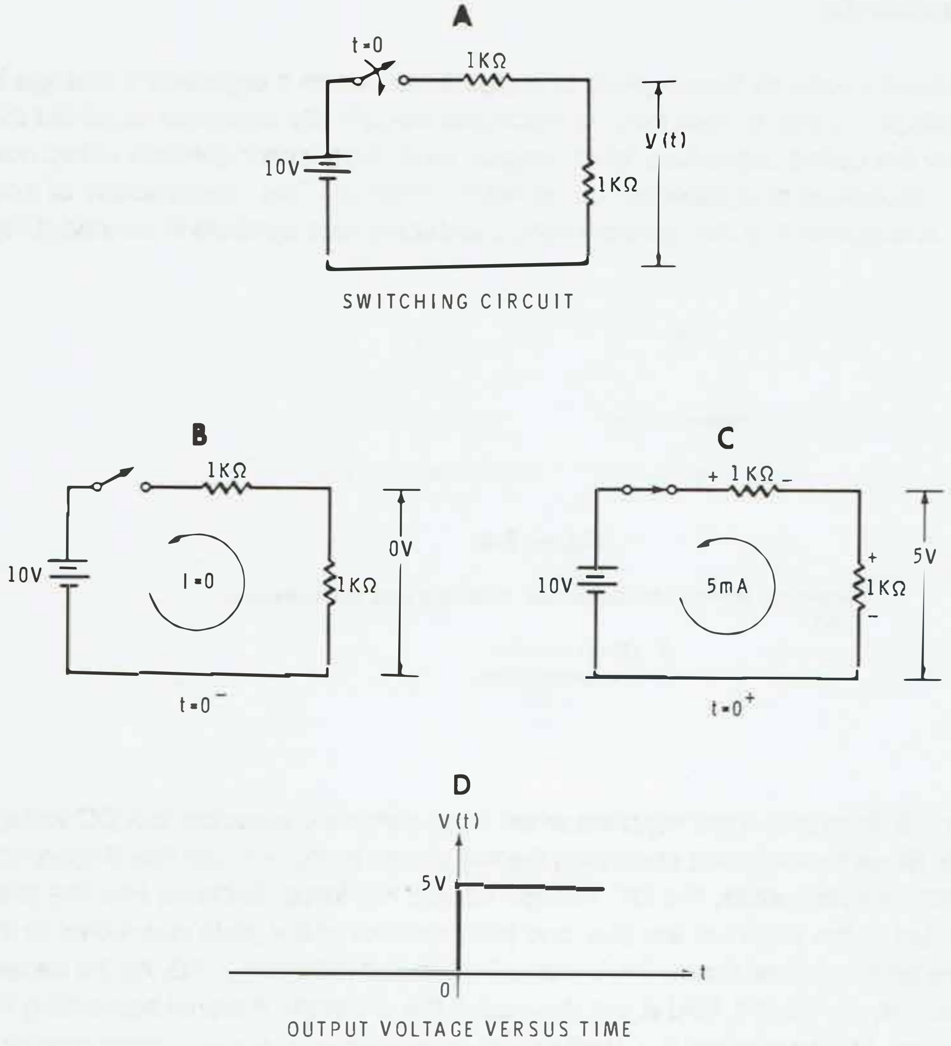


Figure 2-1

Abrupt voltage change due to a switch.
A. Switching circuit.
B. $t = 0^-$.
C. $t = 0^+$.
D. Output voltage versus time.

Capacitance

A component exhibits the property of **capacitance** when it **opposes a change in the voltage across it**. Naturally, components specifically designed to exhibit this property are called capacitors. In its simplist form, a capacitor consists of two conductors separated by a dielectric, or insulating, material. This "construction" of a capacitor is suggested by the two commonly used schematic symbols illustrated in Figure 2-2.

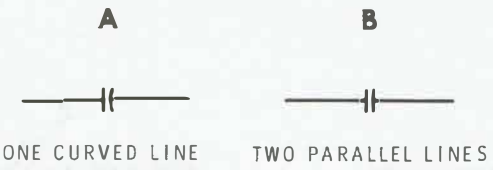


Figure 2-2

Schematic symbols for capacitors. (Both symbols are commonly used.)
A. One curved line.
B. Two parallel lines.

Figure 2-3 illustrates what happens when a capacitor is connected to a DC voltage source. Since the dielectric separates the two plates, no current can flow through the capacitor. Nevertheless, the DC voltage source will force electrons into the plate connected to the negative terminal and pull them out of the plate connected to the positive terminal once the switch is closed, as shown in Figure 2-3 B. As the capacitor charges, an electric field is established in the dielectric material separating the two plates. The strength of the electric field increases as more and more charge is accumulated and, as a result, a voltage builds up between the two plates. This charging process continues until the voltage across the capacitor equals the source voltage (V). At this point the capacitor is fully charged, and no further movement of electrons occurs.

If the capacitor is now disconnected from the charging circuit, as shown in Figure 2-3 C, it will retain its charge. Thus, a capacitor stores electrical energy in the form of an electric field. The amount of charge stored by a capacitor is given by the following formula:

$$Q = CV \qquad \text{(Eq. 2-1)}$$

where: Q = charge stored in coulombs (C)
V = capacitor voltage in volts (V)
C = capacitance in farads (F)

As you can see, the unit of capacitance is the farad, named in honor of Michael Faraday. A **farad** is defined as the amount of capacitance that will store one coulomb of charge when the voltage across the capacitor is one volt. A farad is a **very large** amount of capacitance; typical capacitors have capacitance values in μF, pF, etc.

Figure 2-3D shows how the energy stored in a capacitor can be removed. By connecting a resistor across the plates of a charged capacitor, a discharge path is provided so the excess electrons stored on the negative plate can flow to the positive plate. They will flow until each plate is electrically neutral. At this point, there is no further electron movement and the capacitor is fully discharged.

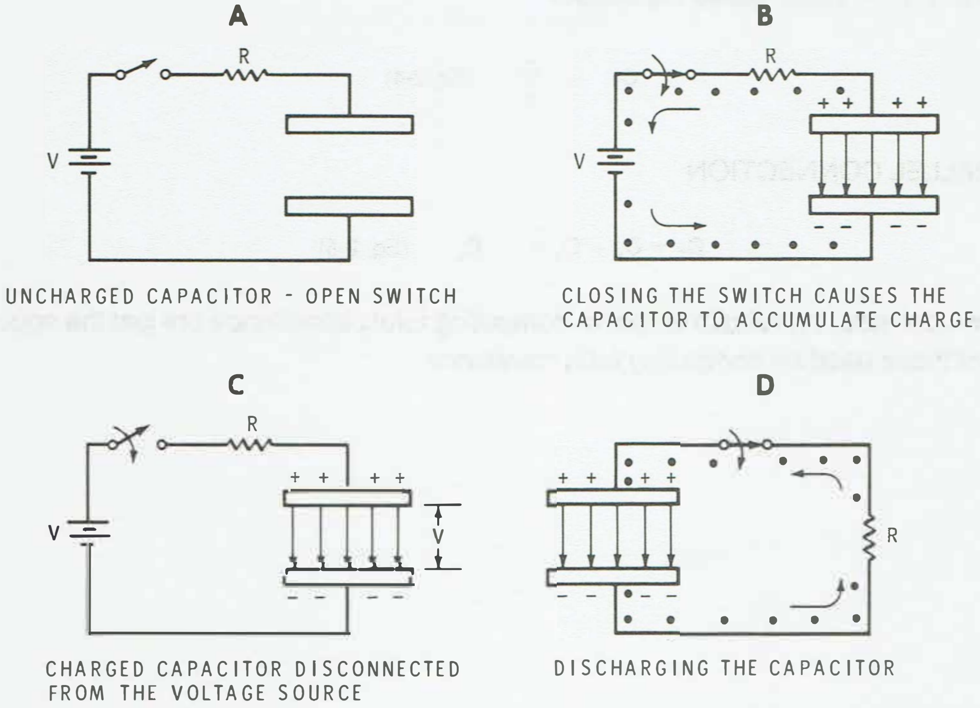

A
R
V
UNCHARGED CAPACITOR - OPEN SWITCH
B
R
V
+ +
+ +
– –
– –
CLOSING THE SWITCH CAUSES THE CAPACITOR TO ACCUMULATE CHARGE
C
R
V
+ +
+ +
V
– –
– –
CHARGED CAPACITOR DISCONNECTED FROM THE VOLTAGE SOURCE
D
+ +
+ +
R
– –
– –
DISCHARGING THE CAPACITOR

Figure 2-3

Charging and discharging a capacitor.

A. Uncharged capacitor — open switch.

B. Closing the switch causes the capacitor to accumulate charge.

C. Charged capacitor disconnected from the voltage source.

D. Discharging the capacitor.

Capacitors, like resistors, may be connected in series, parallel, or series-parallel combinations. To calculate the equivalent capacitance for series and parallel combinations, use the following formulas.

SERIES CONNECTION

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \frac{1}{C_n} \qquad \text{(Eq. 2-2)}$$

For **two** capacitors connected in series, Equation 2-2 reduces to:

$$C_T = \frac{C_1 C_2}{C_1 + C_2} \qquad \text{(Eq. 2-3)}$$

Similarly, for n equal series capacitors:

$$C_T = \frac{C}{n} \qquad \text{(Eq. 2-4)}$$

PARALLEL CONNECTION

$$C_T = C_1 + C_2 + \ldots C_n \qquad \text{(Eq. 2-5)}$$

As you can see, the relationships for computing total capacitance are just the opposite of those used for computing total resistance.

Example 2-1

Compute the total capacitance for the circuits shown in Figure 2-4.

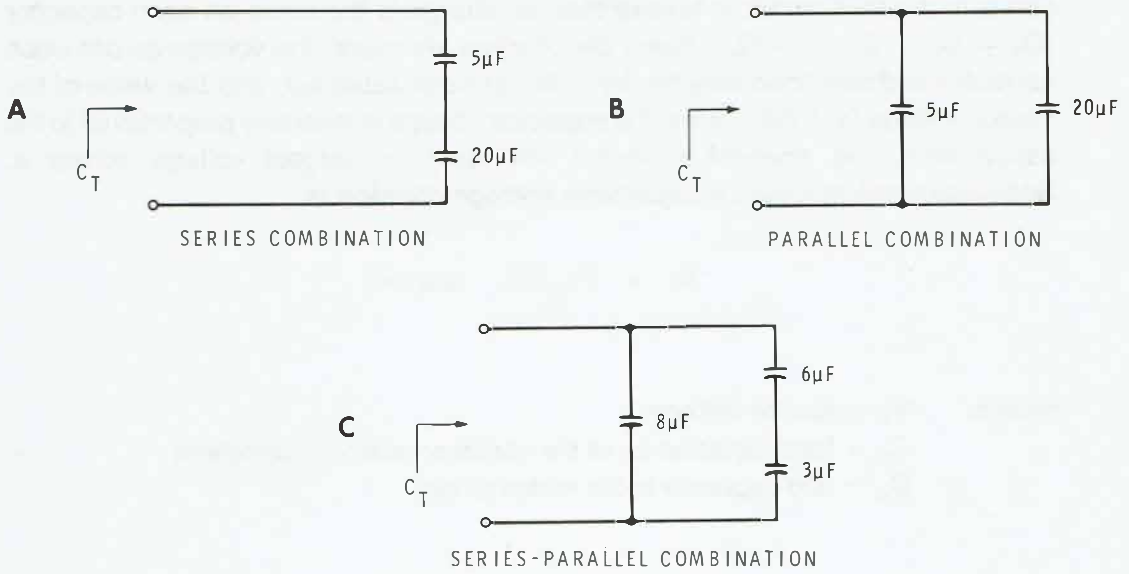


Figure 2-4

Circuits for Example 2-1.
A. Series combination.
B. Parallel combination.
C. Series-parallel combination.

Figure 2-4A:

$$C_T = \frac{C_1 \; C_2}{C_1 + C_2} = \frac{5(20)}{25} = 4\mu F$$

Figure 2-4B:

$$C_T = C_1 + C_2 = 5 + 20 = 25\mu F$$

Figure 2-4C:

The series combination of the 6µF and 3µF capacitors is:

$$C = \frac{(6)(3)}{6+3} = \frac{18}{9} = 2\mu F$$

The equivalent 2µF capacitor is connected in parallel with the 8µF capacitor, giving a total capacitance of:

$$C_T = 2\mu F + 8\mu F = 10\mu F$$

One of the interesting characteristics of series-connected capacitors, such as those shown in Figure 2-5, is the way the source voltage divides between the capacitors once they are fully charged. Recall that $Q = CV$ (Eq. 2-1). Thus, the voltage across a capacitor must equal Q divided by C. Since the charging current is the same at all points in a series circuit, it follows that the charge is the same on each capacitor ($Q_T = Q_1 = Q_2 \ldots = Q_N$). Since the charges are equal, the voltage across each capacitor is determined only by the value of each capacitor, and the value of the source voltage (V_T). Also, since the capacitor voltage is inversely proportional to the capacitance, the smallest capacitor will have the largest voltage across it. Specifically, the formula for **capacitive voltage division** is:

$$V_N = \frac{V_T\ (C_T)}{C_N} \qquad \text{(Eq. 2-6)}$$

where:
V_T = source voltage
C_T = total capacitance of the series connected capacitors
C_N = any capacitor in the series string.

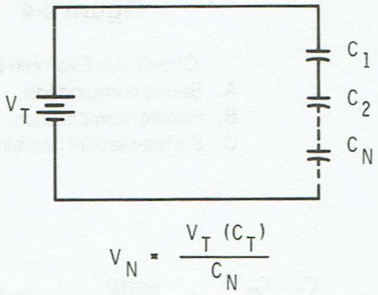


Figure 2-5

Capacitive voltage division.

Example 2-2

Calculate the voltage across each capacitor in Figure 2-6.

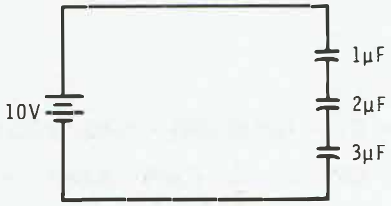


Figure 2-6

Circuit for Example 2-2.

You must first compute the total capacitance (C_T), and then apply the formula for capacitive voltage division.

$$\frac{1}{C_T} = \frac{1}{1} + \frac{1}{2} = \frac{1}{3}$$

$$\frac{1}{C_T} = \frac{11}{6}$$

$$C_T = \frac{6}{11} = 0.546\mu F$$

Therefore:

$$V_{1\mu F} = \frac{0.546\,(10^{-6})\,(10)}{1\,(10^{-6})} = 5.46V$$

$$V_{2\mu F} = \frac{0.546\,(10^{-6})\,(10)}{2\,(10^{-6})} = 2.73V$$

$$V_{3\mu F} = \frac{0.546\,(10^{-6})\,(10)}{3\,(10^{-6})} = 1.82V$$

Incidentally, the amount of **energy** stored in a capacitor is given by:

$$W = \frac{1}{2} CV^2 \qquad \text{(Eq. 2-7)}$$

where: W = energy stored in joules (J)
C = capacitance in farads (F)
V = voltage in volts (V)
Q = charge stored in coulombs

Example 2-3

Calculate the charge and energy stored in each capacitor in Example 2-2.

1μF capacitor

$$Q = CV = 1\mu F\,(5.46V) = 5.46\mu \text{ coulombs}$$
$$W = \frac{1}{2}\,CV^2 = \frac{1}{2}\,(1\mu F)\,(5.46)^2 = 14.9\mu J$$

2μF capacitor

$$Q = CV = 2\mu F\,(2.73V) = 5.46\mu \text{ coulombs}$$
$$W = \frac{1}{2}\,CV^2 = \frac{1}{2}\,(2\mu F)\,(2.73)^2 = 7.45\mu J$$

3μF capacitor

$$Q = CV = 3\mu F\,(1.82V) = 5.46\ \mu \text{ coulombs}$$
$$W = \frac{1}{2}\,CV^2 = \frac{1}{2}\,(3\mu F)\,(1.82)^2 = 4.97\mu J$$

Note that the smallest capacitor (1μF) stores the most energy (14.9 μJ). Also note that each capacitor stores the same charge (5.46μ coulombs).

RC Switching Circuits

Earlier, you saw that abrupt changes in currents and voltages can occur when a switch is opened or closed. In real circuits, instantaneous voltage changes cannot occur because real circuits always contain some capacitance. Consider, for example, two parallel wires separated by air. Since the wires are conductors and air is a dielectric, you have capacitance — whether you want it or not! In most applications such "stray capacitance" is usually, but not always, small enough to neglect. In real circuits the voltages may change very rapidly, but not instantaneously.

Earlier, we briefly outlined how a capacitor charges when it is connected to a DC voltage source. Recall that the initial capacitor voltage, just before the switch was closed ($t = 0^-$), was zero volts. Once the switch was closed ($t = 0$), you learned that the capacitor voltage built up until it equaled the source voltage (V). Similarly, during the time it took for the capacitor voltage to rise from zero volts to V volts, the current decreased from some initial value to zero. We stressed the concept that the voltage across a capacitor cannot change instantaneously. Based on this fact, and our discussion of how a capacitor charges from a DC voltage source, you can deduce the following important characteristics of capacitors in DC circuits:

1. Since capacitor voltage cannot change instantaneously:

   The value of the capacitor voltage at $t = 0^-$ (the instant just before the switch is closed) must equal the value of the capacitor voltage at $t = 0^+$ (the instant just after the switch is closed).

   Clearly, the voltage across an uncharged capacitor is zero volts. Thus, the capacitor voltage must also be zero volts the instant just after the switch is closed. This means **the capacitor acts like a short circuit at t = 0⁺**, as shown in Figure 2-7 B. Also, since the capacitor voltage is zero volts, the entire source voltage appears across R at $t = 0^+$. Thus the **initial values** are:

$$\left.\begin{aligned} I &= \frac{V}{R} \\ V_C &= 0V \end{aligned}\right\} \quad t = 0^+ \text{ (RC charging circuit)}$$

2. Once the capacitor is fully charged ($V_C = V$), the current has decreased to zero. Theoretically, it takes an infinite amount of time for the capacitor to become "fully charged". For this reason, we will say the capacitor is fully charged at $t = \infty$.

   Since no current flows, **the capacitor acts like an open circuit at $t = \infty$**, as illustrated in Figure 2-7C. Thus the final values are:

$$\left.\begin{array}{l} I = 0 \\ \\ V_C = V \end{array}\right\} \quad t = \infty \text{ (RC charging circuit)}$$

Figures 2-7D and 2-7E illustrate how the capacitor current and voltage change with time in an RC charging circuit. Curves like these are called **exponential** curves. We will have more to say about this shortly, but first we'll look at the RC discharging circuit.

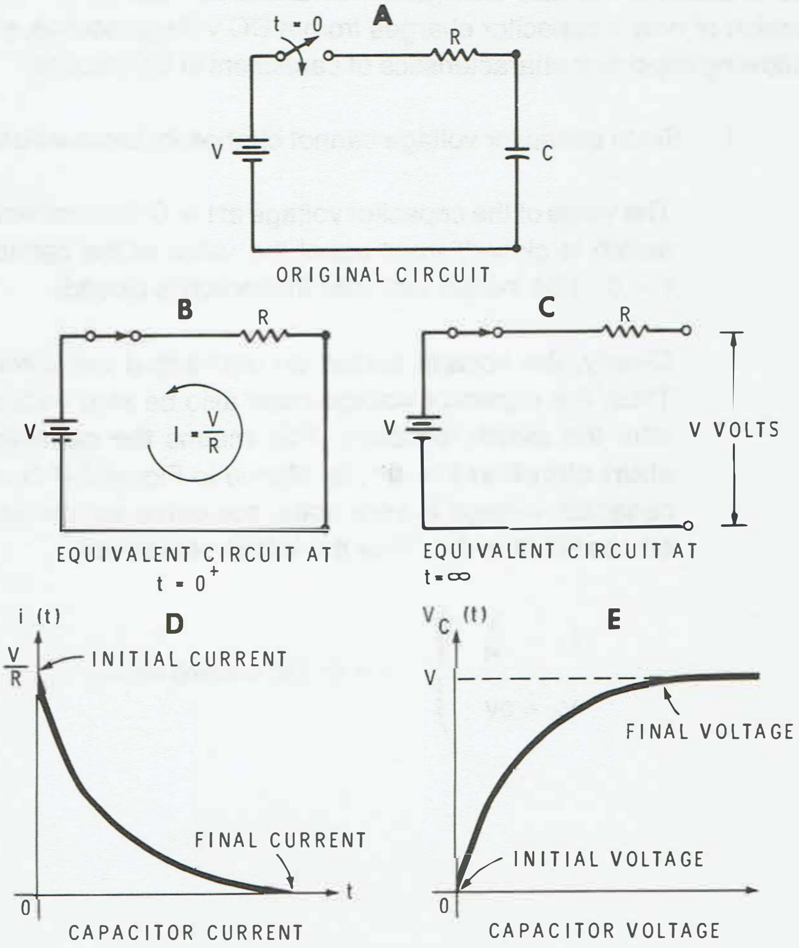


Figure 2-7

The RC charging circuit.

A. Original circuit.
B. Equivalent circuit at $t = 0^+$.
C. Equivalent circuit at $t = \infty$.
D. Capacitor current.
E. Capacitor voltage.

Recall that when a resistor is connected across a charged capacitor (via a switch), current flows from the negative plate to the positive plate until each plate is electrically neutral. Since the capacitor was charged to some voltage (V_o) before the switch is closed, it follows that the capacitor voltage at $t = 0^+$ must also be V_o volts. Thus:

> **The capacitor acts like a voltage source of V_o volts at $t = 0^+$**, as shown in Figure 2-8B. Also, since the resistor is connected across the capacitor, the voltage across the resistor is also V_o volts at $t = 0^+$. Thus, the **initial values** are:

$$\left.\begin{aligned} I &= \frac{V_O}{R} \\ V_C &= V_O \end{aligned}\right\} \quad t = 0^+ \text{ (RC discharging circuit)}$$

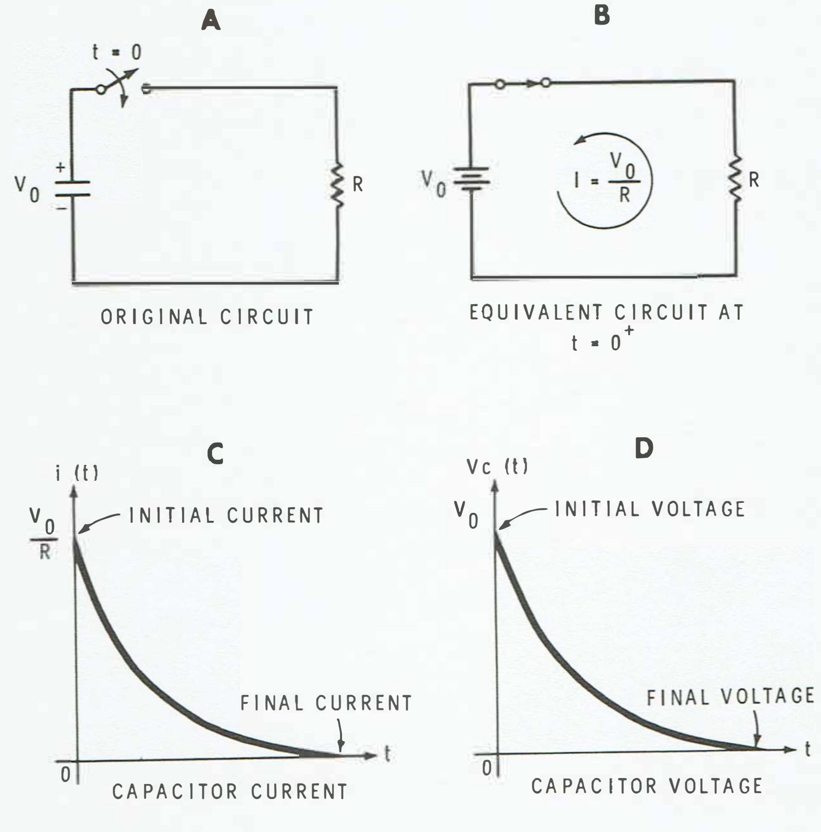


Figure 2-8

The RC discharging circuit.
A. Original circuit.
B. Equivalent circuit at $t = 0^+$.
C. Capacitor current.
D. Capacitor voltage.

Once each plate is electrically neutral, the current and voltage are also zero. Theoretically, an infinite amount of time is required for a capacitor to "completely discharge". Thus, we will say the capacitor is completely discharged at t = ∞. Therefore, the **final values** are:

$$\left.\begin{aligned} I &= 0 \\ V &= 0 \end{aligned}\right\} \quad t = \infty \text{ (RC discharging circuit)}$$

The capacitor current and voltage curves for the RC discharging circuit are given in Figure 2-8 C and Figure 2-8 D respectively. Like the RC charging circuit, these curves are also exponential curves. As a matter of fact, the curve for any current or voltage in a single loop RC circuit, driven from a DC source, will always be exponential. **Thus, every current or voltage starts at some initial value, and then exponentially rises or decays to some final value.**

The following equations were derived using calculus. They describe how an exponential current or voltage rises or decays from the intial value to the final value.

$$i(t) = (I_1 - I_2)\, e^{-t/\tau} + I_2 \qquad \text{(Eq. 2-8)}$$

$$v(t) = (V_1 - V_2)\, e^{-t/\tau} + V_2 \qquad \text{(Eq. 2-9)}$$

where: I_1, V_1 = initial current or voltage
I_2, V_2 = final current or voltage
t = time
e = base of the natural logarithms. This is simply a number which is approximately equal to 2.718
τ = RC and is called the **time constant**.

Recall that it takes an infinite amount of time for a capacitor to "completely" charge or discharge. This is, of course, longer than most experimenters would care to wait before continuing with an experiment! From a practical point of view, then, how long does it take for a capacitor to effectively charge or discharge? To answer this question let's consider two **unit exponential functions**, namely:

$1 - e^{-t/\tau}$ (A) Which has an initial value of 0, and a final value of 1 (rising exponential)

and

$e^{-t/\tau}$ (B) Which has an initial value of 1, and a final value of 0 (decaying exponential).

By substituting different values of t into the two functions, we can predict their values at different times. We will do this for values of t that are multiples of the time constant (τ). For example, after a period of time equal to one time constant we have:

$$1 - e^{-t/\tau} = 1 - e^{-\tau/\tau} = 1 - e^{-1} = 1 - \frac{1}{2.718}$$

$$= 0.632 = 63.2\%$$

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} = \frac{1}{2.718} = 0.368 = 36.8\%$$

Thus, after a period of time equal to one time constant, the rising exponential has reached 63.2% of its final value. Similarly, the decaying exponential has decayed to 36.8% of its initial value. By substituting values of t equal to 2τ, 3τ, 4τ, and 5τ, you can obtain the data given in Table 2-1.

Time in time constants	% rise	% decay
1	63.2	36.8
2	86.5	13.5
3	95	5
4	98.2	1.8
5	99.3	0.7

Table 2-1

% Rise and decay of unit exponential functions.

Table 2-1 states that, after a period of time equal to **five** time constants, the rising exponential has reached 99.3% of its final value. Similarly, the decaying exponential has decayed to 0.7% of the initial value. **Thus, for practical purposes we will assume a capacitor is fully charged, or discharged, after a period of time equal to five time constants**.

The data in Table 2-1 is plotted in Figure 2-9. Since we have expressed time in multiples of time constants, and the value of each function as a percentage, the curves in Figure 2-9 are normalized. This means you can use these curves to predict the current or voltage in any single-loop RC circuit. Incidentally, once the currents and voltages have reached their final values, they are called **steady-state** responses. The currents and voltages in a circuit before the steady-state condition has been reached are called **transient** responses. As you now know, the transient condition lasts for a period of time equal to five time constants. The following examples will help clarify the ideas just discussed.

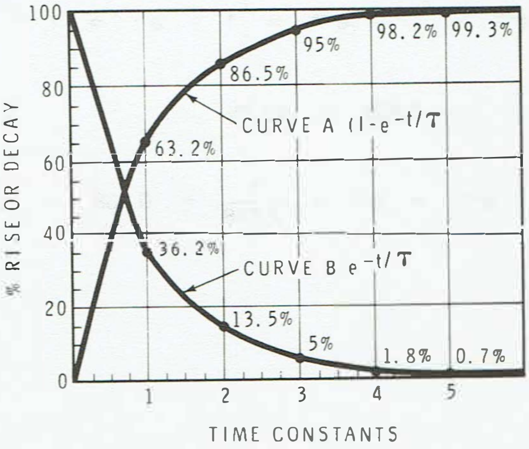


Figure 2-9

Normalized exponential curves.

Example 2-4

The capacitor in Figure 2-10 has 0V across it before the switch is closed. Determine:

- *(a) The voltage across and the current through the capacitor just after the switch is closed.*
- *(b) The final values of capacitor current and voltage.*
- *(c) The approximate amount of time it takes to reach the final values once the switch is closed.*
- *(d) The value of the capacitor current and voltage two seconds after the switch is closed.*
- *(e) Equations that describe the capacitor current and voltage once the switch is closed.*
- *(f) The value of the capacitor current and voltage one-tenth of a second after the switch is closed.*

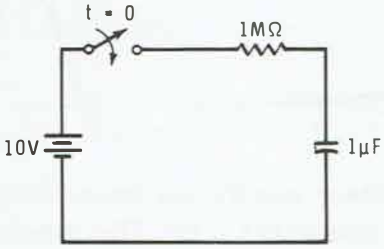


Figure 2-10

Circuit for Example 2-4.

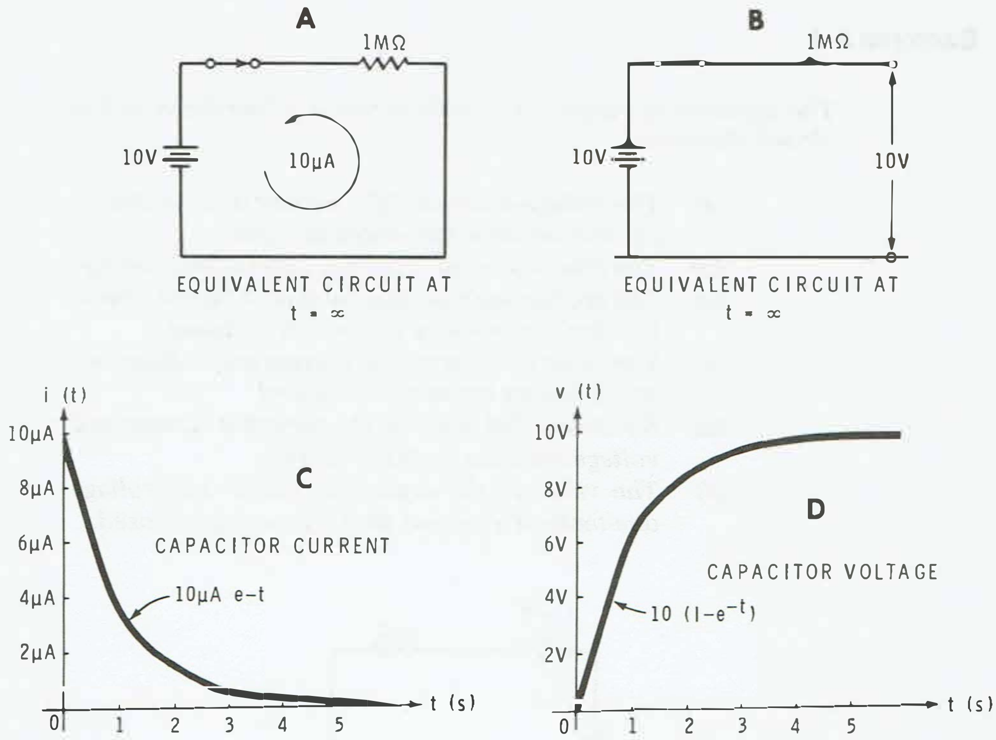


Figure 2-11

Analysis of the circuit in Figure 2-10.
- A. Equivalent circuit at $t = 0^+$.
- B. Equivalent circuit at $t = \infty$.
- C. Capacitor current.
- D. Capacitor voltage.

(a) Since the capacitor voltage was 0V just before the switch was closed, it is 0V just after the switch is closed at $t = 0^+$. This means the capacitor acts like a short circuit, as shown in Figure 2-11A.

Thus:
$$I_1 = \text{initial current} = \frac{10\text{V}}{10\text{M}\Omega} = 10\mu\text{A}$$
$$V_1 = \text{initial voltage} = 0\text{V}$$

(b) Once the capacitor is fully charged, the current is 0. The equivalent circuit is shown in Figure 2-11 B. The final values are, therefore:

$$I_2 = \text{final current} = 0$$
$$V_2 = \text{final capacitor voltage} = 10\text{V}$$

(c) Theoretically, an infinite amount of time is required for the capacitor to completely charge to the source voltage (10V). Practically, however, you can assume the capacitor is completely charged after a period of time equal to five time constants has elapsed. Thus:

$$\tau = RC = 1\text{M}\Omega\,(1\mu\text{F}) = 1\text{s}$$

$5\tau = 5\,(1\text{s}) = 5\text{s}$, which is the approximate amount of time for the current and voltage to reach their final values.

(d) In this case, 2s corresponds to two time constants. From Table 2-1 or Figure 2-9 you can see that the current has decreased to 13.5% of its initial value, and the voltage has risen to 86.5% of its final value. Therefore:

$$\begin{aligned} i\,(2s) &= 0.135\,(10\mu A) = 1.35\mu A \\ \nu_c\,(2s) &= 0.865\,(10V) = 8.65V \end{aligned}$$

The notation i (2s) and ν_c (2s) means the value of i and ν_c at t = 2s.

(e) Since you have already calculated the initial values, final values, and time constant, you can use Equations 2-8 and 2-9 to describe the responses. Thus:

$$\begin{aligned} i(t) &= (I_1 - I_2)\,e^{-t/\tau} + I_2 \\ &= (10\mu A - 0)\,e^{-t/1} + 0 \\ &= 10\mu A\,e^{-t} \end{aligned}$$

$$\begin{aligned} \nu_c\,(t) &= (V_1 - V_2)\,e^{-t/\tau} + V_2 \\ &= (0 - 10)\,e^{-t/1} + 10 \\ &= 10 - 10e^{-t} \\ &= 10\,(1 - e^{-t}) \end{aligned}$$

These responses are shown in Figure 2-11 C and Figure 2-11 D respectively.

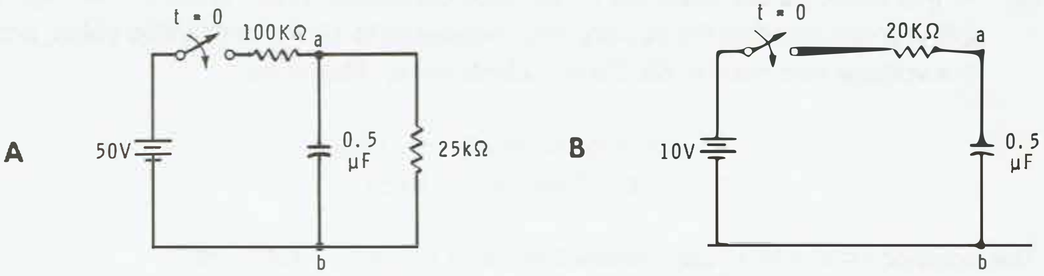
(f) Since 1/10 second is a small portion of the time constant (1s), you cannot read the normalized curves in Figure 2-9 accurately as you did in part (d). In such cases, it is much easier to substitute the value of time (t) directly into the equations for the current and voltage. Thus:

$$\begin{aligned} i\,(0.1s) &= 10\mu A\,e^{-0.1} \\ &= 10\mu A\,(2.718)^{-0.1} \\ &= 10\mu A\,(0.9048) \\ &= 9.048\,\mu A \end{aligned}$$

$$\begin{aligned} \nu_c(0.1s) &= 10\,(1 - e^{-0.1}) \\ &= 10\,(1 - .9048) \\ &= 10\,(0.0952) \\ &= 0.952\,V \end{aligned}$$

If you have a pocket calculator with a y^x function, the above calculations are easily done.

Figure 2-12



ORIGINAL CIRCUIT

THEVENIN EQUIVALENT CIRCUIT

Example 2-5

> *The capacitor in Figure 2-12 A is initially uncharged. The switch is closed at t = 0 and then opened 60 ms later. Sketch the capacitor voltage from t = 0 to t = 150 ms.*

We will divide the problem into two parts. When the switch is closed, the capacitor will begin charging to some voltage. We will examine this case first. Next, we will look at what happens when the switch is opened.

To begin, the circuit in Figure 2-12 A is more complex than the charging circuit in the previous example because it is not a single-loop circuit. However, by considering the capacitor as the load, you can reduce the original circuit to an equivalent single-loop circuit with Thevenin's theorem! Thus, removing the load (C), the open circuit voltage is the voltage across the 25kΩ resistor. The Thevenin voltage is, therefore:

$$V_{TH} = \frac{50V\,(25k\Omega)}{125k\Omega} = 10V$$

Similarly, once the load (C) is removed and the 50V source is reduced to zero, the resistance between terminals a and b equals the parallel combination of the 25 kΩ and 100 kΩ resistors. The Thevenin resistance is, therefore:

$$R_{TH} = 25k\Omega \parallel 100k\Omega = 20k\Omega$$

Figure 2-12 B illustrates the resulting single-loop equivalent circuit.

Since the capacitor is initially uncharged, the initial capacitor voltage just after the switch is closed ($t = 0^+$) is zero. Once the switch is closed, the capacitor begins charging to the **Thevenin equivalent voltage**, which in this example is 10V. The **charging time constant** is determined by the capacitance (C) and Thevenin resistance (R_{TH}). Thus:

$$\tau = R_{TH}\,C = 20k\Omega\,(0.5\mu F) = 10\text{ ms}$$

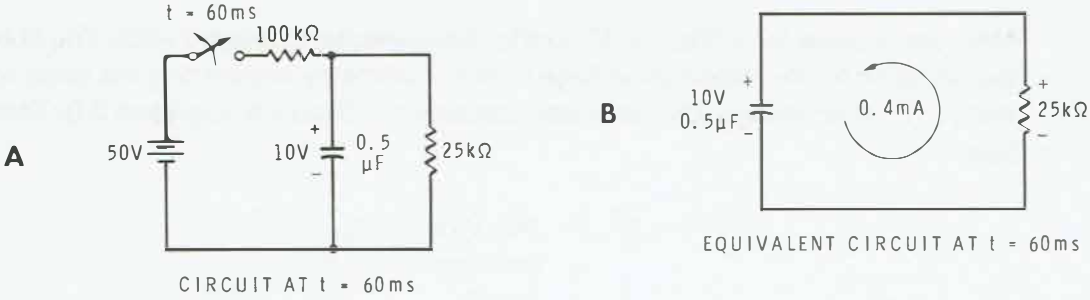


Figure 2-13

Circuits for Example 2-5 (Part two).
A. Circuit at t = 60 ms.
B. Equivalent circuit at t = 60 ms.

After a period of time equal to five time constants (50ms) has elapsed, you can assume that the capacitor is fully charged. Since the switch remains closed for 60ms, you can feel confident that the capacitor voltage has reached the final value of 10V. To derive an equation that describes the capacitor voltage from t = 0 to t = 60ms, all you must do is substitute the initial voltage (0V), final voltage (10V), and time constant (10ms) into Equation 2-9. Thus:

$$\begin{aligned}\nu_c(t) &= (V_1 - V_2)\, e^{-t/\tau} + V_2 \\ &= (0 - 10)\, e^{-t/10\text{ms}} + 10 \\ &= 10 - 10\, e^{-100t} \\ &= 10\,(1 - e^{-100t})\end{aligned}$$

Now let's solve the second part of the problem. At t = 60ms, you found that the capacitor voltage is 10V. At this time, the switch is opened. Once the switch is opened, the 50V source and 100kΩ resistor are effectively removed from the circuit. Therefore, you have a 0.5 μF capacitor, charged to 10V, that is suddenly connected across a 25kΩ resistor, as shown in Figure 2-13 A. By opening the switch you have changed the circuit from an RC charging circuit to an RC discharging circuit, as shown in Figure 2-13 B. Starting at t = 60ms, then, the capacitor voltage will decay from an initial value of 10V to a final value of 0V. Notice that once the switch is opened the only resistance in the loop is the 25kΩ resistor. Thus, the discharging time constant is not the same as the charging time constant. For the discharging circuit:

$$\tau = RC = 25\text{k}\Omega\,(0.5\mu\text{F}) = 12.5\text{ ms}$$

Naturally, it takes 5(12.5ms) or 62.5ms for the capacitor to reach 0 volts. You obtain the equation for the capacitor voltage from t = 60ms by substituting the initial voltage (10V), final voltage (0V), and time constant (12.5ms) into Equation 2-9. Therefore:

$$\begin{aligned} \nu_c(t) &= (V_1 - V_2)\, e^{-t/\tau} + V_2 \\ &= (10 - 0)\, e^{-t/12.5\text{ms}} + 0 \\ &= 10\, e^{-80t} \end{aligned}$$

The capacitor voltage from t = 0 to t = 150ms is shown in Figure 2-14.

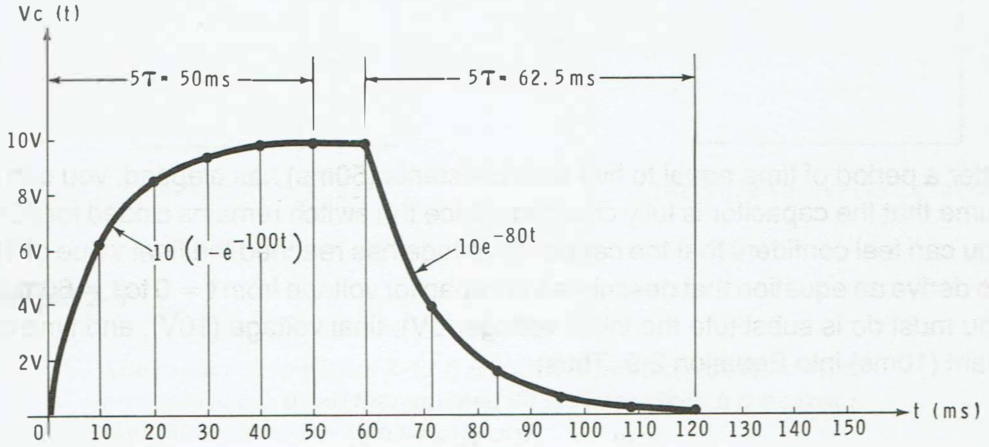


Figure 2-14

Capacitor voltage versus time for Example 2-5.

Self Test Review

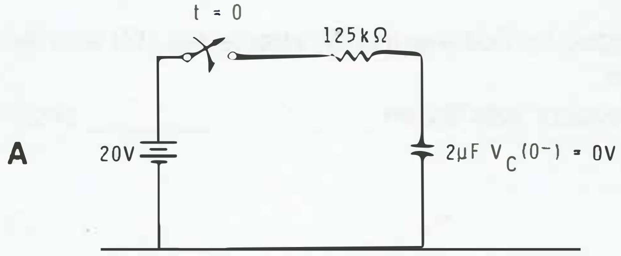
1. The instant just before a switch is thrown is t = ________________ .
2. The instant just after a switch is thrown is t = ________________ .
3. A capacitor consists of two conductors separated by an insulator called a ________________ .
4. A 2μF capacitor with a voltage of 10V stores ____________ μ coulombs of charge. The energy stored in this capacitor is ____________ μJ.
5. Two 10μF capacitors connected in series have a total capacitance of _______ μF. If the capacitors were connected in parallel, the total capacitance would be _______ μF.
6. The voltage across the series combination of a 2μF and 8μF capacitor is 20V. The voltage across the 2μF capacitor is ______ V.
7. Theoretically, it takes an ________________ amount of time for a capacitor to completely charge, or discharge.
8. Practically, you can assume a capacitor has completely charged or discharged after a period of time equal to ________________ time constants has elapsed.
9. At $t = 0^+$ a capacitor that was initially uncharged acts like a ____________ circuit.
10. At $t = 0^+$ a capacitor that was initially charged to 10V acts like a ________ voltage source.
11. At $t = \infty$, a capacitor acts like an ________________ circuit.

Refer to Figure 2-15 A for questions 12 to 14.

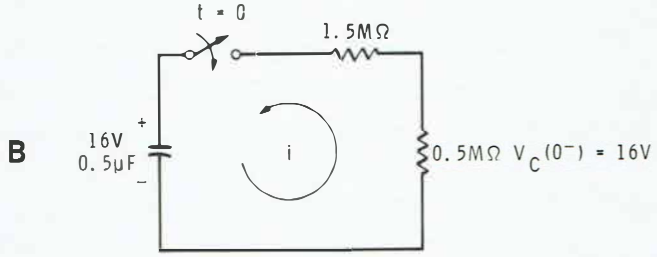
12. The capacitor voltage will equal 20V ________________ seconds after the switch is closed.
13. An equation that describes the capacitor voltage once the switch is closed is ________________.
14. An equation that describes the capacitor current once the switch is closed is ________________.

Refer to Figure 2-15 B for questions 15 to 18.

15. At $t = 0^+$ the current (i) is ________________ μA.
16. Approximately ________________ seconds after the switch is closed, $i = 0$.
17. An equation that describes the capacitor current once the switch is closed is ________________.
18. An equation that describes the capacitor voltage once the switch is closed is ________________.



QUESTIONS 12 TO 14



QUESTIONS 15 TO 18

Figure 2-15

Circuits for Self Test Review.
A. Questions 12 to 14.
B. Questions 15 to 18.

Answers

1. 0^-
2. 0^+
3. dielectric
4. $Q = CV = 2\mu F(10V)$
   $Q = 20\mu$ coulombs
   $W = 1/2\ CV^2$
   $W = 1/2\ (2\mu F)\ (10)^2$
   $W = 100\mu J$
5. $C_T\text{(series)} = \frac{10\mu F}{2} = 5\mu F$
   $C_T\text{(parallel)} = 10 + 10 = 20\mu F$
6. $C_T = \frac{2\ (8)}{10} = 1.6\mu F$
   $V_{2\mu F} = \frac{20V\ (1.6)}{2} = 16V$
7. infinite
8. five
9. short
10. 10V
11. open
12. 1.25
13. $20(1 - e^{-4t})$
14. $160\mu A\ e^{-4t}$
15. $8\mu A$
16. 5
17. $8\mu A\ e^{-t}$
18. $16\ e^{-t}$

The solutions to questions 12 to 18 follow:

12. $\tau = RC = 125k\Omega\ (2\mu F) = 0.25s$
    $5\tau = 5(0.25) = 1.25s$

13. $V_1 = 0V$, $V_2 = 20V$, and $\tau = 0.25s$. Substituting these values into Equation (2-9) yields:

$$\begin{aligned} v_c(t) &= (V_1 - V_2)\ e^{-t/\tau} + V_2 \\ &= (0 - 20)\ e^{-t/0.25} + 20 \\ &= 20 - 20\ e^{-4t} \\ &= 20\ (1 - e^{-4t}) \end{aligned}$$

14. At $t = 0^+$, C acts like a short circuit. Therefore:

$$I_1 = \frac{20V}{125k\Omega} = 0.16mA = 160\mu A$$

At $t = \infty$, C acts like an open circuit. Therefore
$I_2 = 0$
$\tau = 0.25s$ from question 12
Substituting the above values into Equation 2-8 yields:

$$\begin{aligned} i(t) &= (I_1 - I_2)\ e^{-t/\tau} + I_2 \\ &= (160\mu A - 0)\ e^{-t/0.25s} + 0 \\ &= 160\mu A\ e^{-4t} \end{aligned}$$

15. At t = 0^+, C acts like a 16V voltage source. Therefore:

$$I_1 = \frac{16V}{2M\Omega} = 8\mu A$$

16. $\tau = RC = 2M\Omega\,(0.5\mu F) = 1s$
$5\tau = 5(1s) = 5s$

17. Substituting $I_1 = 8\mu A$, $I_2 = 0$, and $\tau = 1s$ into Equation 2-8 yields:

$$\begin{aligned} i_C(t) &= (I_1 - I_2)\,e^{-t/\tau} + I_2 \\ &= (8\mu A - 0)\,e^{-t/1} + 0 \\ &= 8\mu A\,e^{-t} \end{aligned}$$

18. Substituting $V_1 = 16V$, $V_2 = 0V$, and $\tau = 1s$ into Equation 2-9 yields:

$$\begin{aligned} \nu(t) &= (V_1 - V_2)\,e^{-t/\tau} + V_2 \\ &= (16 - 0)\,e^{-t/1} + 0 \\ &= 16\,e^{-t} \end{aligned}$$

RL SWITCHING CIRCUITS

Inductors, like capacitors, have a number of DC characteristics that you should be aware of. In this section you will see what affect an abrupt current change has in an inductive circuit. In addition, you will learn how to describe mathematically the current through and voltage across an inductor in a DC circuit. As with capacitors, specific practical applications for inductors will be provided in a later unit.

Inductance

A component exhibits the property of **inductance** when it **opposes a change in the current through it**. Components that are designed to exhibit this property are called **inductors**. Most inductors consist of a length of wire wound on a core to form a coil. For this reason, inductors are also called **coils.** This construction is suggested by the schematic symbol for an inductor, illustrated in Figure 2-16. To understand why a coil of wire exhibits the property of inductance, recall the following principles from basic electricity:

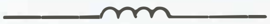


Figure 2-16

Schematic symbol for an inductor.

1. **A magnetic field is generated by moving a charge**. Since current is a moving charge, a magnetic field exists in the space surrounding a current carrying conductor.
2. **Whenever there is relative motion between a magnetic field and conductor, a voltage is induced in the conductor**. The term relative motion means that the conductor can physically move through a stationary magnetic field, or that magnetic lines of force from a moving magnetic field "cut" the conductor. The only restriction is that the magnetic lines of force must not be parallel to the conductor. In this case, the induced voltage is zero.
3. **For a given conductor the amount of induced voltage depends upon how rapidly the magnetic lines of force cut the conductor**. If the relative motion between a conductor and magnetic field is great, the induced voltage will be great and vice versa. Also, if there is no relative motion between the conductor and magnetic field, the induced voltage is zero.
4. **The polarity of the induced voltage is such that it will oppose the motion or change that induced it**. This principle is called Lenz's law. We will discuss it shortly.

Figure 2-17 illustrates what happens when an inductor and series resistance are connected to a DC voltage source. In Figure 2-17 A the switch is open. Since there is no current flow, there is no magnetic field surrounding the conductors. Once the switch is closed, as shown in Figure 2-17 B, the voltage source (V) begins to force current into the circuit. The instant current begins to flow, a magnetic field is generated. As the voltage source forces more and more current into the circuit, the magnetic field expands outward from the center of each conductor.

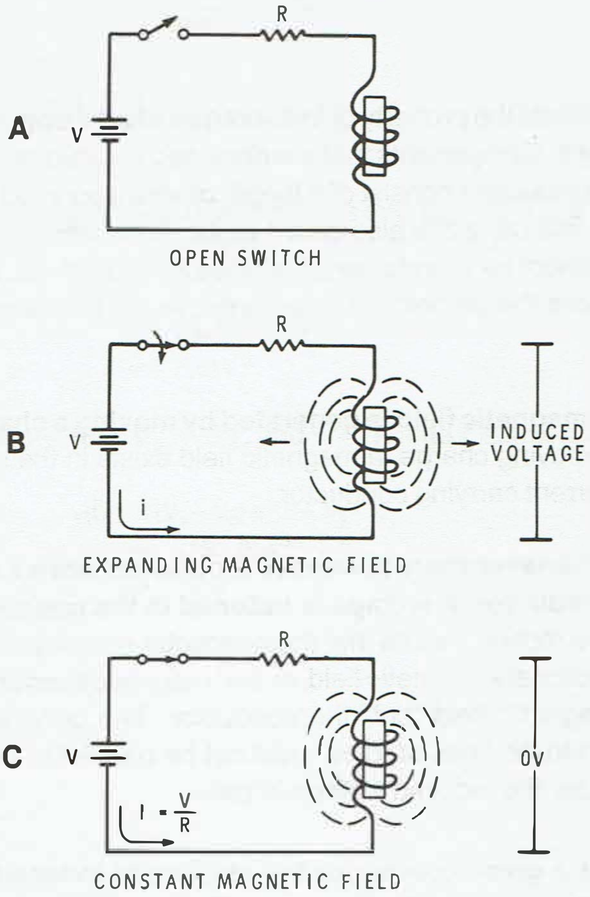


Figure 2-17

Self-induction due to an expanding magnetic field.

A. Open switch.
B. Expanding magnetic field.
C. Constant magnetic field.

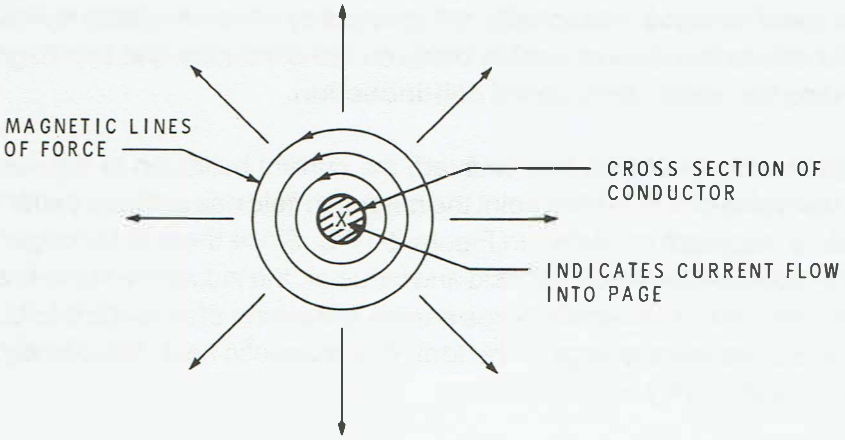


Figure 2-18

The magnetic field surrounding a wire through which the current is increasing.

Figure 2-18 illustrates the expanding magnetic field surrounding a wire. If you have ever dropped a stone into a pond, you can visualize the expanding magnetic lines of force surrounding a wire, since they are analogous to the ripples in the water.

Notice the shape of the magnetic field surrounding the inductor in Figure 2-17 B. Since the inductor consists of many turns of wire wound on a core, the magnetic field surrounding each turn adds to the magnetic fields of adjacent turns. For this reason, the net magnetic field surrounding the inductor is strengthened and is similar to the magnetic field surrounding a bar magnet. In any event, once the magnetic field starts expanding, relative motion exists between the magnetic field and conductors.

As a result of this relative motion, a voltage is induced in the conductors. Experimentally, it has been shown that the induced voltage increases directly with the number of turns. For this reason, the voltage induced in the inductor is much larger than the voltage induced in the wires. For our purposes, we can neglect the voltage induced in the wires, although you should realize that even a short length of wire will exhibit some inductance. Recall that Lenz's law states that the polarity of the induced voltage is such that it opposes the motion or change that induced it. Since the expanding magnetic field was the result of an increasing current, the polarity of the induced voltage is such that it trys to prevent the current from increasing. For this reason:

Current in an inductor cannot change instantaneously.

Since real circuits always contain some inductance, current changes may be very rapid, but not instantaneous. Incidentally, the process by which a voltage is induced in a conductor due to the relative motion between the conductor and the magnetic field surrounding the conductor is called **self-induction**.

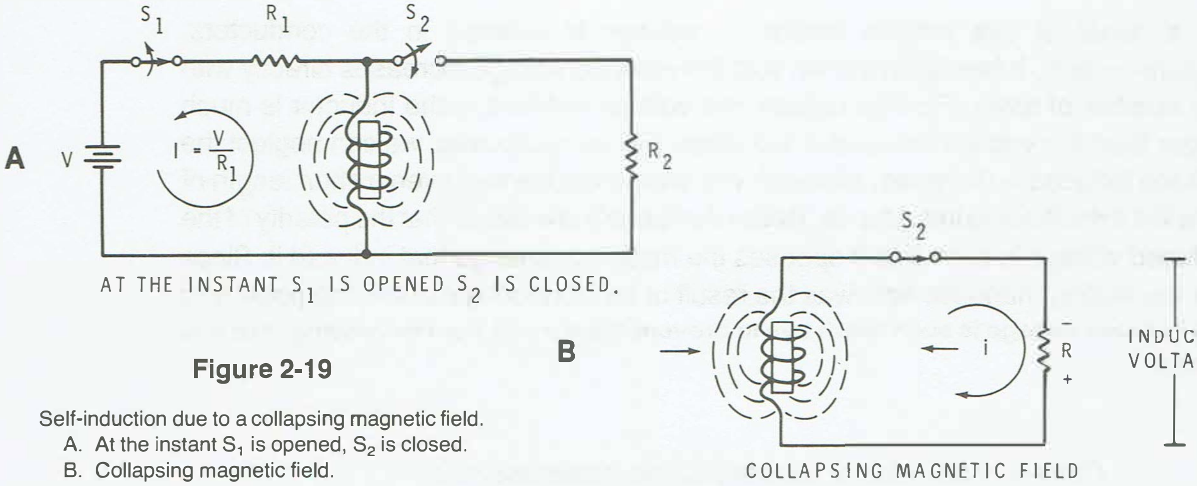
After a sufficient amount of time has elapsed, the current builds up to the steady-state Ohm's law value of V/R. At this point the magnetic field has stopped expanding and is, therefore, constant as shown in Figure 2-17 C. Since there is no longer any relative motion between the magnetic field and inductor, the induced voltage is zero. Earlier, you learned that a capacitor stores energy in the form of an electric field. In a similar way, inductors store energy in the form of a magnetic field. Specifically, the amount of energy stored by an inductor is:

$$W = \frac{1}{2} L I^2 \qquad \text{(Eq. 2-10)}$$

where: W = energy stored in joules (J)
L = inductance in Henries (H)
I = current in amperes (A)

As you can see, the units of inductance are Henries, in honor of Joseph Henry. A **Henry** is defined as the amount of inductance that induces one volt in a conductor when the current through the conductor is changing at the rate of one ampere per second. Typical inductors have inductance values in the µH to mH range, although inductances in unregulated power supplies may be several Henries or more.

Figure 2-19 illustrates how the energy stored in an inductor can be removed. In Figure 2-19A, we assume switch S_1 has been closed long enough for the inductor current to build up to the steady-state value of V/R_1. Switch S_2 is initially assumed to be open.



A
S1
R1
S2
V
I = V/R1
R2
AT THE INSTANT S1 IS OPENED S2 IS CLOSED.
B
S2
i
−
R
+
INDUCED VOLTAGE
COLLAPSING MAGNETIC FIELD

Figure 2-19

Self-induction due to a collapsing magnetic field.
A. At the instant S_1 is opened, S_2 is closed.
B. Collapsing magnetic field.

If we now open switch S_1 and simultaneously close switch S_2, the inductor current will attempt to decrease. As a result, the magnetic field surrounding the inductor starts to collapse and, therefore, induces a voltage in the inductor as shown in Figure 2-19B. Since the collapsing magnetic field was the result of a decreasing current, Lenz's law tells us that the polarity of the induced voltage is such that it trys to prevent the current from decreasing. Thus, current will continue to flow during the time the magnetic field is collapsing. After a sufficient amount of time has elapsed the magnetic field will be fully collapsed, and as a result, the current will be zero.

Inductors, like resistors and capacitors, may be connected in series, parallel, or series-parallel combinations. Such combinations are complicated by the fact that a changing magnetic field surrounding one inductor can induce a voltage in an adjacent inductor and vice versa. This **mutual induction** can alter the equivalent inductance of a particular combination of inductors. If two or more inductors are far enough apart, there is very little mutual induction. In these cases the following formulas are used to compute the equivalent inductance:

Series connection

$$L_T = L_1 + L_2 + \ldots L_N \qquad \text{(Eq. 2-11)}$$

Parallel connection

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots \frac{1}{L_N} \qquad \text{(Eq. 2-12)}$$

For two inductors connected in parallel Equation 2-12 reduces to:

$$L_T = \frac{L_1 \; L_2}{L_1 + L_2} \qquad \text{(Eq. 2-13)}$$

Similarly for n equal parallel inductors:

$$L_T = \frac{L}{n} \qquad \text{(Eq. 2-14)}$$

Incidentally, if the magnetic fields of two or more inductors do not interact, the inductors are said to be shielded from each other. As you can see, the relationships for computing total inductance for shielded inductors are the same as those used for computing total resistance.

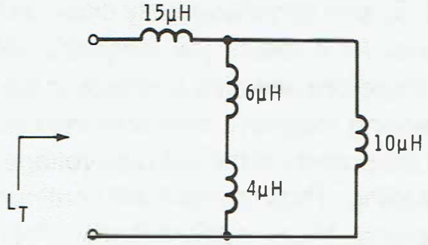


Figure 2-20

Circuit for Example 2-6.

Example 2-6

Calculate the total inductance of the shielded inductor combination in Figure 2-20.

The series combination of the 6μH and 4μH inductors is:

$$L = 6 + 4 = 10\mu H$$

The equivalent 10μH inductor is connected in parallel with 10μH giving a combined inductance of:

$$L = \frac{10}{2} = 5\mu H$$

Finally, the series combination of the equivalent 5μH, and 15μH inductors yields a total inductance of:

$$L_T = 5 + 15 = 20\mu H$$

RL Switching Circuits

In a manner similar to our analysis of RC switching circuits, we will examine the characteristics of the analogous RL switching circuits. To begin, consider the RL charging circuit in Figure 2-21A. Recall that inductor current, like capacitor voltage, cannot change instantaneously. Thus:

> The value of the inductor current at t = 0^-, the instant just before the switch is closed, must equal the value of the inductor current at t = 0^+, the instant just after the switch is closed.

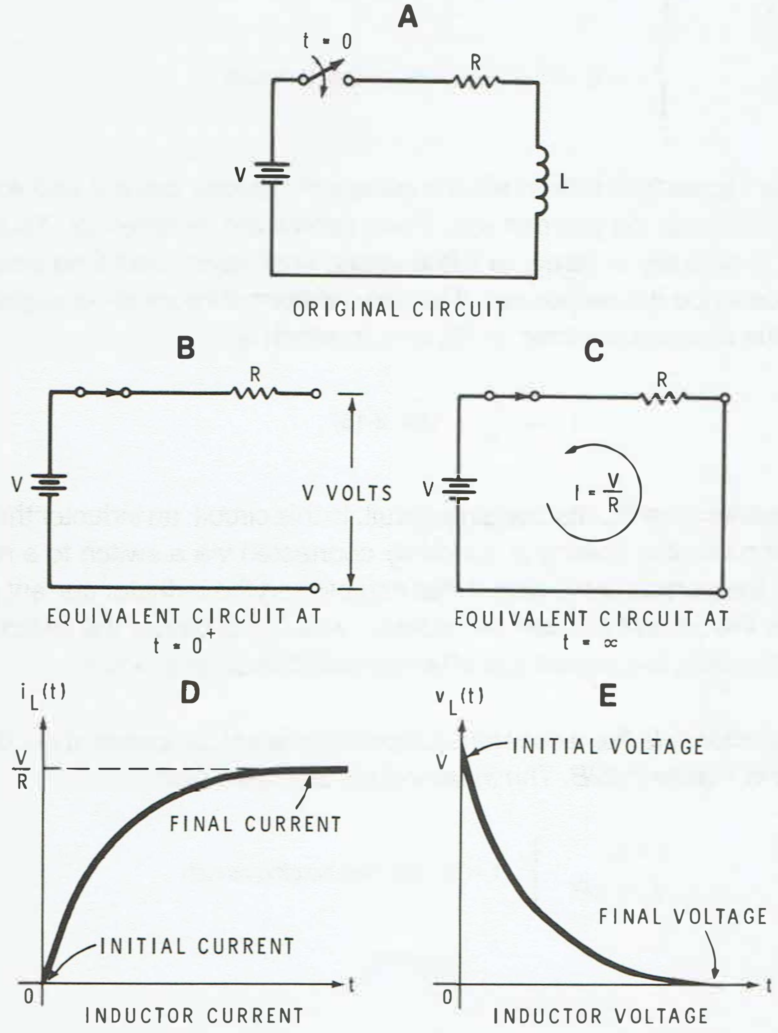


Figure 2-21

The RL charging circuit.
A. Original circuit.
B. Equivalent circuit at t = 0^+.
C. Equivalent circuit at t = ∞.
D. Inductor current.
E. Inductor voltage.

Since the inductor current in Figure 2-21A was zero just before the switch was closed, it must also be zero the instant just after the switch is closed. This means **the inductor acts like an open circuit at t = 0⁺**, as shown in Figure 2-21B. Thus, the initial values are:

$$\left.\begin{array}{l} I = 0 \\ V_L = V \end{array}\right\} t = 0^+ \text{ (RL charging circuit)}$$

As we discussed previously, once the inductor current builds up to the steady-state value of V/R, the inductor voltage has decayed to zero. Since the voltage across the inductor is zero, **the inductor acts like a short circuit at t** = ∞, as shown in Figure 2-21C. Thus, the final values are:

$$\left.\begin{array}{l} I = \dfrac{V}{R} \\ \\ V_L = 0 \end{array}\right\} t = \infty \quad \text{(RL charging circuit)}$$

Figure 2-21D and Figure 2-21E illustrate the curves of inductor current and voltage for the RL charging circuit. As you can see, these curves are exponential. Thus, the equations given previously in terms of initial value, final value, and time constant may be used to describe the responses. The only additional information required is the definition of the time constant for an RL circuit, which is:

$$\tau = \frac{L}{R} \quad \text{(Eq. 2-15)}$$

Figure 2-22A illustrates the RL discharging circuit. In this circuit, an inductor through which a constant current is flowing is suddenly connected via a switch to a resistance. For clarity, the portion of the circuit that established the inductor current (I_o) is not shown. Since the current through the inductor was I_o just before the switch was closed, it must also be I_o the instant just after the switch is closed. Thus:

> *The inductor acts like a constant current source of I_o amperes at t = 0⁺, as shown in Figure 2-22B. The initial values are therefore:*

$$\left.\begin{array}{l} I = I_o \\ V_L = IoR \end{array}\right\} t = 0^+ \text{ (RL discharging circuit)}$$

Once the magnetic field is completely collapsed the inductor voltage is zero. Therefore the final values are:

$$\left.\begin{array}{l} I = 0 \\ \\ V_L = 0 \end{array}\right\} \quad t = \infty \text{ (RL discharging circuit)}$$

Figure 2-22C and Figure 2-22D illustrate the curves of inductor current and voltage for the RL discharging circuit. As you might expect, the responses are exponential. As was the case with RC switching circuits, we will assume for practical purposes that all responses reach their steady-state values after a period of time equal to five time constants has elapsed.

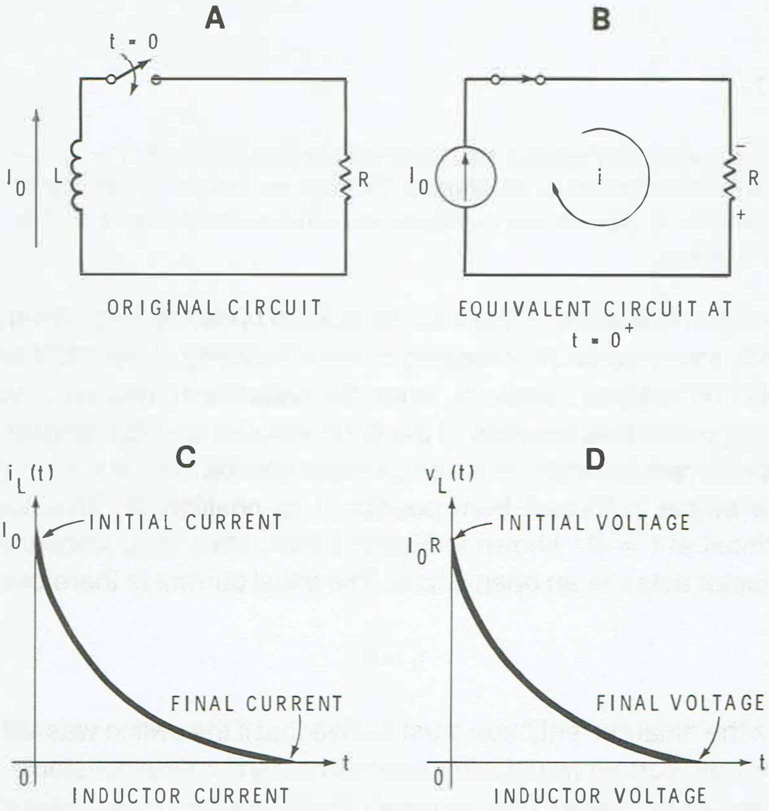


Figure 2-22

The RL discharging circuit.
A. Original circuit.
B. Equivalent circuit at $t = 0^+$.
C. Inductor current.
D. Inductor voltage.

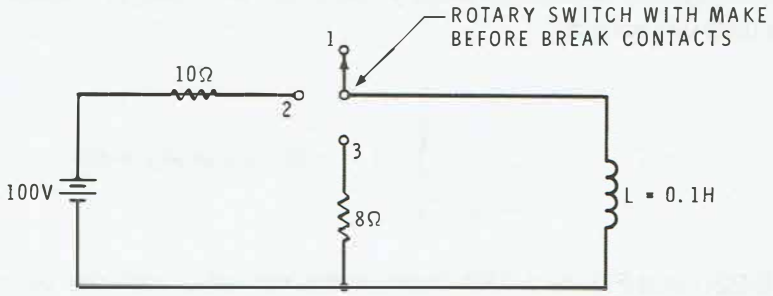


Figure 2-23

Circuit for Example 2-7.
The switch is thrown from position 1 to 2 at t = 0, and from position 2 to 3 at t = 20 ms.

Example 2-7

The switch in Figure 2-23 is initially in position 1. At t = 0, the switch is thrown to position 2. Twenty ms later it is thrown to position 3. Sketch the resulting inductor current from t = 0 to t = 90ms.

When the switch is in position 1, the inductor current is clearly zero. When the switch is in position 2, you have an RL charging circuit consisting of the 100V source, 10Ω resistor, and 0.1H inductor. Similarly, when the switch is in position 3, you have an RL discharging circuit that consists of the 0.1H inductor and 8Ω resistor. Since the initial inductor current (position 1) is zero, it must also be zero at $t = 0^+$ (the instant just after the switch is thrown from position 1 to position 2). This suggests the equivalent circuit at $t = 0^+$ shown in Figure 2-24A. Note that, since the current is zero, the inductor acts like an open circuit. The initial current is therefore:

$$I_1 = 0$$

To determine the "final current," you must realize that if the switch was left in position 2 long enough, the current would, after approximately five time constants, build up to the Ohm's law value of V/R. This suggests the equivalent circuit shown in Figure 2-24B. The final current is therefore:

$$I_2 = \frac{100V}{10\Omega} = 10A$$

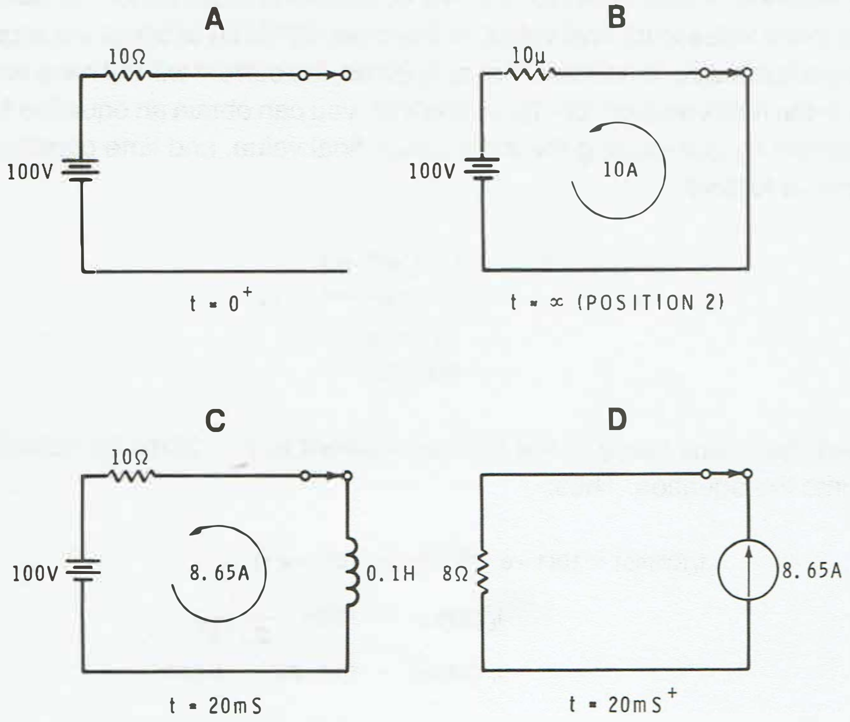


Figure 2-24

Equivalent circuits for Example 2-7.

A. $t = 0^+$.
B. $t = \infty$.
C. $t = 20$ ms.
D. $t = 20$ ms$^+$.

Note in Figure 2-24 B that the inductor acts like a short circuit since the current flowing through it is not changing. To write an equation that describes the inductor current for the RL charging circuit, you also need to know the charging time constant. Thus:

$$\tau = \frac{L}{R} = \frac{0.1\text{H}}{10\Omega} = 0.01\text{s} = 10\text{ms}.$$

Recall that a period of time equal to five time constants is required for the current to rise from its initial value to its final value. In this case, 5(10ms) or 50ms are required. Since the switch remains in position 2 for only 20ms, the current will not have enough time to reach the final value of 10A. Nevertheless, you can obtain an equation for the charging current by substituting the intial value, final value, and time constant into Equation 2-8 as follows:

$$\begin{aligned} i_L(t) &= I_1 - I_2 e^{-t/\tau} + I_2 \\ &= 0 - 10\, e^{-t/10\text{ms}} + 10 \\ &= 10 - 10\, e^{-100t} \\ &= 10(1 - e^{-100t}) \end{aligned}$$

You can find the actual value of the inductor current at t = 20ms by substituting 20ms for t into the equation. Thus:

$$\begin{aligned} i_L(20\text{ms}) = 10(1 - e^{-100(20\text{ms})}) &= 10(1 - e^{-2}) \\ i_L(20\text{ms}) &= 10(1 - \frac{1}{2.718^2}) \\ i_L(20\text{ms}) &= 10\,(0.865) = 8.65\text{A} \end{aligned}$$

Alternately, if you realized 20ms is two time constants, you could have looked at Figure 2-9, which shows that a rising exponential will reach 86.5% of its final value after two time constants. Since 86.5% of the final value (10A) is 8.65A, you can see that both our equation and Figure 2-9 predict the same value of inductor current — 20ms after the switch is thrown from position 1 to position 2. Figure 2-24C illustrates the equivalent circuit at t = 20ms.

At t = 20ms, the switch is thrown from position 2 to position 3. Since the inductor current was 8.65A at t = 20ms, it must also be 8.65A the instant just after the switch is thrown from position 2 to position 3. Thus, at t = 20ms$^+$, the inductor acts like a current source of 8.65A, as shown in Figure 2-24D. For the RL discharging circuit, then, the initial current is:

$$I_1 = 8.65\text{A}$$

When the switch is thrown from position 2 to position 3, the magnetic field begins to collapse. After approximately five time constants, the magnetic field is completely collapsed, resulting in a final inductor current of zero. Thus:

$$I_2 = 0$$

The discharging time constant is:

$$\tau = \frac{L}{R} = \frac{0.1\text{H}}{8\Omega} = 0.125\text{s} = 12.5\text{ms}$$

Substituting the intial value, final value, and time constant into Equation 2-8 provides an equation for the discharging current. Therefore:

$$\begin{aligned} i_L(t) &= (I_1 - I_2)\, e^{-t/\tau} + I_2 \\ &= (8.65 - 0)\, e^{-t/12.5ms} + 0 \\ &= 8.65\, e^{-80t} \end{aligned}$$

Figure 2-25 is a sketch of the inductor current from t = 0 to t = 90ms. The dashed line in Figure 2-25 reminds you that the charging equation $10(1 - e^{-100t})$ is only valid for the first 20ms. After 20ms, the discharging equation $8.65\, e^{-80t}$ describes the inductor current.

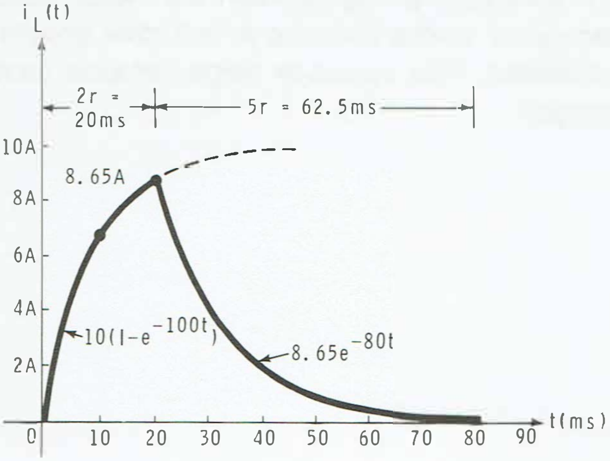


Figure 2-25

Sketch of inductor current verses time for Example 2-7.

Notice in Figure 2-23 that a particular type of (make-before-break) switch is specified. If a regular (break-before-make) switch was used, you would observe arcing between the switch contacts when the switch was thrown from position 2 to position 3. This happens because a very large voltage is induced in the inductor during the time the switch arm moves from position 2 to position 3. In fact, the induced voltage is so great that the air between the switch contacts becomes sufficiently ionized to conduct current!

To understand why this happens, recall that the current through an inductor cannot change instantaneously. When the switch arm leaves position 1, the charging circuit is suddenly opened. As a result, the magnetic field surrounding the inductor begins to collapse at a very rapid rate. The relative motion between the rapidly collapsing magnetic field and inductor induces a very large voltage in the inductor. From Lenz's law, you know that the polarity of the induced voltage is such that it will try to prevent the current from decreasing; hence the arcing between the switch contacts.

The make-before-break switch is constructed so that, when the switch is thrown from one position to another, the new connection is made before the old connection is broken. The use of this type of switch prevents the charging circuit in Figure 2-23 from suddenly being opened during the time the switch arm moves from position 2 to position 3, thus, it minimizes arcing between the switch contacts. Sometimes, capacitors are connected across switches in inductive circuits to also minimize potential arcing problems. This approach helps because capacitance opposes abrupt voltage changes.

Self Test Review

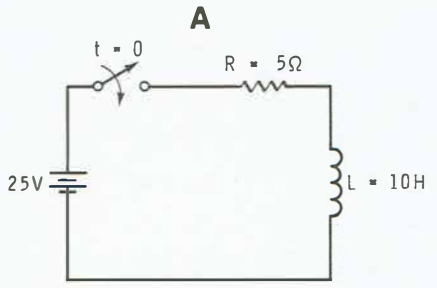
19. Another name for an inductor is a ________________.
20. The opposition to a change in current is ________________.
21. A 10mH inductor has a constant current of 2A flowing through it. The voltage across the inductor is ________________ volts. The energy stored in the inductor is ________________ mJ.
22. Three shielded 9μH inductors connected in series have a total inductance of ________________ μH. If the inductors were connected in parallel, the total inductance would be ________________ μH.
23. The time constant of an RL circuit is ________________.
24. In single-loop RL circuits, approximately ________________ time constants must elapse before the steady-state condition is reached.
25. At $t = 0^+$, an inductor that had an initial current of zero acts like an ________________ circuit.
26. At $t = 0^+$, an inductor that had an initial current of 20mA acts like a ________________ current source.
27. At $t = \infty$, an inductor in an RL charging circuit acts like a ________________ circuit.

Refer to Figure 2-26A for questions 28 to 30

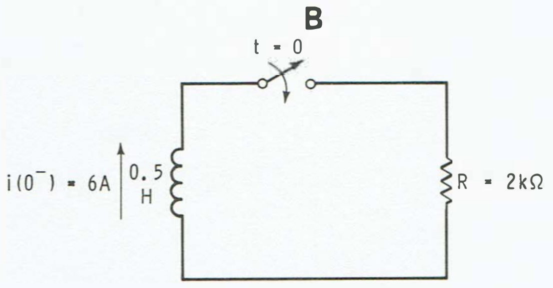
28. The inductor current will equal 5A ____________________ seconds after the switch is closed.
29. An equation that describes the inductor current once the switch is closed is ____________________.
30. An equation that describes the inductor voltage once the switch is closed is ____________________.

Refer to Figure 2-26B for questions 31 to 34

31. At $t = 0^+$, the inductor current is ____________________ A.
32. Approximately ____________________ ms after the switch is closed, the inductor current will be zero.
33. An equation that describes the inductor current once the switch is closed is ____________________.
34. An equation that describes the inductor voltage once the switch is closed is ____________________.



QUESTIONS 28 TO 30



QUESTIONS 31 TO 34

Figure 2-26

Circuits for questions 28 to 34.
A. Questions 28 to 30.
B. Questions 31 to 34.

Answers

19. coil
20. Inductance
21. 0 - inductor voltage exists only when the current is changing.
    $W = 1/2\,LI^2$
    $W = 1/2\,(10mH)\,(2)^2$
    $W = 20mJ$
22. $L_T\,(\text{series}) = 9 + 9 + 9 = 27\mu H$
    $L_T\,(\text{parallel}) = 9/3 = 3\mu H$
23. L/R
24. five
25. open
26. 20mA
27. short
28. 10
29. $5(1 - e^{-0.5t})$
30. $25\,e^{-0.5t}$
31. 6
32. 1.25
33. $6\,e^{-4000t}$
34. $12\,000\,e^{-4000t}$

The solution to questions 28 to 34 follow:

28. $$\tau = \frac{L}{R} = \frac{10H}{5\Omega} = 2s$$

$$5\tau = 5(2) = 10s$$

29. $I_1 = 0, I_2 = \dfrac{25V}{5\Omega} = 5A$, and $\tau = 2s$. Substituting these values into Equation 2-8 yields:

$$\begin{aligned} i_L(t) &= (I_1 - I_2)\,e^{-t/\tau} + I_2 \\ &= (0 - 5)\,e^{-t/2} + 5 \\ &= 5 - 5\,e^{-0.5t} \\ &= 5\,(1 - e^{-0.5t}) \end{aligned}$$

30. At $t = 0^+$, L acts like an open circuit. Therefore:
$V_1 = 25V$

At $t = \infty$, L acts like a short circuit. Therefore:
$V_2 = 0$

Substituting the above values into Equation 2-9 yields:

$$\begin{aligned} v_L(t) &= (V_1 - V_2)\,e^{-t/\tau} + V_2 \\ &= (25 - 0)\,e^{-t/2} + 0 \\ &= 25\,e^{-0.5t} \end{aligned}$$

31. At $t = 0^+$, L acts like a 6A current source. Therefore: $I_L = 6A$

32. $\tau = \frac{L}{R} = \frac{0.5H}{2k\Omega} = 0.25ms$

$5\tau = 5(0.25ms) = 1.25ms$

33. Substituting $I_1 = 6A$, $I_2 = 0$, and $\tau = 0.25ms$ into Equation 2-8 yields:

$$\begin{aligned} I_L(t) &= (I_1 - I_2)\, e^{-t/\tau} + I_2 \\ &= (6 - 0)\, e^{-t/0.25ms} + 0 \\ &= 6\, e^{-4000t} \end{aligned}$$

34. At $t = 0^+$, $i_L = 6A$. Therefore, $V_1 = 6A(2K\Omega) = 12000V$ clearly at $t = \infty$ $\nu_L = V_2 = 0$. Substituting these values into Equation 2-9 yields:

$$\begin{aligned} \nu_L(t) &= (V_1 - V_2)\, e^{-t/\tau} + V_2 \\ &= (12000 - 0)\, e^{-t/0.25ms} + 0 \\ &= 12000\, e^{-4000t} \end{aligned}$$

LOGARITHMS

The use of logarithms and logarithmic notation is commonplace in the electronics industry. Numerous instruments, for example, have logarithmic scales. In addition, electronic manufacturers frequently provide component and system specifications in terms of logarithmic ratios. For these reasons, it is important that you learn what logarithms are and how to work with them.

Basic Concepts

In the previous sections, you learned that the DC currents and voltages in single-loop RC and RL networks are exponential functions. The basic exponential function is by definition of the form:

$$y = b^x \qquad \text{(Eq. 2-16)}$$

where: y is called the dependent variable
b is a constant called the base
x (the independent variable) is called the exponent of the base b.

In equation 2-16, y is a function of x, since the value of y depends upon the particular value assigned to x. For example, if the base is 2 and the exponent 3, the corresponding value of y is:

$$y = b^x = 2^3 = 8$$

By introducing the concept of a **logarithm,** it is also possible to express x as a function of y. Specifically:

The exponent x is the logarithm of the number y to the base b.

Stated mathematically:

$$x = \log_b y \qquad \text{(Eq. 2-17)}$$

It is important to realize that both Equation 2-16 and Equation 2-17 summarize the same relationship between the variables x and y — only the manner in which the relationship is stated is different. We will refer to Equation 2-16 as the **exponential form**, and Equation 2-17 as the **logarithmic form**. Incidentally, in Equation 2-17, since x is expressed as a function of y, x is now considered to be the dependent variable and y the independent variable.

Example 2-8

Given that $y = 2^3$, *find the logarithm of y to the base 2.*

If $y = 2^3$, then from the definition of a logarithm (Equation 2-17), $\log y_2 = 3$. Remember a logarithm is an exponent; specifically, it is the power the base b must be raised to so that $b^x = y$.

Example 2-9

(a) *Convert* $y = 3^x$ *from exponential form to logarithmic form.*
(b) *Convert* $\log_2 y = 5$ *to exponential form.*
(a) *If* $y = 3^x$, *then* $\log_3 y = x$.
(b) *If* $\log_2 y = 5$, *then* $y = 2^5$.

In electronics two bases are used with logarithms, base e and base 10. Logarithms whose base is e are called **natural** logarithms, while logarithms whose base is 10 are called **common** logarithms. To avoid confusion the symbol log x (without any base indicated) refers to base 10 common logarithms. Similarly the symbol ln x (without any base indicated) refers to base e natural logarithms. Occasionally, you may want to convert a base e logarithm to a base 10 logarithm and vice versa. To do this, you can use the following approximate formula:

$$\ln y = 2.303 \log y \qquad \text{(Eq. 2-18)}$$

Example 2-10

(a) *Find* $\ln y$ *if* $\log y = 4$
(b) *Find* $\log y$ *if* $\ln y = 10$

(a) $\ln y = 2.303 \log y$
$\ln y = 2.303\,(4) = 9.212$

Check: If $\log y = 4$, then $y = 10^4 = 10000$.
If $\ln y = 9.212$, then $y = (2.718)^{9.212} = 10007$, which ≈ 10000.

(b) $\log y = \dfrac{\ln y}{2.303} = \dfrac{10}{2.303} = 4.342$

Check: If $\ln y = 10$, then $y = (2.718)^{10} = 22004$.
If $\log y = 4.342$, then $y = 10^{4.342} = 21979$, which ≈ 22004.

Since logarithms are exponents; they obey the rules of exponents encountered in algebra. From such rules and the definition of a logarithm provided earlier, the following properties of logarithms have been established.

Product of two numbers

$\log A B = \log A + \log B$ (Eq. 2-19)

Division of two numbers

$\log \frac{A}{B} = \log A - \log B$ (Eq. 2-20)

Number raised to a power

$\log A^n = n \log A$ (Eq. 2-21)

For natural (base e) logarithms, you would substitute ln for log in the relationships just given.

Example 2-11

Write equivalent expressions for:

(a) $\log (2)(5)$

(b) $\log \frac{10}{2}$

(c) $\ln 2^3$

(a) $\log 2(5) = \log 2 + \log 5$

(b) $\log \frac{10}{2} = \log 10 - \log 2$

(c) $\ln 2^3 = 3 \ln 2$

How can you determine the value of expressions like log 2, ln 2, etc? For many years, the standard method was to use tables of logarithms and interpolation techniques for numbers not listed in the tables. Considering the low cost, and convenience, of scientific - type pocket calculators that provide logarithmic functions, their use is highly recommended!

Example 2-12

Show the steps necessary to determine log 2 and ln 2 with a typical pocket calculator.

Depressing the appropriate buttons in the sequence shown is all that's required!

Button	Display	Button	Display
2	2	2	2
log	0.30103	ln	0.69315

Thus: $\log 2 = 0.30103$ since $10^{0.30103} = 2$
$\ln 2 = 0.69315$ since $e^{0.69315} = 2$

Different calculators display different numbers of digits. In this example, we have assumed a six-digit display.

Given the logarithm of a number, how can you determine the number? Again, we recommend the use of a pocket calculator as shown in the following example.

Example 2-13

Assume a six-digit pocket calculator is available. Show the steps necessary to determine x if:
(a) $\log x = 0.30103$
(b) $\ln x = 0.69315$

In this example, you must determine the inverse, or antilog, functions. Most calculators have a button marked 2nd, INV, etc., for this type of calculation. Thus, you would first enter the logarithm of the number, and then depress the following buttons in the sequence shown:

(a)	Button	Display	(b)	Button	Display
	.30103	0.30103		.69315	0.69315
	2nd(INV)	0.30103		2nd(INV)	0.69315
	log	2		ln	2

Thus: If $\log x = 0.30103$ $x = 10^{0.30103} = 2$
If $\ln x = 0.69315$ $x = e^{0.69315} = 2$

Applications For Natural Logs

Circuits that are designed to generate a pulse train, or a single pulse of specific duration, are called **timing circuits**. Many timing circuits consist of an RC charging and discharging circuit. The time constants of these circuits control the frequency of the pulse train, or the width of a single pulse. In any event, the rising and decaying portions of the output voltage are exponential and, therefore, are described by the exponential equation:

$$\nu(t) = (V_1 - V_2)\, e^{-t/\tau} + V_2 \qquad \text{(Eq. 2-9)}$$

You already know how to use this equation to determine the value of the voltage $\nu(t)$ at a specific time t. Suppose, however, you want to reverse this process. Specifically, you want to know at what time (t) the voltage is a specific value. What you need then is an equation that says t equals "such and such". In order to derive the desired equation, we will first solve Equation 2-9 for the exponential term $e^{-t/\tau}$. Thus:

$$\begin{aligned} \nu(t) &= (V_1 - V_2)\, e^{-t/\tau} + V_2 \\ (V_1 - V_2)\, e^{-t/\tau} &= \nu(t) - V_2 \\ e^{-t/\tau} &= \frac{\nu(t) - V_2}{V_1 - V_2} \end{aligned}$$

At this point, we can take the natural logarithm of both sides of the equation.

$$\ln e^{-t/\tau} = \ln \frac{\nu(t) - V_2}{V_1 - V_2}$$

By definition, the natural log of $e^{-t/\tau}$ is $-t/\tau$. Therefore:

$$\frac{-t}{\tau} = \ln \frac{\nu(t) - V_2}{V_1 - V_2}$$

Finally, solving for the time t we have:

$$t = -\tau \ln \frac{\nu(t) - V_2}{V_1 - V_2} \qquad \text{(Eq. 2-22)}$$

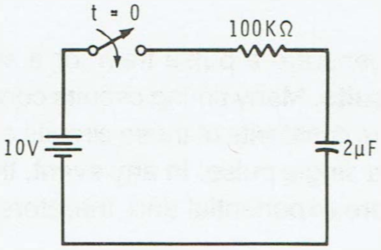


Figure 2-27

Circuit for Example 2-14.

Example 2-14

The capacitor in Figure 2-27 is initially uncharged. How long after the switch is closed will it take for the capacitor voltage to reach 5V?

$$\tau = RC = 100k\Omega(2\mu F) = 200\text{ ms}$$
$$V_1 = 0V$$
$$V_2 = 10V$$
$$v(t) = 5V$$

Substituting the values for τ, V_1, V_2, and $v(t)$ into Equation 2-22 yields:

$$t = -200\text{ms} \ln \frac{5-10}{0-10}$$
$$t = -200\text{ms} \ln 0.5$$
$$t = -200\text{ms} \, (-0.6931)$$
$$t = 138.6\text{ ms}$$

Applications For Common Logs — Decibels

The **gain** of a circuit or system is a ratio of the output signal to the input signal. In electronics, for example, we frequently speak of the current, voltage, or power gain of a circuit or system. To avoid confusing one type of gain with another, we will employ the following notation:

Current Gain $A_i = \frac{i_o}{i_{in}}$ (Eq. 2-23)

Voltage Gain $A_v = \frac{v_o}{v_{in}}$ (Eq. 2-24)

Power Gain $A_p = \frac{P_o}{P_{in}} = AiAv$ (Eq. 2-25)

It has become standard industrial practice to express current, voltage, and power gains as logarithmic ratios in units of **decibels**, abbreviated dBs. Specifically, for a current or voltage ratio the equivalent dB gain is:

$$A_{dB} = 20 \log A \quad \text{(Eq. 2-26)}$$

Similarly, for a **power ratio** the equivalent dB gain is:

$$Ap_{dB} = 10 \log Ap \quad \text{(Eq. 2-27)}$$

The following examples illustrate the conversions of gains to their dB equivalents and vice versa.

Example 2-15

A voltage amplifier has an input voltage of 10mV and an output voltage of 1V. What is the dB voltage gain?

$$A_v = \frac{v_o}{v_{in}} = \frac{1\text{V}}{10\text{mV}} = 100$$

$$Av_{dB} = 20 \log Av$$

$$Av_{dB} = 20 \log 100 = 40\text{dB}$$

Example 2-16

An amplifier consists of two cascaded stages. The voltage gain of the first stage is 20dB and the voltage gain of the second stage is 6dB. Determine:

(a) The gain of each stage.
(b) The total gain of the two-stage amplifier.
(c) The total dB gain of the two-stage amplifier.

(a) $A_{1dB} = 20 \log A_1 = 20$ $\qquad$ $A_{2dB} = 20 \log A_2 = 6$

$\log A_1 = \frac{20}{20} = 1$ $\qquad$ $\log A_2 = \frac{6}{20} = 0.3$

$A_1 = 10^1 = 10$ $\qquad$ $A_2 = 10^{0.3} = 2$

(b) $A_T = A_1 A_2$
$A_T = 10(2) = 20$

(c) Since $A_T = A_1 A_2$
$A_{TdB} = 20 \log [A_1 A_2]$
$A_{TdB} = 20 \log A_1 + 20 \log A_2$
$A_{TdB} = A_{1dB} + A_{2dB}$

Thus, for cascaded sections the total dB gain is the sum of the dB gains of each stage. Thus:

$$A_{TdB} = 20 + 6 = 26dB$$

Figure 2-28 illustrates the concept of total gain for cascaded stages.

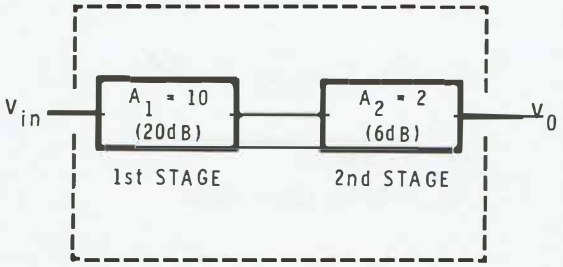


Figure 2-28

Cascaded stages.
The total gain is the product of each stage gain ($A_T = 10(2) = 20$).
Due to the properties of logarithms, the total dB gain equals the sum of the dB gains of each stage ($A_{TdB} = 20 + 6 = 26dB$).

Reference Levels — dBm

Some instruments are calibrated to indicate **relative** current, voltage, or power levels rather than absolute values. In such instruments, the measured value indicates how much larger or smaller the measured signal is compared to the reference value. To illustrate this concept, we will consider the measurement of power with a reference level of 1mW. When a reference level is used for a power measurement, the instrument reading is given by:

$$P_{dBm} = 10 \log \frac{P}{P_{ref}} \qquad \text{(Eq. 2-28)}$$

where P_{dBm} = relative power in dBm. The m is added to indicate that a reference level is used.
P = measured power
P_{ref} = reference level (1mW in this case)

For example, for a measured power of 0.5mW:

$$P_{dBm} = 10 \log \frac{0.5\text{mW}}{1\text{mW}} = -3\text{dBm}$$

By substituting powers of 0.25mW, 0.5mW, 1mW, 2mW, and 4mW into Equation 2-28, we obtained the data in Table 2-2.

Absolute Power (mW)	Relative Power (dBm)
0.25	–6
0.5	–3
1	0
2	+3
4	+6

Table 2-2

dBm Calculations - P_{ref} = 1mW.

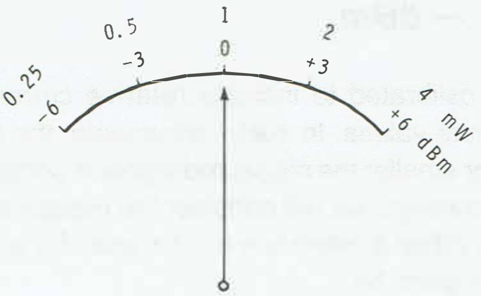


Figure 2-29

The concept of dBm. Note that the reference level is 1mW.

Figure 2-29 shows how the dBm scale is calibrated.

In concluding our discussion of decibels, it should be pointed out that you may encounter reference levels other than 1mW. In any event, for a dBm value to have meaning, you **must** specify the reference level.

Self Test Review

35. The equation $y = 10^3$ is said to be in ________________ form.
36. The equation $\log y = 3$ is said to be in ________________ form.
37. Given $y = 3^2$, we can say that the logarithm of y to the base ____________ is ________________.
38. The base of natural logarithms is ________________, while the base of common logarithms is ________________.
39. If $\log y = 2$, then $\ln y =$ ________________.
40. The log of 10^n is ________________.
41. The ln of $(2.718)^{-3}$ is ________________.
42. Once a switch is closed, the voltage across a capacitor in an RC discharging circuit is given by $v_c(t) = 100\ e^{-t/20\mu s}$. How long after the switch is closed will it take for the capacitor voltage to reach 50V? t = ________________ μs.
43. A voltage gain of 50 is equivalent to a dB voltage gain of ____________.
44. An amplifier has a dB power gain of 32dB. This is equivalent to a regular power gain of ________________.
45. A dBm power meter whose reference level is 1mW would read __________ dBm if the measured signal was 3mW.
46. The cascaded sections of an electronic system have dB voltage gains of +10dB, –6dB, and +3dB respectively. The overall dB gain is therefore ____________ dB.

Answers

35. exponential
36. logarithmic
37. 3, 2
38. e or 2.718
39. $\ln y = 2.303 \log y$
    $\ln y = 2.303\,(2)$
    $\ln y = 4.606$
40. n
41. –3
42. $13.86\mu s$
43. 33.99dB
44. 1585
45. 4.77dBm
46. 7dB

The solutions to questions 42 to 46 follow.

42. Since $\nu_c(t) = 100\, e^{-t/20\mu s}$, we see that $V_1 = 100V$, $V_2 = 0V$, and $\tau = 20\mu s$. To determine the value of t that corresponds to $\nu_c(t) = 50V$ we can employ Equation 2-22. Thus,

$$t = -\tau \ \ln \frac{\nu(t) - V_2}{V_1 - V_2}$$

$$t = -20\mu s \ \ln \frac{50 - 0}{100 - 0}$$

$$t = -20\mu s \ln 0.5$$

$$t = -20\mu s\,(-0.693) = 13.86\mu s$$

43.
$$A_{vdB} = 20 \log A_v$$
$$A_{vdB} = 20 \log 50$$
$$A_{vdB} = 20\,(1.699) \approx 33.99dB$$

44.
$$A_{pdB} = 10 \log A_p$$
$$32 = 10 \log A_p$$
$$\log A_p = \frac{32}{10} = 3.2$$
$$A_p = 10^{3.2} = 1585$$

45.
$$P_{dBm} = 10 \log \frac{P}{P_{ref}}$$
$$P_{dBm} = 10 \log \frac{3mW}{1mW}$$
$$P_{dBm} = 10 \log 3$$
$$P_{dBm} = 10\,(0.477) = 4.77\ dBm$$

46.
$$A_{vdB_{(Total)}} = +\,10 - 6 + 3 = 7dB$$

RECTANGULAR COMPLEX NUMBERS

The mastery of complex arithmetic is essential for the analysis and design of AC circuits. Once you can handle complex numbers, you can extend the techniques of DC circuit analysis and design to include AC circuits. As you will see, there is really nothing "complex" about complex numbers except the word complex.

Real, Imaginary, and Complex Numbers

Integers represent whole quantities, and may be either positive or negative. The typical counting numbers –2, –1, +1, +2, etc., are examples of integers. A **rational** number is a number you obtain by dividing one integer by another. For example, the number 0.5 is an integer because the number 1 (an integer) divided by the number 2 (also an integer) equals 0.5. **Irrational** numbers are numbers you cannot obtain by dividing one integer by another. Numbers like π, $\sqrt{2}$, and nonrepeating nonterminating decimals are examples of irrational numbers. Collectively, all of the integers, rational, and irrational numbers are refered to as **real** numbers.

Real numbers can be represented as points on a horizontal line called the **real number line**, as shown in Figure 2-30. As you can see, equal intervals are marked off to the left (negative numbers) and right (positive numbers) of the point corresponding to zero. In Figure 2-30, we have labeled only the positive and negative integers, although it should be obvious that the points between the integers represent both rational and irrational numbers. Thus any real number can be represented as a unique point on the real number line.

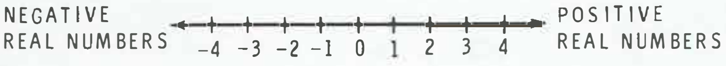


Figure 2-30

The real number line.
All real numbers can be represented as points on a horizontal line.

At one time, mathematicians thought that any number could be represented as a point on the real number line. For example, if you wanted to locate the point corresponding to the $\sqrt{4}$ you would move two intervals to the right of zero, since the $\sqrt{4} = 2$. If the number inside the square-root sign was not a perfect square, you could use a pocket calculator to estimate the square root. For example $\sqrt{5} \approx 2.236$. Thus, the point on the real number line corresponding to the $\sqrt{5}$ would be located approximately 2.236 intervals to the right of zero. If you needed more accuracy you could carry the square root calculation out to the required number of decimal places. For example, $\sqrt{5} = 2.236068$ would certainly be more than accurate enough for most calculations.

Now, suppose you wanted to locate the point corresponding to the $\sqrt{-4}$. What is the $\sqrt{-4}$? Clearly the $\sqrt{-4}$ is not $+2$, since $+2\,(+2)$ equals $+4$ not -4. Similarly, the $\sqrt{-4}$ is **not** -2, since $-2\,(-2)$ also equals $+4$, not -4. Eventually, mathematicians realized the square root of a negative number could not be represented as a point on the real number line! Therefore the square root of a negative number is not a real number. Mathematicians decided to call this newly discovered number imaginary. Thus:

An imaginary number is the square root of a negative real number.

It is important to realize that there is nothing "make believe", "fictitious", or "imaginary" about imaginary numbers. The word imaginary is simply the word chosen for the name of a particular type of number. To extract the square root of an imaginary number, the product rule for square roots is employed. Recall from elementary algebra that:

$$\sqrt{AB} = \sqrt{A}\,\sqrt{B} \qquad \text{(Eq. 2-29)}$$

For imaginary numbers, you can apply Equation 2-29 as follows:

$$\begin{aligned}\sqrt{-n} &= ?\\ \sqrt{-n} &= \sqrt{-1(n)}\\ \sqrt{-n} &= \sqrt{-1}\,\sqrt{n} \qquad \text{(Eq. 2-30)}\end{aligned}$$

Since it is awkward to continue to prefix the $\sqrt{n}$ by the $\sqrt{-1}$, we will use the shorthand symbol j to mean $\sqrt{-1}$. By definition, then:

$$j = \sqrt{-1} \qquad \text{(Eq. 2-31)}$$

Therefore:

$$\sqrt{-n} = j\sqrt{n} \qquad \text{(Eq. 2-32)}$$

Example 2-17

Find the square roots of the following numbers:

(a) $\sqrt{-4}$
(b) $-\sqrt{-9}$

(a) $\sqrt{-4} = \sqrt{-1(4)} = \sqrt{-1}\,\sqrt{4} = j2$
(b) $-\sqrt{-9} = -\sqrt{-1(9)} = -\sqrt{-1}\,\sqrt{9} = -j3$

Example 2-18

Determine the values of j^2, $-j^2$, j^3, *and* $-j^3$

$$j^2 = \sqrt{-1}\,\sqrt{-1} = -1$$
$$-j^2 = -1\,(-1) = +1$$
$$j^3 = j\,j^2 = j(-1) = -j$$
$$-j^3 = -1(-j) = +j$$

As you can see, multiplying j terms follow the rules for exponents of elementary algebra. The terms j^2 and $-j^2$ occur so frequently that you should memorize them. If you need other powers of j, you can work them out by following the procedure outlined in Example 2-18.

Just as real numbers can be represented at points on the real number line, imaginary numbers can be represented as points on an imaginary number line. The imaginary number line is drawn vertically, rather than horizontally, to distinguish it from the real number line as shown in Figure 2-31. Thus, any imaginary number can be represented as a unique point on the imaginary number line. The $\sqrt{-4}$ for example, would be represented by the point two intervals above zero, since the $\sqrt{-4}$ equals j2. Similarly, $-\sqrt{-9}$ would correspond to the point three intervals below zero since $-\sqrt{-9}$ equals –j3.

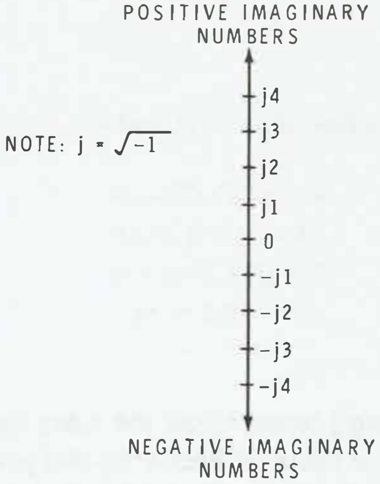


Figure 2-31

A number line for imaginary numbers
All imaginary numbers can be represented as points on a vertical line.

When you combine real and imaginary numbers you get **complex** numbers. Specifically:

A complex number is the algebraic sum of a real and imaginary number.

We will call the real part of a complex number "a" and the imaginary part "b". Thus, in general, a complex number is of the form:

$$A = a \pm jb \qquad \text{(Eq. 2-33)}$$

where: A = complex number
a = real part
b = imaginary part
j = $\sqrt{-1}$

Some examples of complex numbers include 3 + j4, –2 + j4, –3 – j2, and –4 + j5. Incidentally, a complex number written in the form a ± jb is in **rectangular form**. As you might suppose, complex numbers (like real and imaginary numbers) can also be represented graphically. All you have to do is draw the imaginary and real number lines at right angles to each other to form a two-dimensional plane called the **complex plane**, as shown in Figure 2-32. Any complex number, then, can be represented as a point in the complex plane.

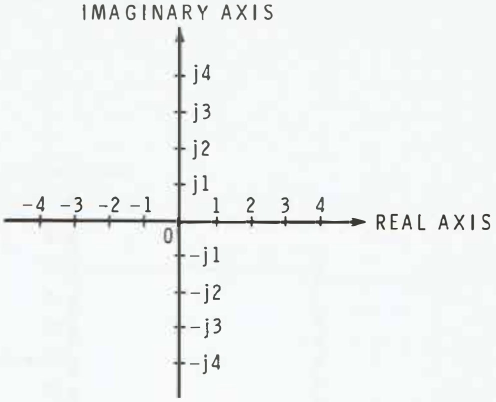


Figure 2-32

The complex plane.
All complex numbers can be represented as points in the complex plane.

Example 2-19

> *Locate the points in the complex plane that represent the following numbers:* $A = -8 + j7$, $B = 3 + j4$, $C = 6 - j6$, *and* $D = -4 - j4$.

Recall that positive real numbers are located to the right of zero, and negative real numbers to the left of zero. Similarly, positive imaginary numbers are located above zero and negative imaginary numbers below zero. Thus, the point corresponding to A = (–8 + j7) is located 8 units, or intervals, to the left of zero, and 7 units above zero. Similarly, the points corresponding to B, C, and D are located as shown in Figure 2-33.

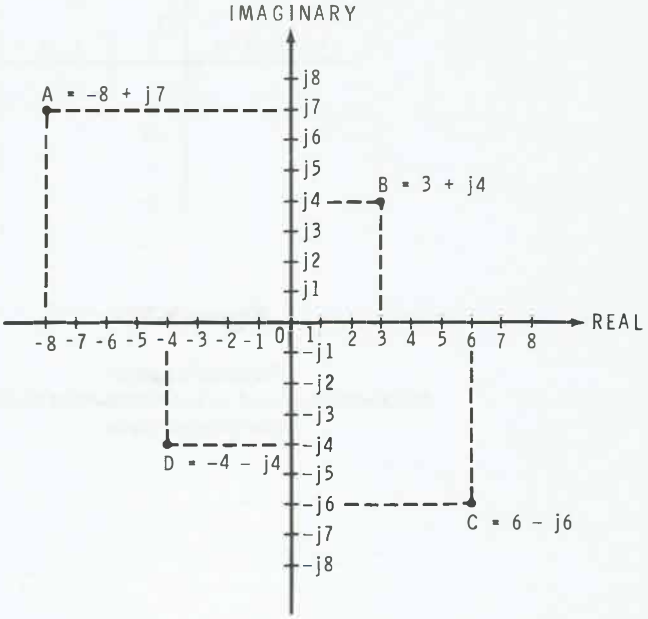


Figure 2-33

Solutions to Example 2-19.

Operations With Rectangular Numbers

You already know how to perform the mathematical operations of addition, subtraction, multiplication, and division on real numbers. We will now extend these operations to include complex numbers.

ADDITION

The sum of two complex numbers equals the algebraic sum of their real and imaginary parts.

Stated mathematically:

Sum of imaginary parts

$$(a + jb) + (c + jd) = (a + c) + j(b + d) \qquad \text{(Eq. 2-34)}$$

Sum of real parts

The following examples illustrate the process

$$(3 + j4) + (2 + j5) = (3 + 2) + j(4 + 5) = 5 + j9$$

$$(-3 + j4) + (2 + j5) = (-3 + 2) + j(4 + 5) = -1 + j9$$

$$(3 - j4) + (2 + j5) = (3 + 2) + j(-4 + 5) = 5 + j1$$

$$(-3 - j4) + (2 + j5) = (-3 + 2) + j(-4 + 5) = -1 + j1$$

$$(3 + j4) + (-2 + j5) = (3 - 2) + j(4 + 5) = 1 + j9$$

$$(3 + j4) + (2 - j5) = (3 + 2) + j(4 - 5) = 5 - j1$$

$$(3 + j4) + (-2 - j5) = (3 - 2) + j(4 - 5) = 1 - j1$$

SUBTRACTION

The difference of two complex numbers equals the algebraic difference of their real and imaginary parts.

Stated mathematically:

Difference of imaginary parts

$$(a + jb) - (c + jd) = (a - c) + j(b - d) \qquad \text{(Eq. 2-35)}$$

Difference of real parts

Just as Equation 2-34 was used to add two complex numbers, Equation 2-35 may be used to subtract two complex numbers. In effect, you subtract by converting the given subtraction problems into an equivalent addition problem. This works because the difference A − B is equal to the sum of A and the negative of B. Specifically:

$$A - B = A + (-B) \qquad \text{(Eq. 2-36)}$$

The negative of a number is obtained by multiplying the number by −1. For an expression contained in parentheses, this is equivalent to changing the sign of each term within the parentheses. In general, then, the difference of two complex numbers is:

$$(a + jb) - (c + jd) = (a + jb) + (-c - jd) \qquad \text{(Eq. 2-37)}$$

Equation 2-37 is equivalent to Equation 2-35. Since Equation 2-37 is easier to "visualize", we will use it to subtract complex numbers. The following examples illustrate the process.

$$\begin{aligned}
(5 + j9) - (2 + j5) &= (5 + j9) + (-2 - j5) \\
&= (5 - 2) + j(9 - 5) = 3 + j4 \\[1em]
(-1 + j9) - (2 + j5) &= (-1 + j9) + (-2 - j5) \\
&= (-1 - 2) + j(9 - 5) = -3 + j4 \\[1em]
(5 + j1) - (2 + j5) &= (5 + j1) + (-2 - j5) \\
&= (5 - 2) + j(1 - 5) = 3 - j4 \\[1em]
(-1 + j1) - (2 + j5) &= (-1 + j1) + (-2 - j5) \\
&= (-1 - 2) + j(1 - 5) = -3 - j4 \\[1em]
(1 + j9) - (-2 + j5) &= (1 + j9) + (2 - j5) \\
&= (1 + 2) + j(9 - 5) = 3 + j4 \\[1em]
(5 - j1) - (2 - j5) &= (5 - j1) + (-2 + j5) \\
&= (5 - 2) + j(-1 + 5) = 3 + j4 \\[1em]
(1 - j1) - (-2 - j5) &= (1 - j1) + (2 + j5) \\
&= (1 + 2) + j(-1 + 5) = 3 + j4
\end{aligned}$$

MULTIPLICATION

You can employ the following steps to multiply two complex numbers.

1. Multiply the second number by the real and imaginary parts of the first number. Specifically, (a + jb) (c + jd) = a(c + jd) + jb(c + jd).
2. Replace j^2 by –1 and $-j^2$ by + 1.
3. Collect like terms.

The following examples illustrate this three-step procedure.

$$\begin{aligned}(2 + j3)(5 + j4) &= 2(5 + j4) + j3(5 + j4)\\ &= 10 + j8 + j15 + j^2 12\\ &= 10 + j8 + j15 - 12\\ &= -2 + j23\end{aligned}$$

$$\begin{aligned}(3 - j2)(2 + j5) &= 3(2 + j5) - j2(2 + j5)\\ &= 6 + j15 - j4 - j^2 10\\ &= 6 + j15 - j4 + 10\\ &= 16 + j11\end{aligned}$$

$$\begin{aligned}(-2 - j4)(3 - j5) &= -2(3 - j5) - j4(3 - j5)\\ &= -6 + j10 - j12 + j^2 20\\ &= -6 + j10 - j12 - 20\\ &= -26 - j2\end{aligned}$$

$$\begin{aligned}(2 - j3)(3 - j2) &= 2(3 - j2) - j3(3 - j2)\\ &= 6 - j4 - j9 + j^2 6\\ &= 6 - j4 - j9 - 6\\ &= 0 - j13\\ &= -j13\end{aligned}$$

DIVISION

Given a complex number of the form $A = a \pm jb$, the **conjugate**, A^*, is defined as $A^* = \pm jb$. Therefore, to obtain the conjugate, all you have to do is change the sign separating the real and imaginary terms. For example, if $A = 3 + j4$, the conjugate is $A^* = 3 - j4$. Similarly, if $A = 2 - j5$, the conjugate would be $A^* = 2 + j5$. Here's what happens if a complex number is multiplied by its conjugate:

$$\begin{aligned} A\,A^* &= (a + jb)(a - jb) \\ &= a(a - jb) + jb(a - jb) \\ &= a^2 - jab + jab - j^2b^2 \\ &= a^2 - jab + jab + b^2 \\ &= a^2 + b^2 \end{aligned}$$

Therefore: $$A\,A^* = a^2 + b^2 \qquad \text{(Eq. 2-37)}$$

Consequently, whenever a complex number is multiplied by its conjugate, the result equals the sum of the squares of the real and imaginary parts. This is important because you will use this idea to divide two complex numbers. Specifically:

> To divide two complex numbers, multiply the numerator and denominator by the conjugate of the denominator.

The following examples illustrate the process.

$$\begin{aligned} \frac{-2 + j23}{5 + j4} &= \frac{(-2 + j23)}{(5 + j4)} \cdot \frac{(5 - j4)}{(5 - j4)} \\ &= \frac{-2(5 - j4) + j23(5 - j4)}{5^2 + 4^2} \\ &= \frac{-10 + j8 + j115 - j^2 92}{25 + 16} \\ &= \frac{-10 + j8 + j115 + 92}{41} \\ &= \frac{82 + j123}{41} = 2 + j3 \end{aligned}$$

$$\begin{aligned} \frac{16 + j11}{2 + j5} &= \frac{(16 + j11)}{(2 + j5)} \cdot \frac{(2 - j5)}{(2 - j5)} \\ &= \frac{16(2 - j5) + j11(2 - j5)}{2^2 + 5^2} \\ &= \frac{32 - j80 + j22 - j^2 55}{4 + 25} \\ &= \frac{32 - j80 + j22 + 55}{29} \\ &= \frac{87 - j58}{29} = 3 - j2 \end{aligned}$$

As you can see, the process of division involves adding, subtracting, and multiplying complex numbers as intermediate steps. For this reason, division problems make excellent practice to nail down rectangular complex arithmetic.

Self Test Review

47. A number that you obtain by dividing one integer by another is called a ______________ number.
48. A number that you cannot obtain by dividing one integer by another is called an __________________ number.
49. The square root of a negative real number is called an ________________ number.
50. The algebraic sum of a real and imaginary number is a ________________ number.
51. The symbol __________________ equals the square root of –1.
52. j^2 = __________________ and $-j^2$ = __________________.
53. The number 2 – j3 corresponds to a point __________________ intervals to the ________________ of zero, and __________________ intervals ______________ zero on the complex plane.
54. The sum of (2 – j3) and (–3 + j2) is __________________.
55. (10 – j5) – (2 – j5) equals __________________.
56. The product of (–2 + j3) and (2 – j7) equals __________________.
57. The conjugate of (–2 + j3) is __________________.
58. (–26 – j2) divided by (3 – j2) equals __________________.

Answers

47. rational
48. irrational
49. imaginary
50. complex
51. j
52. $j^2 = -1, -j^2 = +1$
53. two, right, three, above
54. $-1 - j1$
55. 8
56. $17 + j20$
57. $-2 - j3$
58. $-5.69 - j4.46$

Here are the solutions to questions 54 to 58.

54. $(2 - j3) + (-3 + j2) = (2 - 3) + j(-3 + 2) = -1 - j1$

55.
$$\begin{aligned}(10 - j5) - (2 - j5) &= (10 - j5) + (2 + j5) \\ &= (10 - 2) + j(-5 + 5) = 8 + j0 = 8\end{aligned}$$

56.
$$\begin{aligned}(-2 + j3)(2 - j7) &= -2(2 - j7) + j3(2 - j7) \\ &= -4 + j14 + j6 - j^2 21 \\ &= -4 + j14 + j6 + 21 \\ &= 17 + j20\end{aligned}$$

57. $A = -2 + j3$

Therefore $A^* = -2 - j3$

58.
$$\begin{aligned}\frac{(-26 - j2)}{(3 - j2)} &= \frac{(-26 - j2)}{(3 - j2)} \cdot \frac{(3 + j2)}{(3 + j2)} \\ &= \frac{-26\,(3 + j2) - j2\,(3 + j2)}{3^2 + 2^2} \\ &= \frac{-78 - j52 - j6 - j^2 4}{9 + 4} \\ &= \frac{-78 - j52 - j6 + 4}{13} \\ &= \frac{-74 - j58}{13} = -5.69 - j4.46\end{aligned}$$

POLAR COMPLEX NUMBERS

As you know, a complex number can be expressed in rectangular form, or as a point in the complex plane. In addition there are several other ways to describe a complex number. In this section, we will discuss the polar representation of a complex number and show you how to convert between the polar and rectangular forms. Before doing this, however, we will review some basic terminology and properties of right triangles.

Basic Trigonometry

Trigonometry deals with the relationships that exist between the angles and sides of triangles; and of special interest are the properties of right triangles. A right triangle, of course, is one that contains a right, or 90°, angle. The side opposite the right angle is called the **hypotenuse**, and is the longest side of the triangle. The remaining two sides of a right triangle are called the **legs** of the triangle. Figure 2-34 illustrates a right triangle whose hypotenuse is c, and whose legs are a and b. The famous **Pythagorean theorem** states that in every right triangle the square of the hypotenuse equals the sum of the squares of the legs. Stated mathematically:

$$c^2 = a^2 + b^2 \qquad \text{(Eq. 2-38)}$$

where: c is the hypotenuse, and a and b are the legs.

By taking the square root of each side of Equation 2-38 you obtain

$$c = \sqrt{a^2 + b^2} \qquad \text{(Eq. 2-39)}$$

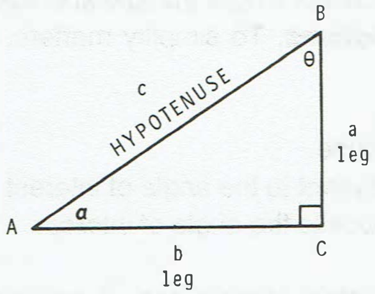


Figure 2-34

The right triangle.
In every right triangle $c^2 = a^2 + b^2$ therefore $c = \sqrt{a^2 + b^2}$.

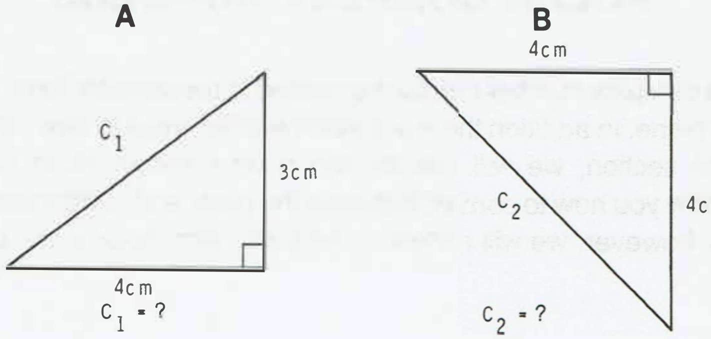


Figure 2-35

Triangles for Example 2-20.
A. C_1 = ?
B. C_2 = ?

Example 2-20

Find the length of the hypotenuse c in Figure 2-35 A and Figure 2-35 B.

Figure 2-35 A
$$c_1 = \sqrt{a^2 + b^2}$$
$$c_1 = \sqrt{3^2 + 4^2}$$
$$c_1 = \sqrt{9 + 16}$$
$$c_1 = \sqrt{25} = 5\text{cm}$$

Figure 2-35 B
$$c_2 = \sqrt{a^2 + b^2}$$
$$c_2 = \sqrt{4^2 + 4^2}$$
$$c_2 = \sqrt{16 + 16}$$
$$c_2 = \sqrt{32} \approx 5.66\text{cm}$$

The ratios of the various sides of a right triangle are used to define a number of very useful **trigonometric functions**. To simplify matters, we will adopt the following shorthand notation.

H = hypotenuse
A = side adjacent to the angle of interest
O = side opposite the angle of interest

In terms of H, A, and O, the **sine**, abbreviated sin; **cosine**, abbreviated cos; and **tangent**, abbreviated tan; of any angle θ are defined as follows:

$$\sin\theta = \frac{O}{H}, \cos\theta = \frac{A}{H} \tan\theta = \frac{O}{A}$$

You can memorize the above relationships by remembering the name of a fictitious indian princess named **SOH-CAH-TOA**.

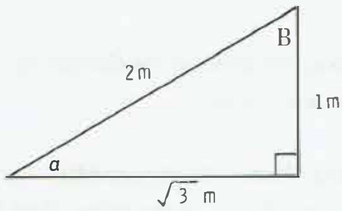


Figure 2-36

Triangle for Example 2-21.

Example 2-21

Determine the sin, cos, and tan for angle α, and angle β in Figure 2-36.

$$\sin\alpha = \frac{O}{H} = \frac{1}{2} = 0.5$$
$$\cos\alpha = \frac{A}{H} = \frac{\sqrt{3}}{2} \approx 0.866$$
$$\tan\alpha = \frac{O}{A} = \frac{1}{\sqrt{3}} \approx 0.577$$
$$\sin\beta = \frac{O}{H} = \frac{\sqrt{3}}{2} \approx 0.866$$
$$\cos\beta = \frac{A}{H} = \frac{1}{2} = 0.5$$
$$\tan\beta = \frac{O}{A} = \frac{\sqrt{3}}{1} \approx 1.73$$

As you can see, the “opposite side” and “adjacent side” depend on which angle you’re looking at. There are many different types of problems you can solve using the Pythagorean theorem and the trigonometric functions just defined. For design purposes, the ability to solve the following kinds of problems is especially useful.

1. Given an angle, determine the sin, cos, and or tan of the angle.
2. Given the sin, cos, or tan of an angle, determine the angle.
3. Given the hypotenuse and one angle (in addition to the right angle), determine the legs of the right triangle.
4. Given the legs, determine the hypotenuse and interior angles of a right triangle.

Let’s look at each type of problem with an example.

Example 2-22

Show the steps necessary to determine the sin, cos, and tan of 30° with a typical pocket calculator.

Most calculators can determine these trigonometric functions if the angle is in degrees or radians (2π radians $= 360°$). Assuming that the calculator is set for a degree calculation, all you need to do is enter the angle and depress the appropriate button, as follows:

Button	Display	Button	Display	Button	Display
30	30	30	30	30	30
sin	0.5	cos	0.86603	tan	0.57735

Therefore, $\sin 30° = 0.5$, $\cos 30° \approx 0.86603$, and $\tan 30° = 0.57735$.

If you know the sin, cos, or tan of an angle and must determine the angle, the process is referred to as determining the **inverse trigonometric function**. The following notation is used to indicate inverse trigonometric functions.

arc sin or $\sin^{-1}$ means "the angle whose sin is"
arc cos or $\cos^{-1}$ means "the angle whose cos is"
arc tan or $\tan^{-1}$ means "the angle whose tan is"

Note that there are two ways to indicate the inverse trigonometric functions. If the -1 notation is used you must be careful. The -1 is **not** an exponent; it is just a symbol that indicates "the angle whose (sin, cos, or tan) is". Some calculators employ the arc type notation, while others employ the -1 type notation. In addition, many calculators simply have a second button you can use for inverse functions.

Example 2-23

Determine the values of the following inverse trigonometric functions with a typical pocket calculator. Arc sin $\theta = 0.5$, arc cos $\theta = 0.5$, and arc tan $\theta = 1$.

In these cases, all you do is enter the functions and then depress the appropriate buttons.

Button	Display	Button	Display	Button	Display
.5	0.5	.5	0.5	.5	1
2nd	0.5	2nd	0.5	2nd	1
sin	30	cos	60	tan	45

Therefore, if arc sin $\theta = 0.5$, $\theta = 30°$; similarly if arc cos $\theta = 0.5$, $\theta = 60°$; finally if arc tan $\theta = 1$, $\theta = 45°$.

Example 2-24

Determine the length of each leg a and b in Figure 2-37.

You can use the sin and cos "trig" functions to determine a and b as follows:

$$\sin 30° = \frac{O}{H} = \frac{b}{20\text{cm}}$$

Therefore, $b = 20 \sin 30° = 20\,(0.5) = 10\text{cm}$.

$$\cos 30° = \frac{A}{H} = \frac{a}{20\text{cm}}$$

Therefore, $a = 20 \cos 30° \approx 20(0.866) = 17.32\text{cm}$.

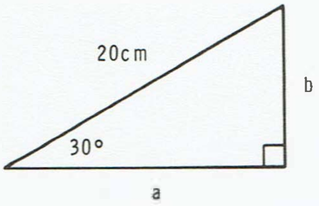


Figure 2-37

Triangle for Example 2-24.

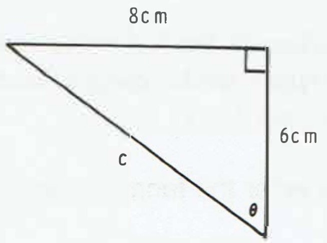


Figure 2-38

Triangle for Example 2-25.

Example 2-25

Determine the length of the hypotenuse c and the angle θ in Figure 2-38.

You can easily determine the length of c with the Pythagorean theorem.

$$c = \sqrt{a^2 + b^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100}$$
$$c = 10 \text{ cm}$$

Similarly, you can calculate angle θ by first calculating the value of the tan of angle θ.

$$\tan\theta = \frac{O}{A} = \frac{8\text{cm}}{6\text{cm}} \approx 1.333$$

Consequently, with the help of a calculator, you find $\theta = 53.13°$.

Polar Numbers

Consider point P in Figure 2-39 A. As you can see, it is located three units to the right of zero and four units above zero. In rectangular form, we would say P = 3 + j4. This is shown in Figure 2-39 B. Note that the real part of P, 3, defines the horizontal coordinate; and the imaginary part of P, 4, defines the vertical coordinate.

Figure 2-39 C illustrates a third way to describe a complex number. In this case, the distance between point P and the origin (A) is provided, as well as the angle (θ) between the positive real axis and the line joining point P and the origin. This type of representation of a complex number is called **polar form**.

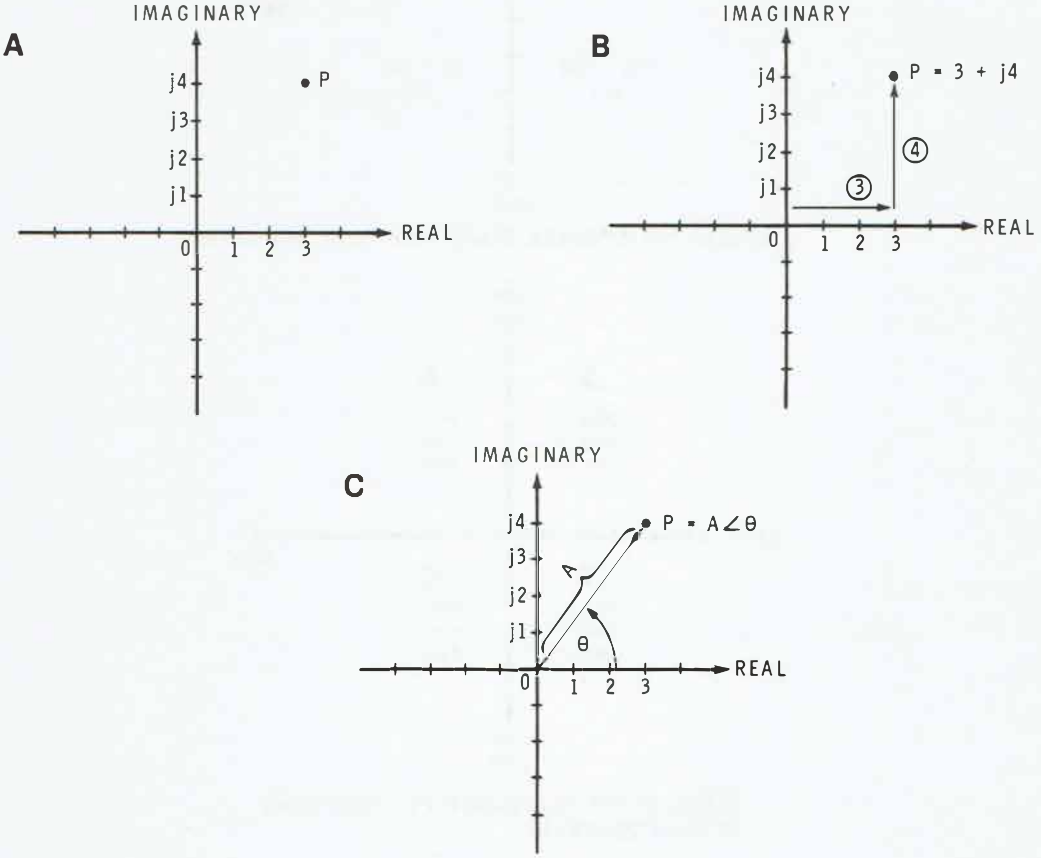


Figure 2-39

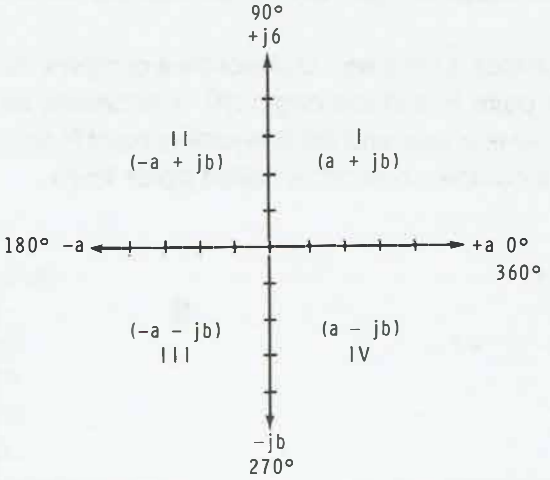
Three ways to describe the same complex number.

A. A point in the complex plane.

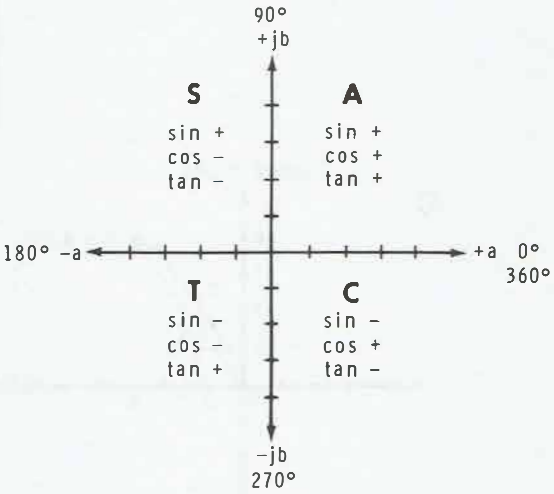
B. In rectangular notation (3 + j4), the horizontal coordinate is the real part, and the vertical coordinate the imaginary part of the complex number.

C. In polar notation, the distance from the origin (A) and the angle (θ) measured in a counterclockwise direction from the positive real axis are stated.

Figure 2-40 shows how the complex plane is divided into four **quadrants**. Quadrant I includes rectangular numbers with both positive real and imaginary parts. A polar number in quadrant I would be characterized by an angle between 0° and 90°. Table 2-3 summarizes the characteristics of both rectangular and polar numbers for each quadrant.



DIVIDING THE COMPLEX PLANE INTO FOUR QUADRANTS



SIGNS OF THE TRIGONOMETRIC FUNCTIONS IN EACH QUADRANT

Figure 2-40

Additional features of the complex plane.
A. Dividing the complex plane into four quadrants.
B. Signs of the trigonometric functions in each quadrant.

Quadrant	Rectangular #	Polar #
I	a + jb	$A\angle\theta, \theta =$ 0° to 90°
II	–a + jb	$A\angle\theta, \theta =$ 90° to 180°
III	–a – jb	$A\angle\theta, \theta =$ 180° to 270°
IV	a – jb	$A\angle\theta, \theta =$ 270° to 360°

Table 2-3

Rectangular And Polar Numbers in Each Quadrant of The Complex Plane.

Because the real and imaginary parts are positive in some quadrants and negative in others, the various trigonometric functions may be positive or negative — depending upon which quadrant a particular complex number is in. Figure 2-40 B indicates the sign of each trig function in each quadrant. You can memorize the various signs by recalling the sentence "**A**ll **S**tudents **T**ake **C**alculus". Thus, all trig functions are positive in the first quadrant, only sin is positive in the second quadrant, and so on.

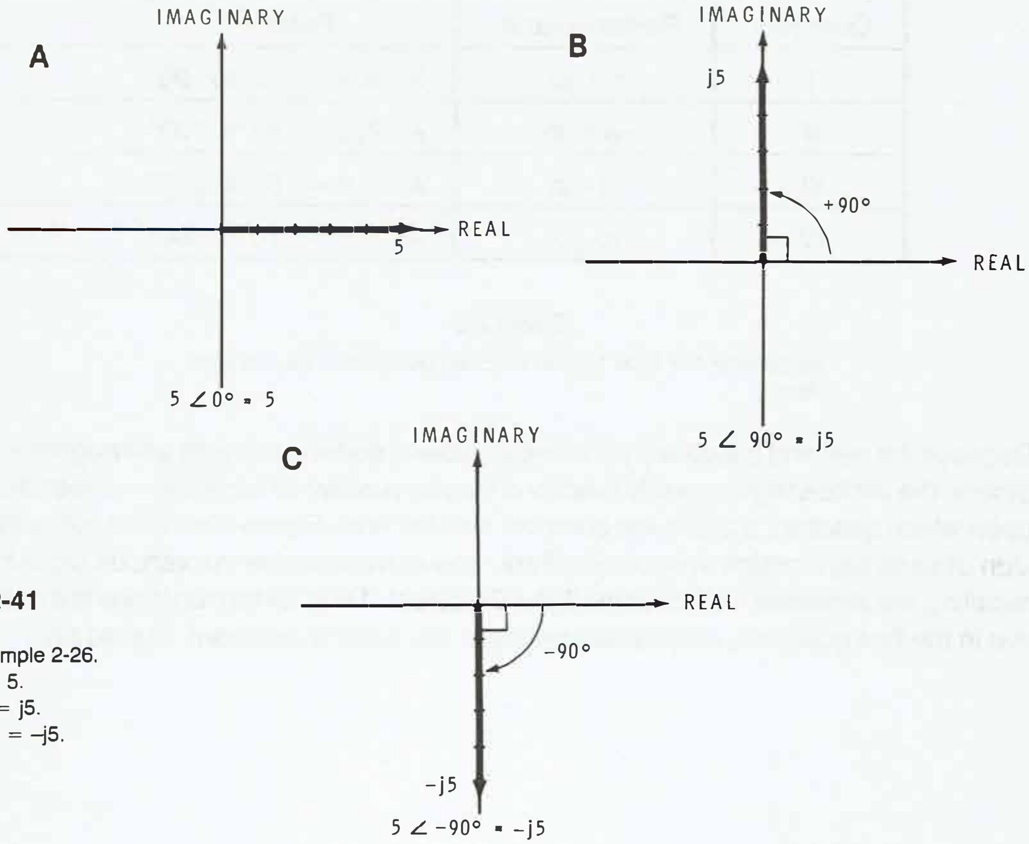


Figure 2-41

Sketches for Example 2-26.
A. $5\angle 0° = 5$.
B. $5\angle 90° = j5$.
C. $5\angle -90° = -j5$.

Example 2-26

Sketch the following polar numbers: $5\angle 0°$, $5\angle 90°$, and $5\angle -90°$.

The sketches are shown in Figure 2-41 A, B, and C. Since positive angles are measured in a counterclockwise direction from the positive real axis, a negative angle corresponds to a clockwise rotation as shown in Figure 2-41 C. Note that multiplying a real quantity by j is equivalent to rotating the real quantity 90° counterclockwise. Similarly, multiplying a real quantity by –j is equivalent to rotating the real quantity 90° clockwise. In general, then, you can conclude that:

$$jA = A\angle 90° \qquad \text{(Eq. 2-40)}$$

and

$$-jA = A\angle -90° \qquad \text{(Eq. 2-41)}$$

Equations 2-40 and 2-41 are important. Memorize them for future use. Incidentally, any positive angle can be expressed as an equivalent negative angle and vice versa. For example, $5\angle -90°$ is equivalent to $5\angle 270°$ and vice versa. In general, you express angles in the first three quadrants as positive angles, while angles in the fourth quadrant are usually expressed as negative angles.

Example 2-27

Sketch polar numbers 4 ∠ 120° and 3 ∠ 220°. Express the given positive angles as equivalent negative angles.

To determine the equivalent negative angle, all you do is subtract the given positive angle from 360°. Therefore:

$$4\angle 120^\circ = 4\angle -(360^\circ - 120^\circ) = 4\angle -240^\circ$$
$$3\angle 220^\circ = 3\angle -(360^\circ - 220^\circ) = 3\angle -140^\circ$$

The sketches are shown in Figure 2-42 A and B respectively.

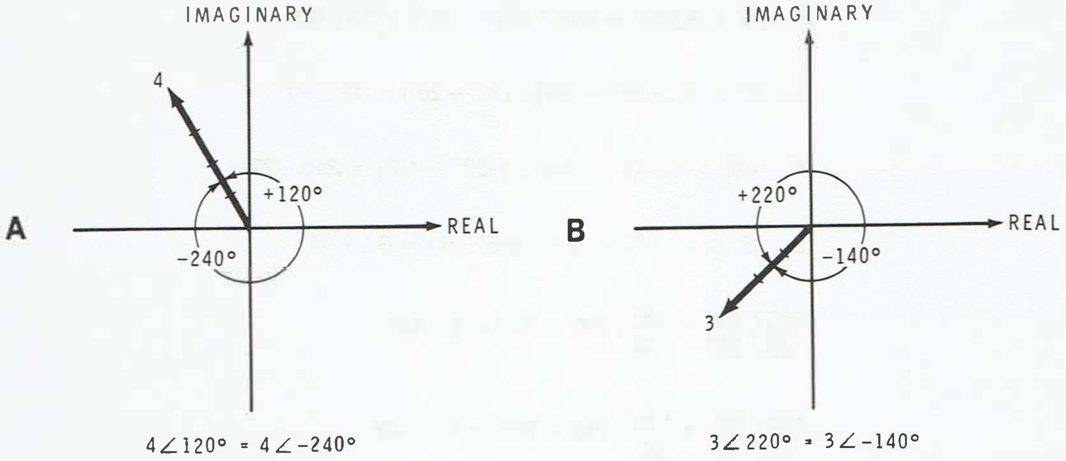


Figure 2-42

Sketches for Example 2-27.
A. 4∠120° = 4∠–240°.
B. 3∠220° = 3∠–140°.

Multiplication and Division of Polar Numbers.

The multiplication and division of polar numbers is really quite simple; the following rules are employed:

Multiplication

$$A\angle\theta_1 \times B\angle\theta_2 = A\,B\angle(\theta_1 + \theta_2) \qquad \text{(Eq. 2-42)}$$

Division

$$\frac{A\angle\theta_1}{B\angle\theta_2} = \frac{A}{B}\angle(\theta_1 - \theta_2) \qquad \text{(Eq. 2-43)}$$

The following examples illustrate the use of these rules.

$$5\angle 30^\circ \times 6\angle 60^\circ = 5(6)\angle(30^\circ + 60^\circ) = 30\angle 90^\circ$$

$$3\angle 30^\circ \times 4\angle -20^\circ = 3(4)\angle(30^\circ - 20^\circ) = 12\angle 10^\circ$$

$$4\angle -60^\circ \times 6\angle 40^\circ = 4(6)\angle(-60^\circ + 40^\circ) = 24\angle -20^\circ$$

$$2\angle -5^\circ \times 4\angle -6^\circ = 2(4)\angle(-5^\circ - 6^\circ) = 8\angle -11^\circ$$

$$\frac{100\angle 60^\circ}{20\angle 20^\circ} = \frac{100}{20}\angle(60^\circ - 20^\circ) = 5\angle 40^\circ$$

$$\frac{100\angle 30^\circ}{25\angle 70^\circ} = \frac{100}{25}\angle(30^\circ - 70^\circ) = 4\angle -40^\circ$$

$$\frac{80\angle 40^\circ}{40\angle -20^\circ} = \frac{80}{40}\angle(40^\circ - (-20^\circ)) = 2\angle 60^\circ$$

$$\frac{60\angle -30^\circ}{10\angle -10^\circ} = \frac{60}{10}\angle(-30^\circ - (-10^\circ)) = 6\angle -20^\circ$$

As you can see, when you are multiplying polar numbers, multiply the magnitudes and add the angles. Similarly, when you are dividing polar numbers, divide the magnitudes and subtract the angles.

Rectangular ⇌ Polar Conversions

You add and subtract polar numbers by first converting them to rectangular numbers. You can then add or subtract the rectangular numbers and convert the result back to polar form. For this reason, you need to know how to convert polar numbers to rectangular numbers and vice versa. Such conversions are based on Pythagorean theorem and the trig functions discussed earlier. First, we will examine the method used to convert polar numbers to rectangular numbers; and then illustrate the reverse, rectangular to polar, conversions.

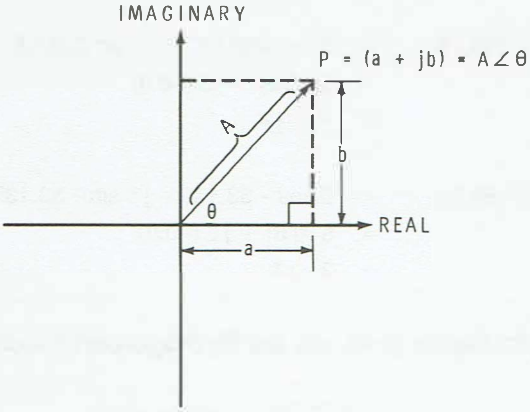


Figure 2-43

Diagram used to derive polar to rectangular and rectangular to polar conversions.

Figure 2-43 shows a right triangle where A is the hypotenuse and a and b are the legs. The cos of the angle θ is:

$$\cos\theta = \frac{a}{A}$$

Therefore, $a = A\cos\theta$.

Similarly, the sin of the angle θ is:

$$\sin\theta = \frac{b}{A}$$

Thus:

$$b = A\sin\theta$$

In general, then, we can state:

$$A\angle\theta = A\cos\theta + jA\sin\theta \qquad \text{(Eq. 2-44)}$$

Equation 2-44 permits you to convert polar numbers to rectangular numbers. The following examples illustrate this process.

$$\begin{aligned}
5\angle 53.13^\circ &= 5\cos 53.13^\circ + j\,5\sin 53.13^\circ \\
&= 5\,(0.6) + j\,5\,(0.8) \\
&= 3 + j\,4
\end{aligned}$$

$$\begin{aligned}
5\angle 126.87^\circ &= 5\cos 126.87^\circ + j\,5\sin 126.87^\circ \\
&= 5\,(-0.6) + j\,5\,(0.8) \\
&= -3 + j\,4
\end{aligned}$$

$$\begin{aligned}
5\angle 233.13^\circ &= 5\cos 233.13^\circ + j\,5\sin 233.13^\circ \\
&= 5(-0.6) + j\,5\,(-0.8) \\
&= -3 - j\,4
\end{aligned}$$

$$\begin{aligned}
5\angle -53.13^\circ &= 5\cos -53.13^\circ + j\,5\sin -53.13^\circ \\
&= 5\,(0.6) + j\,5\,(-0.8) \\
&= 3 - j\,4
\end{aligned}$$

Once again, referring to Figure 2-43, via the Pythagorean theorem, we see that:

$$a = \sqrt{a^2 + b^2} \qquad \text{(Eq. 2-45)}$$

and

$$\theta = \arctan\frac{b}{a} \qquad \text{(Eq. 2-46)}$$

Equation 2-45 is valid for any quadrant. Equation 2-46 on the other hand, is only valid for the first quadrant. Figure 2-44 shows you how to calculate angles in all four quadrants. For example, in the second quadrant, angle θ is clearly less than 180° by an amount equal to angle α. By looking at the other sketches in Figure 2-44, you can see how angle θ is calculated in each quadrant. Table 2-4 summarizes this important information.

Quadrant	Formula For θ
I	$\theta = \arctan\frac{b}{a}$
II	$\theta = 180^\circ - \arctan\frac{b}{a}$
III	$\theta = 180^\circ + \arctan\frac{b}{a}$
IV	$-\theta = \arctan\frac{b}{a}$

Table 2-4

Calculating the Angle θ For Rectangular To Polar Conversions. In each case, the values of a and b used are the absolute values.

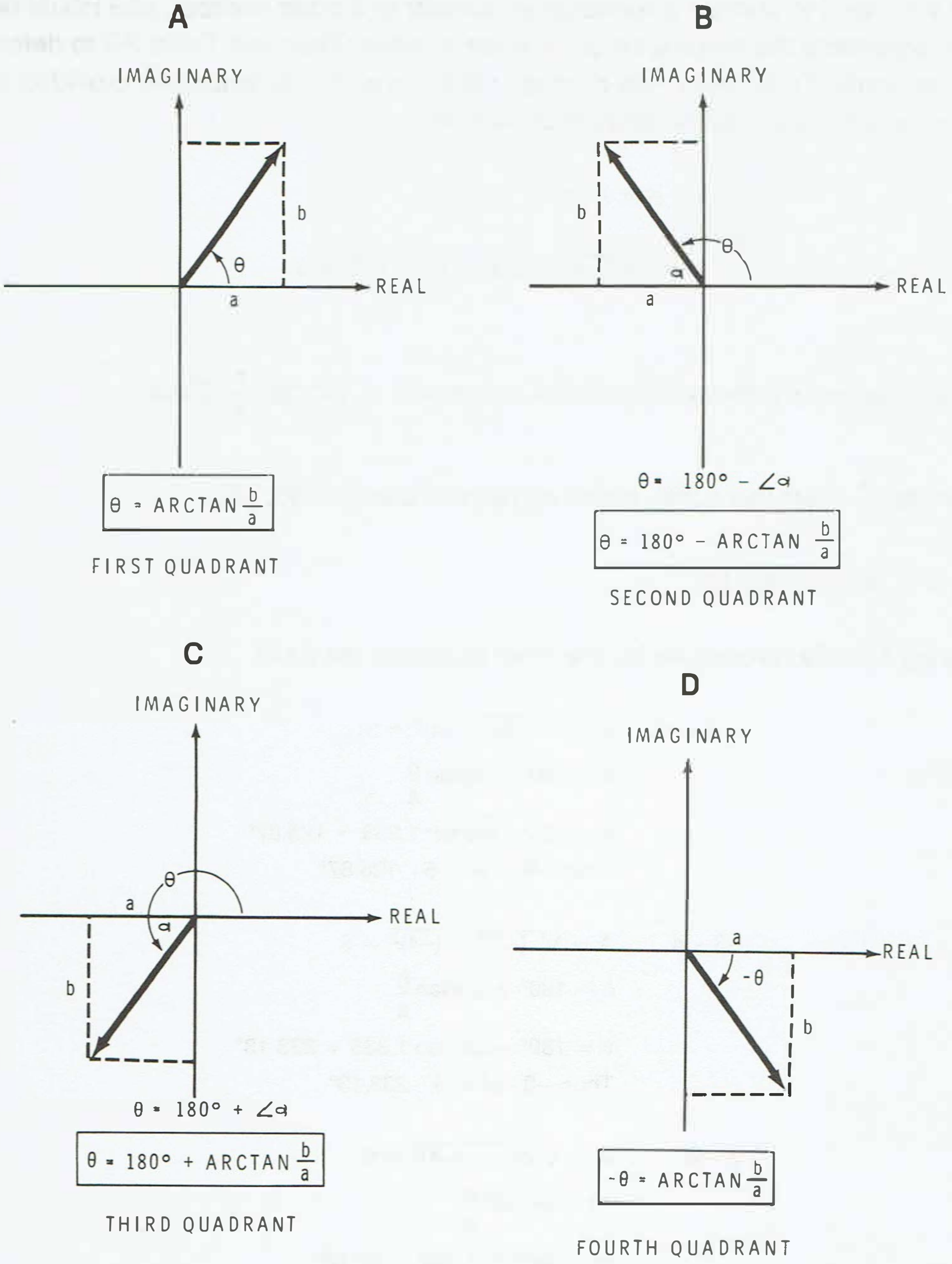


Figure 2-44

Calculating angles in the four quadrants.
A. First quadrant.
B. Second quadrant.
C. Third quadrant.
D. Fourth quadrant.

When you need to convert a rectangular number to a polar number, use Equation 2-45 to determine the magnitude of the polar number. Then use Table 2-7 to determine the angle. To illustrate this process, let's "reverse" the examples provided in the discussion of polar to rectangular conversions.

$$3 + j4 = A\angle\theta$$

$$A = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

For first quadrant numbers in Table 2-4 indicates $\theta = \text{arc tan}\,\frac{b}{a}$. Thus:

$\theta = \text{arc tan}\,\frac{4}{3} = \text{arc tan}\ 1.333$, which corresponds to $\theta = 53.13°$.

Thus, $3 + j4 = 5\angle 53.13°$.

Following a similar procedure for the other numbers, we have

$-3 + j4$ $\quad A = \sqrt{(-3)^2 + (4)^2} = 5$

$\theta = 180° - \text{arc tan}\,\frac{b}{a}$

$\theta = 180° - \text{arc tan}\ 1.333 = 126.87°$

Thus, $-3 + j4 = 5\angle 126.87°$

$-3 - j4$ $\quad A = \sqrt{(-3)^2 + (-4)^2} = 5$

$\theta = 180° + \text{arc tan}\,\frac{b}{a}$

$\theta = 180° + \text{arc tan}\ 1.333 = 233.13°$

Thus, $-3 - j4 = 5\angle 233.13°$

$3 - j4$ $\quad A = \sqrt{(3)^2 + (-4)^2} = 5$

$-\theta = \text{arc tan}\,\frac{b}{a}$

$-\theta = \text{arc tan}\ 1.333 = 53.13°$

Thus $3 - j4 = 5\angle -53.13°$

With a little practice, and the help of a calculator, you will find it relatively easy to convert polar numbers to rectangular numbers and vice versa.

Self Test Review

59. A right triangle has legs of 5cm and 12cm. The hypotenuse is ________ cm.
60. Assuming H = hypotenuse, O = opposite side, and A = adjacent side, sin θ = ________________, cos θ = ________________ and tan θ = ________________.
61. ________________ or ________________ means "the angle whose cosine is".
62. The tangent function is positive in the ____________ and ____________ quadrants.
63. Multiplying a real quantity by j is equivalent to a ________________ degree ________________ rotation.
64. Multiplying a real quantity by –j is equivalent to a ________________ degree ________________ rotation.
65. $8\angle -16^\circ \times 3\angle 32^\circ =$ ________________ $\angle$ ________________.
66. $\frac{25\angle \ 30^\circ}{5\ \angle -15^\circ} =$ ________________ $\angle$ ________________.
67. $20\angle 60^\circ =$ ________________ + j ________________. $30\angle -45^\circ =$ ________________ – j ________________. Therefore, $20\angle 60^\circ + 30\angle -45^\circ =$ ________________ j ________________.
68. If you convert the rectangular equivalent of $20\angle 60^\circ + 30\angle -45^\circ$ to polar form, the result is ________________ $\angle$ ________________.

Answers

59. 13cm
60. $\sin\theta = O/H$, $\cos\theta = A/H$
    $\tan\theta = O/A$
61. arc cos, or $\cos^{-1}$
62. first, third
63. 90°, counterclockwise
64. 90°, clockwise
65. $24, \angle 16°$
66. $5, \angle 45°$
67. $20\angle 60° = 10 + j17.32$
    $30\angle -45° = 21.21 - j21.21$
    $31.21 - j3.89$
68. $31.45\angle -7.1°$

Here are the solutions to question 59 and 65 to 68.

59. $\text{cm} = \sqrt{a^2 + b^2} = \sqrt{(5)^2 + (12)^2} = \sqrt{169} = 13\text{ cm}$

65. $8\angle -16° \times 3\angle 32° = 8(3)\angle -16 + 32) = 24\angle 16°$

66. $\dfrac{25\angle\ 30°}{5\ \angle -15°} = \dfrac{25}{5}\angle(30° - (-15°)) = 5\angle 45°$

67.
$$\begin{aligned} 20\angle 60° &= 20\,(\cos 60°)\ j\,20 \sin 60° \\ &= 20(0.5) + j\,20\,(0.866) \\ &= 10 + j\,17.32 \end{aligned}$$

$$\begin{aligned} 30\angle -45° &= 30 \cos -45° + j\,30 \sin -45° \\ &= 30(0.707) + j\,30\,(-0.707) \\ &= 21.21 - j\,21.21 \end{aligned}$$

$$\begin{aligned} 20\angle 60° + 30\angle -45° &= (10 + j17.32) + (21.21 - j21.21) \\ &= (10 + 21.21) + j(17.32 - 21.21) \\ &= 31.21 - j3.89 \end{aligned}$$

68.
$$\begin{aligned} A &= \sqrt{(31.21)^2 + (3.89)^2} \\ A &= \sqrt{974.06 + 15.13} \\ A &= \sqrt{989.19} = 31.45 \end{aligned}$$

From Table 2-4

$$-\theta = \text{arc tan}\,\frac{a}{b} = \text{arc tan}\,\frac{3.89}{31.21} = \text{arc tan } 0.1246$$

which corresponds to $\theta = -7.1°$

SUMMARY

Here are the key ideas we presented in this unit

The opening or closing of a switch produces abrupt changes in the currents and voltages in a circuit. In physical circuits, instantaneous changes are not possible because physical circuits always exhibit some capacitance and inductance.

The currents and voltages in single loop RC and RL circuits driven from a DC source are exponential in nature. By substituting the initial value, final value, and time constant into the exponential equation, you can obtain an equation for the particular current, or voltage of interest. You can determine the initial values by realizing that capacitor voltage and inductor current cannot change instantaneously. Thus, the value of capacitor voltage or inductor current change the instant just after a switch is opened or closed ($t = 0^+$) must equal the value of capacitor voltage or inductor current the instant just before the switch was thrown ($t = 0^-$).

You can determine the final values by realizing that a capacitor acts like an open circuit, and an inductor acts like a short circuit, once the steady-state values are reached. The time constant of an RC circuit is $R \times C$, and the time constant of an RL circuit is L/R. For practical reasons, we assume that it takes five time constants to reach the steady-state values. Be careful when you are opening inductive circuits because of the possibility of a large self-induced voltage.

The basic exponential function is of the form $y = b^x$. The logarithm of y to the base b is the exponent x. In logarithmic form, $x = \log_b y$. In electronics, both natural (base e) and common (base 10) logarithms are encountered. The symbol used for base e logarithms is ln, while the symbol log is used for base 10 logarithms. Since $\ln y = 2.303 \log y$, it is relatively easy to convert from one base to the other. Some of the properties that logarithms obey include:

$$\log AB = \log A + \log B$$

$$\log \frac{A}{B} = \log A - \log B$$

$$\log A^n = n \log A.$$

Natural logarithms are useful when you want to determine how long it will take for the voltage across a capacitor to reach a particular value. Common logarithms are used to express current, voltage, and/or power gains as logarithmic ratios in units of decibels (dBs). For a current or voltage ratio $A_{dB} = 20 \log A$, and for a power ratio $A_{PdB} = 10 \log A_P$. Since dB notation is used extensively, it is important to be able to convert from, and to, dB notation.

One advantage of dB notation for a series of cascaded sections is the fact that the net dB gain equals the sum of the dB gains of each section. Some instruments have dB scales calibrated to indicate relative values rather than absolute values. In such cases, the subscript m is added to the dB notation to indicate that a reference level is being used.

Real numbers are represented as points on a horizontal line called the real number line. The square root of a negative real number is called an imaginary number. Imaginary numbers are represented as points on a vertical line called the imaginary number line.

A complex number is the algebraic sum of a real and imaginary number. Complex numbers are represented as points in the complex plane formed by the intersection of the real and imaginary number lines. Rectangular complex numbers are of the form $a \pm jb$, where a is the real part, b the imaginary part, and j represents the $\sqrt{-1}$.

You add rectangular numbers by simply adding the real and imaginary parts. You subtract rectangular numbers by converting the given subtraction problem into an equivalent addition problem. The multiplication of two rectangular numbers is similar to multiplying binomial expressions in algebra. The major difference is that j^2 is replaced by -1, and $-j^2$ by $+1$ once the product is obtained. You form the conjugate of a complex number by changing the sign separating the real and imaginary parts. This concept is important, since you divide rectangular numbers by multiplying the numerator and denominator by the conjugate of the denominator.

You can also express complex numbers in polar form. In this case, the distance between the origin and the point in the complex plane is given, as well as the angle between the positive real axis, and a line joining the point and origin. In working with polar numbers, the basic trig functions of sine, cosine, and tangent are employed in conjunction with the Pythagorean theorem.

Multiplication and division of polar numbers is relatively simple. To add and subtract polar numbers, however, you have to first convert the polar numbers to rectangular numbers. Then add or subtract the rectangular numbers and convert the result back to polar form. Convert from polar to rectangular by utilizing the relationship $A\angle\theta = A\cos\theta + jA\sin\theta$. To convert a rectangular number to a polar number, calculate the magnitude of the polar number from $A = \sqrt{a^2 + b^2}$. Determine the angle θ by referring to Table 2-4. Use a scientific calculator when you are working with complex numbers.

At this time, complete Experiment 2 in Unit 8.

UNIT EXAMINATION

The following multiple choice examination is designed to test your understanding of the material presented in this unit. Place a check beside the multiple choice answer (A, B, C, or D) that you feel is most correct. When you have completed the examination, compare your answers with the correct ones that appear after the exam.

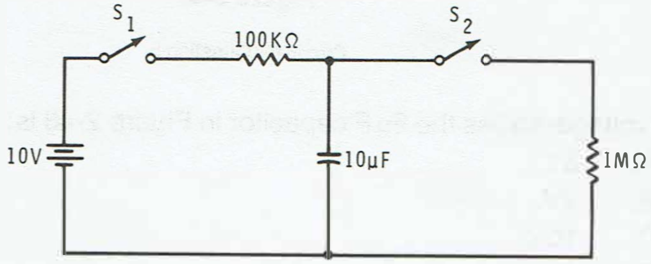


Figure 2-45

Circuit for Questions 1 to 4.

Refer to Figure 2-45 for questions 1 to 4.

1. Initially both switches are open and the capacitor is uncharged. Switch S_1 is closed at $t = 0$. The capacitor voltage reaches 10V after approximately:
   - A. 1 second.
   - B. 5 seconds.
   - C. 25 seconds.
   - D. 25 μs.

2. At $t = 127$ seconds, switch S_1 is opened. The energy stored in the capacitor is:
   - A. 500 μJ.
   - B. 5J
   - C. 500 mJ.
   - D. Zero

3. Ten seconds after switch S_1 is opened, swtich S_2 is closed. The equation for the capacitor voltage from this point on is:
   - A. $10\,(1-e^{-t})$.
   - B. $v_c(t) = 10V$.
   - C. $10\,(1 + e^{-t})$.
   - D. $10\,e^{-t/10}$.

4. Once switch S_2 is closed, the period of time that must elapse before the capacitor voltage equals 6V is:
   - A. 10 seconds.
   - B. 50 seconds.
   - C. 5.108 seconds.
   - D. 76.073 μs.

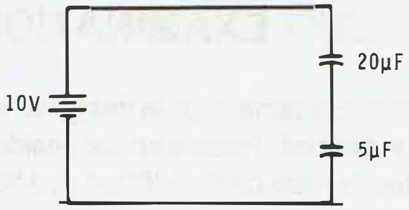


Figure 2-46

Circuit for Question 5.

5. The voltage across the 5μF capacitor in Figure 2-46 is:
   - A. 8V.
   - B. 2V.
   - C. 10V.
   - D. 5V.

Refer to Figure 2-47 for questions 6 to 8.

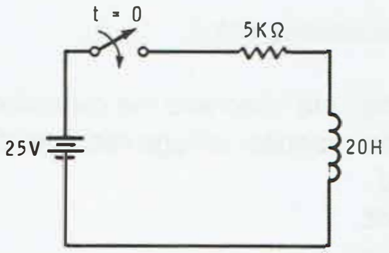


Figure 2-47

Circuit for Questions 6 to 8.

6. Once the switch is closed, the equation that describes the inductor current is:
   - A. $5\text{mA}\,(1-e^{-4000t})$.
   - B. $5\text{mA}\,(1-e^{-250t})$.
   - C. $i_L(t) = 5\text{mA}$.
   - D. $1\text{mA}\,(1-e^{-250t})$.

7. Once the switch is closed, the equation that describes the inductor voltage is:
   - A. $25\,(1-e^{-250t})$.
   - B. $25\,(1-e^{-4000t})$.
   - C. $25\,e^{-4000t}$.
   - D. $25\,e^{-250t}$.

8. In the steady-state, the energy stored in the inductor is:
   - A. 100μJ.
   - B. 75mJ.
   - C. 750μJ.
   - D. 250μJ.

9. If $y = 4^6$, then the logarithm of y to the base 4 is:
   - A. y.
   - B. 4.
   - C. 6.
   - D. 10.

10. If log x = 2.5, then ln x equals:
    - A. 2.718.
    - B. 10.
    - C. 1.0855.
    - D. 5.7575.

11. The dB gain that corresponds to a voltage gain of 100 is:
    - A. 40dB.
    - B. 100dB
    - C. 20dB
    - D. A reference level is needed.

12. An amplifier consists of two cascaded stages whose voltage gains are 4 and 20, respectively. The overall gain of the amplifier is:
    - A. 24dB
    - B. 38dB
    - C. 5dB.
    - D. 20dB.

13. If 20 log x = 100, then x equals:
    - A. 5.
    - B. 100,000.
    - C. 20.
    - D. 100.

14. dBm notation implies:
    - A. Natural logs.
    - B. A reference level.
    - C. Power gain.
    - D. Voltage gain.

15. $10 \angle 90°$ equals:
    - A. j10.
    - B. –j10.
    - C. 10 + j0.
    - D. 10 – j0.

16. $(3 - j2) + (-5 + j6)$ equals
    - A. $2 - j4$.
    - B. $-2 - j4$
    - C. $-2 + j4$.
    - D. $8 + j8$.

17. $(10 - j6) - (3 + j5)$ equals
    - A. $7 - j1$.
    - B. $7 + j1$.
    - C. $7 - j11$.
    - D. $13 - j1$.

18. $(2 - j5)(3 + j4)$ equals:
    - A. $26 - j7$.
    - B. $-14 - j7$.
    - C. $-14 + j7$.
    - D. $14 + j7$.

19. $(26 - j7)$ divided by $(3 + j4)$ equals:
    - A. $2 + j5$.
    - B. $2 - j5$.
    - C. $14 + j7$.
    - D. $-2 + j5$.

20. $3\angle 60 \times 10\angle -20^\circ$ equals:
    - A. $30\angle 40^\circ$.
    - B. $30\angle 80^\circ$.
    - C. $30\angle -40^\circ$.
    - D. $30\angle 60^\circ$.

21. $100\angle 60^\circ$ divided by $20\angle -20^\circ$ equals:
    - A. $5\angle 40^\circ$.
    - B. $5\angle 60^\circ$.
    - C. $5\angle -40^\circ$.
    - D. $5\angle 80^\circ$.

22. $2 - j5$ equals:
    - A. $5.385\angle 68.2^\circ$.
    - B. $5.385\angle -68.2$.
    - C. $5.385\angle 248.2^\circ$.
    - D. $5.385\angle 111.8^\circ$.

23. $100\angle 60^\circ$ equals:
    - A. $50 + j86.6$.
    - B. $86.6 + j50$.
    - C. $50 - j86.6$.
    - D. $86.6 - j50$.

EXAMINATION ANSWERS

1. B — The charging time constant is 100kΩ(10μF) or 1s, since five time constants are required to reach the steady state t = 5(1) = 5s.

2. A — The energy stored in a capacitor is

$W = \frac{1}{2}CV^2$. Thus, $W = \frac{1}{2}(10\mu F)(100V)^2 = 500\mu J$.

3. D — The discharging time constant is 1MΩ(10μF), or 10s. Substituting $\tau = 10s$, $V_1 = 10V$, and $V_2 = 0$ into the exponential equation yields:

$$\nu_c(t) = (V_1 - V_2)\,e^{-t/\tau} + V_2$$
$$\nu_c(t) = (10 - 0)\,e^{-t/10} + 0$$
$$\nu_c(t) = 10\,e^{-t/10}$$

4. C — Substituting $\tau = 10s$, $V_1 = 10V$, $V_2 = 0$, and $\nu(t) = 6V$ into Equation 2-22 yields:

$$t = -\tau \ln \frac{\nu(t) - V_2}{V_1 - V_2}$$
$$t = -10 \ln \frac{6-0}{10-0} = -10 \ln 0.5 = 5.108s$$

5. A — The series 20μF and 5μF capacitors have a total capacitance of C_T = 20(5)/20 + 5, or 4μF. Using the formula for capacitive voltage division, you obtain:

$$V_N = V_T\,(C_T)/C_N = 10V(4\mu F)/5\mu F = 8V$$

6. B — The time constant is L/R or 20H/5kΩ = 4ms. The initial inductor current I_1 is 0, and the final inductor current I_2 is V/R or 25V/5kΩ = 5mA. Substituting these values into the exponential equation yields:

$$i_L(t) = (I_1 - I_2)\,e^{-t/\tau} + I_2$$
$$i_L(t) = (0 - 5mA)\,e^{-t/4ms} + 5mA$$
$$i_L(t) = 5mA - 5mA\,e^{-250t} = 5mA\,(1 - e^{-250t})$$

7. D — τ = 4ms. The initial inductor voltage V_1 is 25V, and the final inductor voltage V_2 is 0. Substituting these values into the exponential equation yields:

$$\begin{aligned}\nu_L(t) &= (V_1 - V_2)\,e^{-t/\tau} + V_2\\ &= (25 - 0)\,e^{-t/4ms} + 0\\ &= 25\,e^{-250t}\end{aligned}$$

8. D — The energy stored in an inductor is $W = 1/2\ LI^2$. Thus, $W = 1/2\ (20H)(5mA)^2 = 250\mu J$.

9. C — The logarithm of 4^6 to the base 4 is the exponent; which in this case is 6.

10. D — To convert from common logs to natural logs, you use the following formula: $\ln y = 2.303 \log y$. Thus, $\ln x = 2.303(2.5) = 5.7575$

11. A — For a voltage or current ratio, $A_{dB} = 20 \log A$. Thus, $A_{dB} = 20 \log 100 = 40dB$.

12. B — The total gain is 4(20) or 80. Since this is a voltage ratio, $A_{dB} = 20 \log 80 \approx 38dB$.

13. B —

$$
\begin{aligned}
&20 \log x = 100 \\
&\log x = 100/20 = 5 \\
&\text{Therefore, } x = 10^5 \text{ or } 100\,000
\end{aligned}
$$

14. N — The m is added to indicate that a reference level is being used.

15. A — Multiplying a real number by j is equivalent to a 90° counterclockwise rotation. Thus, $10 \angle 90° = j10$.

16. C —

$$
\begin{aligned}
(3 - j2) + (-5 + j6) &= (3 - 5) + j(-2 + 6) \\
&= -2 + j4
\end{aligned}
$$

17. C —

$$
\begin{aligned}
(10 - j6) - (3 + j5) &= (10 - j6) + (-3 - j5) \\
&= (10 - 3) + j(-6 - 5) \\
&= 7 - j11
\end{aligned}
$$

18. A —

$$
\begin{aligned}
(2 - j5)(3 + j4) &= 2(3 + j4) - j5(3 + j4) \\
&= 6 + j8 - j15 - j^2 20 \\
&= 6 + j8 - j15 + 20 \\
&= 26 - j7
\end{aligned}
$$

19. B —

$$
\begin{aligned}
\frac{26 - j7}{3 + j4} &= \frac{(26 - j7)}{(3 + j4)} \cdot \frac{(3 - j4)}{(3 - j4)} \\
&= \frac{26(3 - j4) - j7(3 - j4)}{3^2 + 4^2} \\
&= \frac{78 - j104 - j21 - j^2 28}{9 + 16} \\
&= \frac{78 - j104 - j21 - 28}{25} \\
&= \frac{50 - j125}{25} = 2 - j5
\end{aligned}
$$

20. A — $3\angle 60° \times 10\angle -20° = 3(10)\angle(60° - 20°) = 30\angle 40°$

21. D — $\dfrac{100\angle\ 60°}{20\ \ \angle -20°} = \dfrac{100}{20}\angle(60° - (-20°)) = 5\angle 80°$

22. B — $A = \sqrt{a^2 + b^2} = \sqrt{(2)^2 + (-5)^2} = \sqrt{29} = 5.385$

2 – j5 lies in the fourth quadrant. Thus, $-\theta = \arctan \frac{b}{a} = \arctan \frac{5}{2} =$ arc tan 2.5 which corresponds to $\theta = -68.2°$

23. A —

$$\begin{aligned} 100\angle 60° &= 100 \cos 60° + j100 \sin 60° \\ &= 100(0.5) + j100(0.866) \\ &= 50 + j86.6 \end{aligned}$$

Unit 3

AC CIRCUIT ANALYSIS

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INTRODUCTION

Generally speaking, the currents and voltages in DC circuits are constant. In RC and RL DC circuits, abrupt changes due to the signal source, or a switch, produce response that exponentially rise or decay from an initial constant value to a final constant value. This transient condition lasts for a relatively short time, equal to approximately five time constants. Recall that the effects of capacitance and inductance were significant **only** during the relatively brief transient period — when the responses were changing.

An AC source is one that periodically reverses polarity. Thus, current flows first in one direction, then the other. Since the responses are continually changing, both capacitance and inductance oppose the flow of AC current. By using complex numbers, the techniques of DC circuit analysis can be extended to include the analysis of AC circuits. In this unit, you will learn how complex numbers are used to determine the currents, and voltages in AC circuits.

UNIT OBJECTIVES

When you have completed this unit, you should be able to:

1. Define: peak value, peak-to-peak value, average value, period, cyclic frequency, radian frequency, and phase angle.
2. Distinguish between pulsating DC, pure DC, and AC waveforms.
3. Convert time domain to frequency domain expressions, and vice versa.
4. Apply the techniques of DC circuit analysis to elementary AC circuits by using complex notation.
5. Calculate the power supplied by a source to an AC circuit.
6. Define circuit phase angle and power factor.
7. Determine the load impedance required for maximum power transfer.
8. Summarize the characteristics of series and parallel resonant circuits.
9. Construct the frequency spectrum of a complex wave when given the Fourier series for the wave.

UNIT ACTIVITY GUIDE

☐ Read section on "Sinusoidal Notation".

☐ Answer Self-Test Review questions 1-18.

☐ Read section on "Reactance and Impedance".

☐ Answer Self-Test Review questions 19-34.

☐ Perform Experiment 3 in Unit 8.

☐ Read section on "Selected AC Topics".

☐ Answer Self-Test Review questions 35-42.

☐ Study Summary.

☐ Complete Unit Examination.

☐ Check Examination Answers.

SINUSOIDAL NOTATION

A **periodic** signal is one that repeats its basic pattern at regular time intervals, called a period (abbreviated *T*). There are many types of periodic waveforms that are important in electronics. Some examples include square, sawtooth, and triangular waveforms. The sine wave, however, is without question the single most important waveform you will encounter in your study of electronics. As the electrical industry developed, it was quickly discovered that it is much easier to generate and economically transmit sinusoidal voltages than DC voltages. In addition, non-sinusoidal waveforms, like square waves, sawtooths, and triangular waves, can be represented as the sum of a series of sine waves. For these reasons, it is essential that you learn how to describe sine waves, and how to analyze circuits with sinusoidal sources.

Time Varying Signals

The term DC stands for Direct Current. This simply means that the current flows in one direction. The term **pure DC** is used to describe a current that flows in one direction and always has the same value. Similarly, a voltage would be described as pure DC, if it always had the same polarity and magnitude. If a current flows in one direction and varies in value (or if a voltage has one polarity and varies in magnitude), the term **pulsating DC** is used to describe the current (or voltage). Figure 3-1 illustrates some examples of pure and pulsating DC waveforms. Note that the values of the pulsating DC waveforms change with time. For this reason, the pulsating DC waveforms are considered to be **time-varying** signals. Also note that the period of the pulsating DC waveforms in Figure 3-1B is 1ms, since this is the amount of time required for the waveforms to complete one **cycle**.

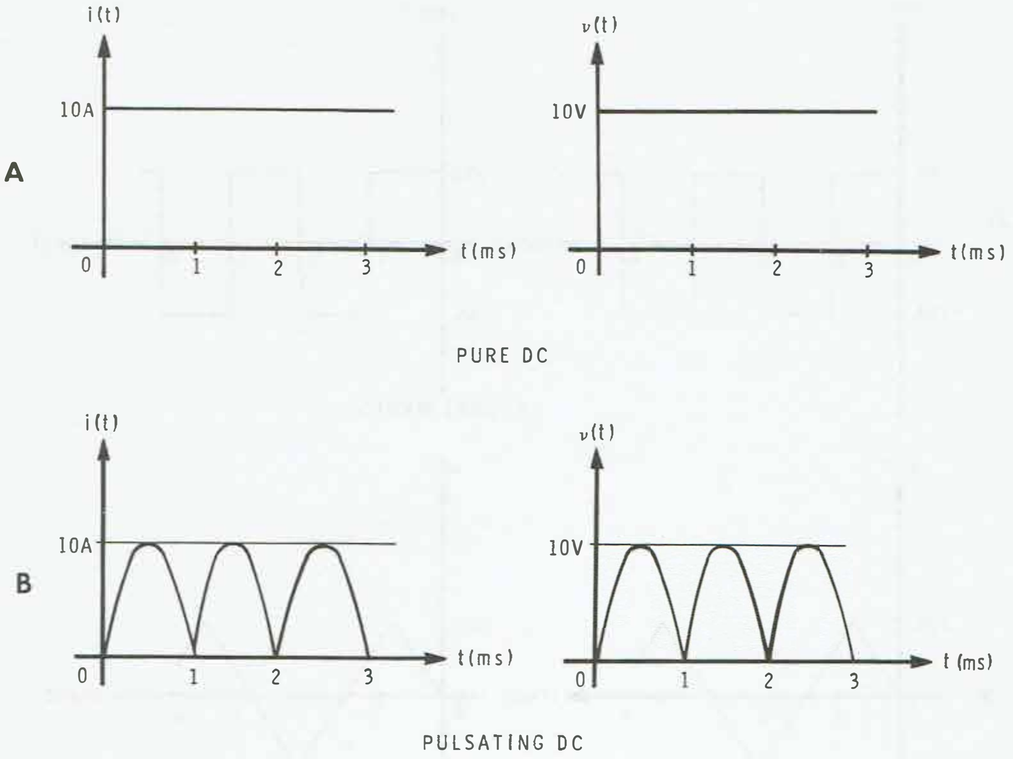
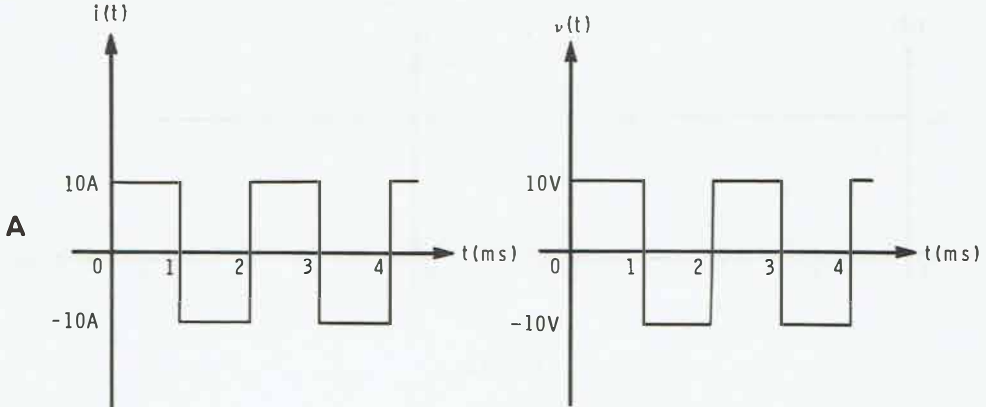
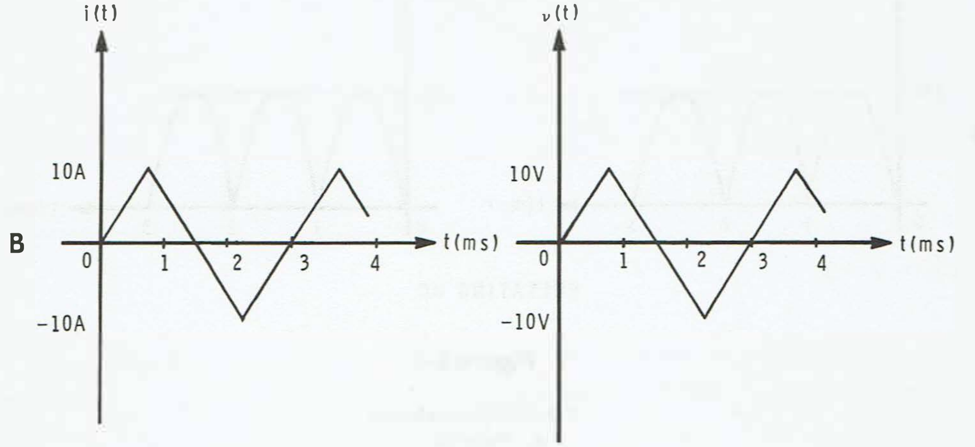


Figure 3-1

Typical DC waveforms
A Pure DC
B Pulsating DC



SQUARE WAVES



TRIANGULAR WAVES

Figure 3-2

Typical AC waveforms
A Square waves
B Triangular waves

AC is an abbreviation for alternating current. An AC voltage is one whose polarity reverses each half cycle. Thus, current flows first in one direction, then the other. Some examples of typical AC waveforms are illustrated in Figure 3-2. You should note that the period of the square waves is 2mS, and the period of the triangular waves is 3mS.

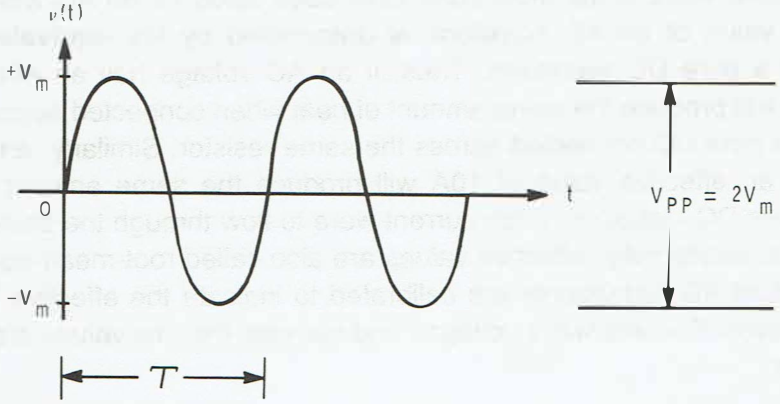


Figure 3-3

The sine wave

The Sine Wave

Figure 3-3 illustrates a sine wave voltage. This is the type of AC waveform that dominates the electronics industry. Since the value of a sinusoidal current or voltage continually changes, you must be specific when describing the value of the waveform. In order to do this, you should be familiar with the following terms.

PEAK VALUE

This is the maximum positive or negative value of the waveform. As shown in Figure 3-3, the positive and negative peak values of the sine wave are equal. The term maximum value is also used to indicate the peak value. Thus, $V_p = V_m$.

PEAK-TO-PEAK VALUE

This is the value from the positive peak to the negative peak. Thus, $V_{pp} = 2V_p = 2V_m$.

EFFECTIVE VALUE (rms)

The effective value is the most commonly used value for an AC waveform. The effective value of an AC waveform is determined by the **equivalent heating effect** of a pure DC waveform. Thus, if an AC voltage has an effective value of 10V, it will produce the same amount of heat when connected across a resistor as 10V of pure DC connected across the same resistor. Similarly, an AC current that has an effective value of 10A will produce the same amount of heat as 10A of pure DC, assuming each current were to flow through the same (or equal) resistance. Incidentally, effective values are also called root-mean-square or rms values. Most AC instruments are calibrated to indicate the effective (rms) value of sine waves. For sine wave voltages and currents the rms values are calculated as follows:

$$V_{rms} = 0.707\,V_m \qquad \text{(Eq. 3-1)}$$

$$I_{rms} = 0.707\,I_m \qquad \text{(Eq. 3-2)}$$

where V_{rms} and I_{rms} denote effective values
V_m and I_m denote peak values.

If you wish to convert rms values to peak values, use the following formulas.

$$V_m = \frac{V_{rms}}{0.707} = 1.414\,V_{rms} \qquad \text{(Eq. 3-3)}$$

$$I_m = \frac{I_{rms}}{0.707} = 1.414\,I_{rms} \qquad \text{(Eq. 3-4)}$$

AVERAGE (DC) VALUES

Graphically, the average value of a waveform equals the area under the waveform divided by the base of the waveform. The **true average value** of a sine wave is zero, because the waveform has the same amount of positive (above zero) area and negative (below zero) area. Thus, the net area is zero, which makes the true average value zero. The average value of either the positive, or negative portion of a sine wave is given by:

$$V_{avg} = 0.636\,V_m \qquad \text{(Eq. 3-5)}$$

$$I_{avg} = 0.636\,I_m \qquad \text{(Eq. 3-6)}$$

Naturally, for the negative portion of a sine wave, the average value is negative. Also the average value is positive for the positive portion of the sine wave. Figure 3-4 illustrates the concept of average values for a sine wave. The term DC value is often used to mean average value since a DC instrument will provide a reading equal to the **true** average value if it is used to measure an AC waveform. For this reason, a DC voltmeter will read 0V if it is used to measure a sine wave voltage.

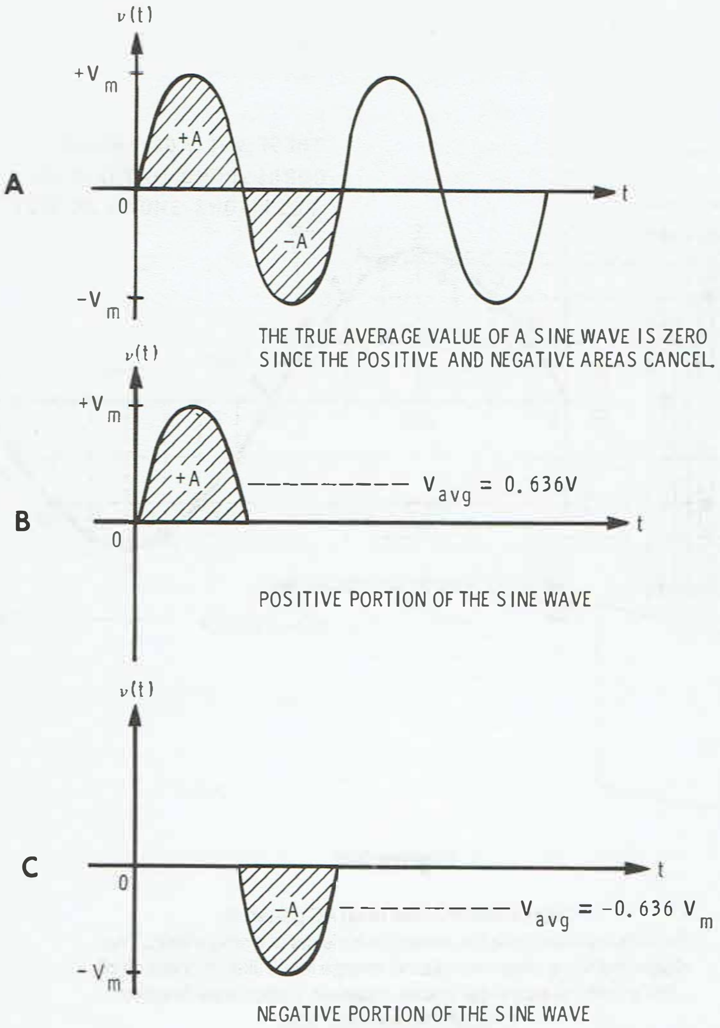

A
v(t)
+V_m
+A
-A
0
t
-V_m
THE TRUE AVERAGE VALUE OF A SINE WAVE IS ZERO SINCE THE POSITIVE AND NEGATIVE AREAS CANCEL.
B
v(t)
+V_m
+A
V_avg = 0.636V
0
t
POSITIVE PORTION OF THE SINE WAVE
C
v(t)
0
t
-A
V_avg = -0.636 V_m
-V_m
NEGATIVE PORTION OF THE SINE WAVE

Figure 3-4

The concept of average value

A The true average value of a sine wave is zero since the positive and negative areas cancel

B Positive portion of the sine wave

C Negative portion of the sine wave

PERIOD

This is the amount of time required to generate one complete cycle of the waveform. The symbol T is used to indicate the period of the waveform.

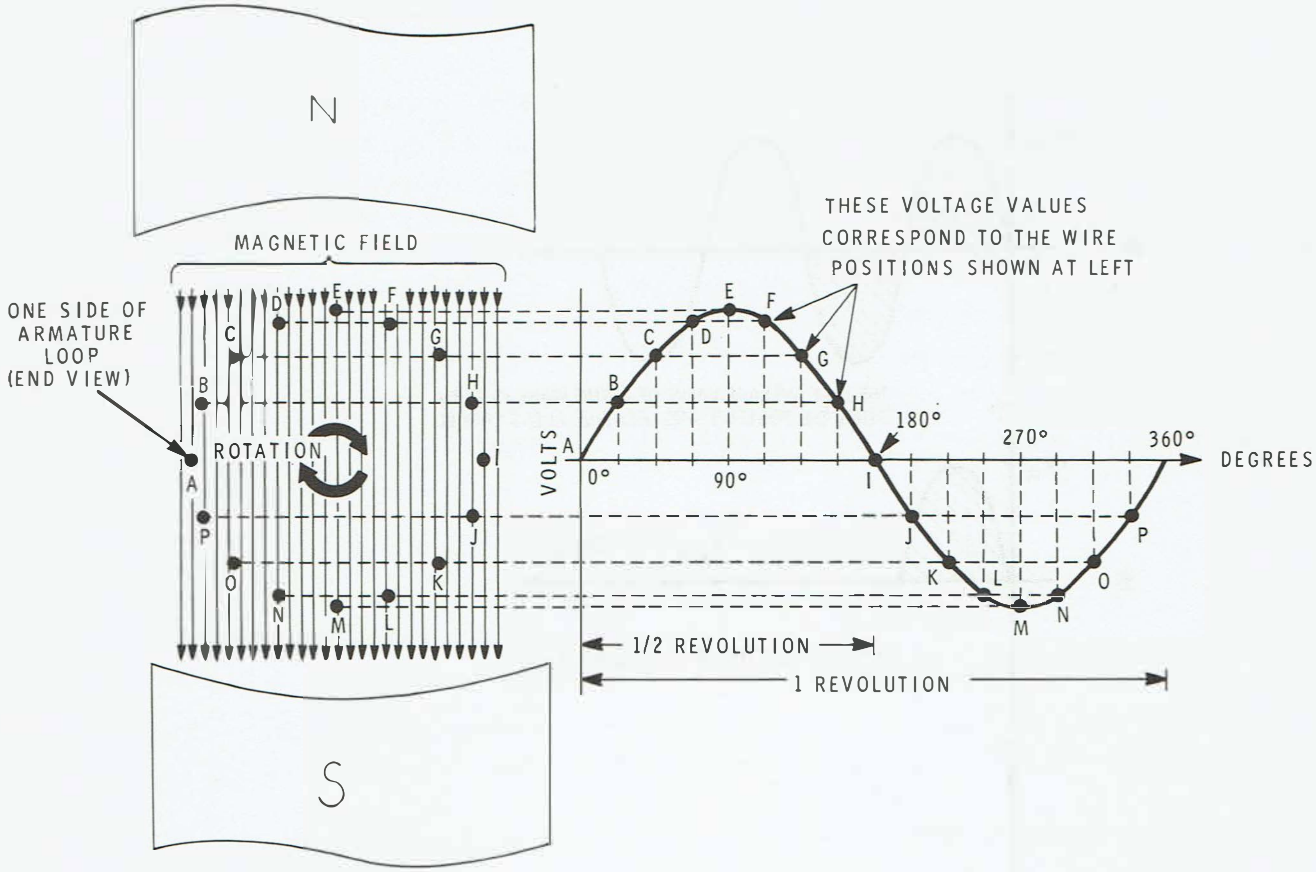

N
S
MAGNETIC FIELD
ONE SIDE OF ARMATURE LOOP (END VIEW)
ROTATION
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
THESE VOLTAGE VALUES CORRESPOND TO THE WIRE POSITIONS SHOWN AT LEFT
VOLTS
0°
90°
180°
270°
360°
DEGREES
1/2 REVOLUTION
1 REVOLUTION

Figure 3-5

Cross sectional view of an AC generator
In this simplified case the armature consists of a single loop. The diagram shows what the induced voltage looks like (in one side of the armature) as the conductor rotates in a clockwise direction through the magnetic field.

FREQUENCY (CYCLIC)

The cyclic frequency indicates the number of cycles generated each second. Usually the word cyclic is deleted. Thus, the term frequency means cyclic frequency. The units of frequency are normally given in Hertz (abbreviated Hz). One Hz is equivalent to one cycle per second. The relationship between frequency and period is given by the following formula:

$$f = \frac{1}{T} \quad \text{(Eq. 3-7)}$$

For example, a sine wave whose period is 1ms has a frequency of 1/1ms or 1kHz.

RADIAN FREQUENCY

A sine wave voltage can be generated by rotating a conductor, in the form of a loop of wire called an armature, in a magnetic field. Figure 3-5 illustrates a simple AC generator, and the resulting sine wave output voltage. This particular generator employs a single pair of north-south magnetic poles. Naturally, practical AC generators have armatures consisting of many loops of wire, and several pairs of north-south magnetic poles. Generally speaking, the frequency of the generated voltage equals the product of the angular velocity of the armature in revolutions per second and the number of pairs of north-south magnetic poles. Since one complete cycle corresponds to 2π radians (360 degrees), the angular velocity in radians per second equals 2π times the frequency. Another name for angular velocity is **radian frequency**. Thus:

$$\omega = 2\pi f \qquad \text{(Eq. 3-8)}$$

where ω = radian frequency in rad/s
f = frequency in Hz.

A **radian** is simply a unit of angular measure. An angle of one radian is equivalent to (approximately) 57.3°.

Notice in Figure 3-5 that the horizontal axis is labeled in degrees rather than units of time. Since radians are also units of angular measure, the horizontal axis could also have been labeled in units of radians. The sine wave derives its name from the fact that it changes in value according to the trigonometric sine function. Thus, the horizontal axis of any sine curve may be labeled in units of time, degrees, or radians as illustrated in the following example.

Example 3-1

A 60 Hz, 10V peak sine wave voltage is available from a signal generator. Sketch the voltage with the horizontal axis labeled in (a) units of time, (b) units of degrees, and (c) units of radians.

(a) Since the frequency is 60 Hz, the period is:

$$T = \frac{1}{f} = \frac{1}{60\ \text{Hz}} = 16.67\ \text{ms.}$$

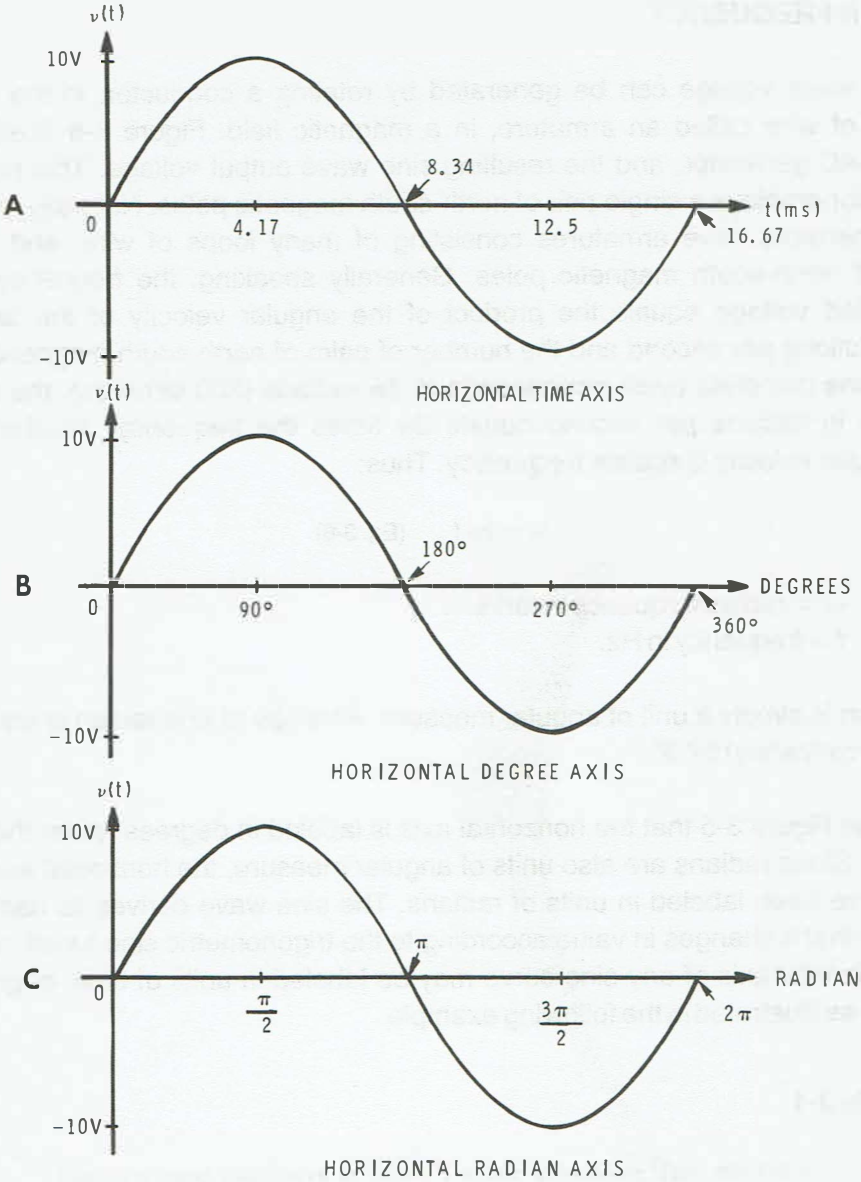


Figure 3-6

Sketches for Example 3-1
A Horizontal time axis
B Horizontal degree axis
C Horizontal radian axis

Clearly, the time required to complete one-fourth, one-half, and three-fourths of a cycle equals the period (16.67ms) times 0.25, 0.5, and 0.75 respectively as shown in Figure 3-6A.

(b) Since $T = 360°$

$$T/4 = 0.25\,(360°) = 90°$$
$$T/2 = 0.5\,(360°) = 180°$$
$$3T/4 = 0.75\,(360°) = 270°$$

(c) Similarly, since $T = 360° = 2\pi$ radians

$$T/4 = \pi/2 \text{ rad}$$
$$T/2 = \pi \text{ rad}$$
$$3T/4 = 3\pi/2 \text{ rad}$$

Figure 3-6B and Figure 3-6C illustrate the respective sketches.

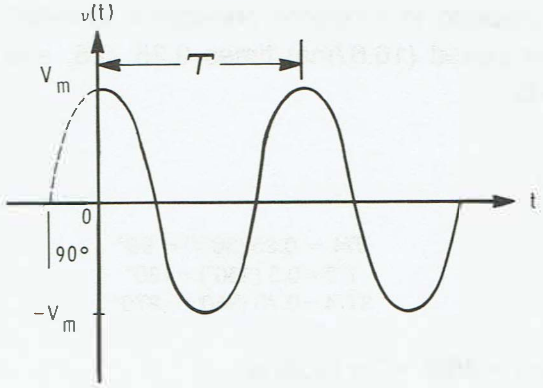


Figure 3-7

The cosine wave

PHASE ANGLE

A cosine wave changes in value according to the trigonometric cosine function. Figure 3-7 illustrates a cosine wave voltage. Note that the cosine wave has the **same shape** as a sine wave. In fact, the only difference between a sine and cosine wave is that a cosine wave starts 90° earlier than a sine wave. For this reason, a cosine wave is said to **lead** a sine wave by 90°. Conversely a sine wave is said to **lag** a cosine wave by 90°. Figure 3-8 illustrates a sine ($\nu_1(t)$) and cosine ($\nu_2(t)$) wave sketched on the same axis. Note that the cosine wave reaches its maximum value 90° earlier than the sine wave. Since sine and cosine waves have the same shape, the more general term **sinusoid** is used to describe either function. The mathematical description of a sinusoid is given by:

$$\nu(t) = V_m \sin(\omega t + \theta) \qquad \text{(Eq. 3-9)}$$

where V_m = peak value
ω = radian frequency
θ = phase angle

In electronics, the phase angle is usually expressed in degrees (mathematicians prefer units of radians). Naturally, the equation for a sinusoidal current is similiar to Equation 3-9; the only difference is that V_m is replaced by I_m. Equation 3-9 indicates that sinusoids may differ from each other in either **peak value** (V_m), **radian frequency** (ω), **phase angle** (θ), or some combination of the three. In most cases the frequency of the sources is constant. Thus, in most circuits sinusoidal currents and voltages will differ only in their peak values, and/or phase angles.

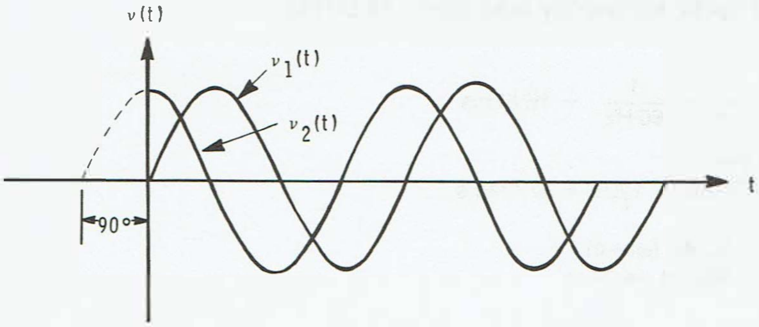


Figure 3-8

Sine and cosine waves
The cosine wave $v_2(t)$ **leads** the sine wave $v_1(t)$ by 90°. Alternately you can say the sine wave $v_1(t)$ **lags** the cosine wave $v_2(t)$ by 90°. Both sine and cosine waves may be described by the more general term sinusoid.

Example 3-2

Most appliances in the United States are designed to operate from a 115V rms, 60 Hz source. From this information determine:

(a) The peak voltage.
(b) The peak-to-peak voltage.
(c) The effective voltage.
(d) The average voltage.
(e) The cyclic frequency.
(f) The period.
(g) The radian frequency.
(h) An equation that describes the voltage. Assume the phase angle is 0°.

(a) $V_m = 1.414\, V_{rms}$
$V_m = 1.414\,(115V) = 162.61V$

(b) $V_{pp} = 2V_m$
$V_{pp} = 2(162.61V) = 325.22V$

(c) The effective voltage is just another name for the rms voltage. Therefore $V_{eff} = V_{rms} = 115V$

(d) Since the voltage is sinusoidal its true average is zero. Thus, $V_{avg} = 0$

(e) The cyclic frequency was given as 60 Hz.

(f) $T = \frac{1}{f} = \frac{1}{60\text{ Hz}} = 16.67\text{ms}$

(g) $\omega = 2\pi f$
$\omega = 2\pi(60) = 120\pi = 377\text{ rad/s}$

(h) $\nu(t) = V_m \sin(\omega t + \theta)$
$\nu(t) = 162.61 \sin 377\, t$

Figure 3-9 illustrates a sketch of the voltage.

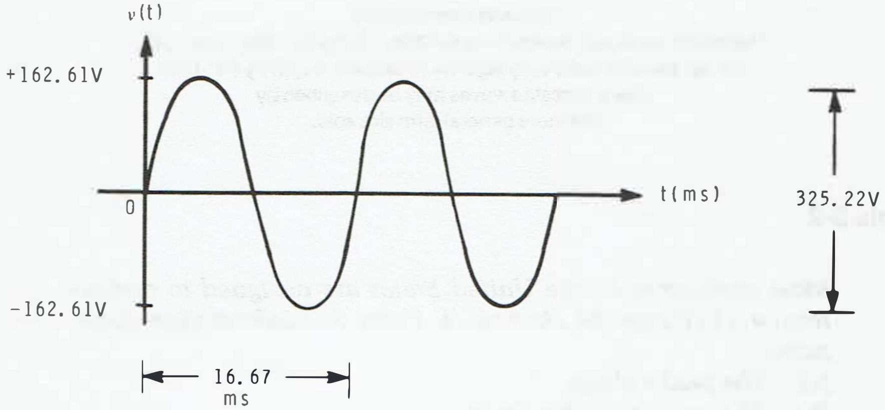


Figure 3-9

Sketch of outlet voltage for Example 3-2

Example 3-3

Sketch the following voltages:
(a) $\nu_1(t) = 10 \sin 1885\, t$
(b) $\nu_2(t) = 10 \sin(1885t + 45°)$
(c) $\nu_3(t) = 10 \sin(1885t - 45°)$
How do these voltages differ from each other?

In each case, the peak voltage (10V), and radian frequency (1885 rad/s) is the same. Thus, the voltages are identical except for their respective phase angles. Figure 3-10 illustrates a sketch of the three sinusoids. Note that ν_2 (t) leads ν_1 (t) by 45° while ν_3 (t) lags ν_1 (t) by 45°.

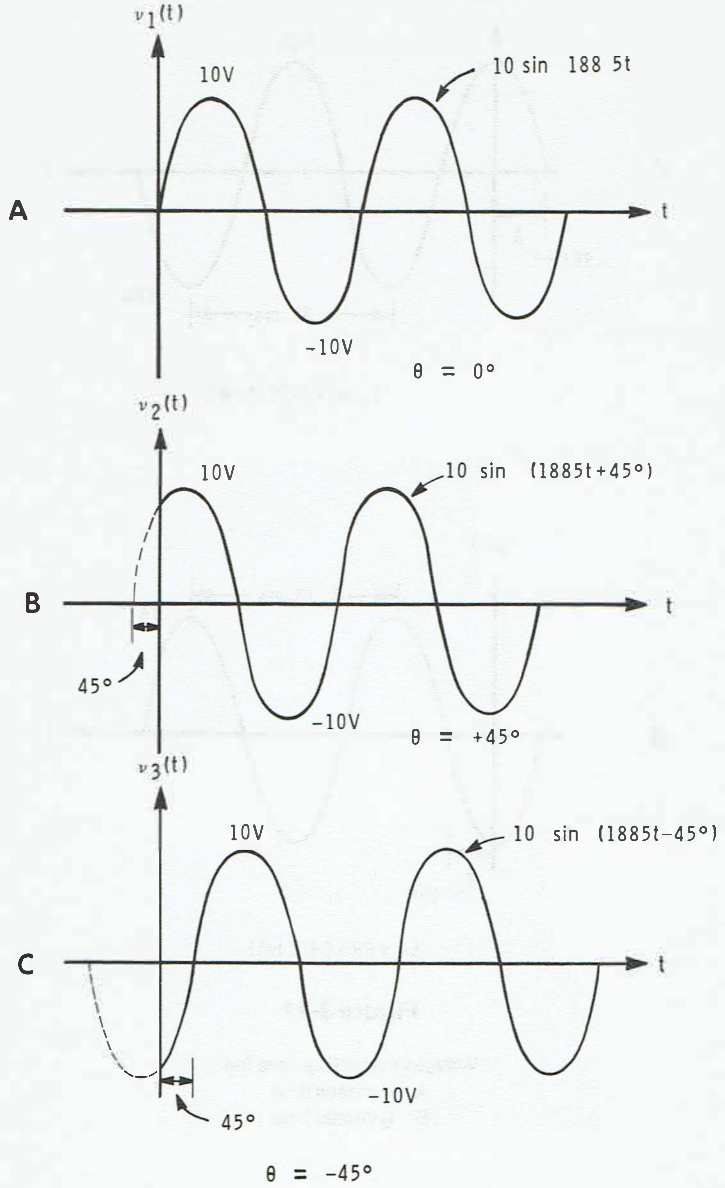
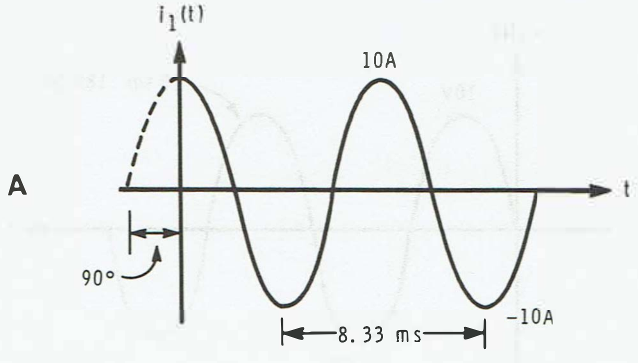
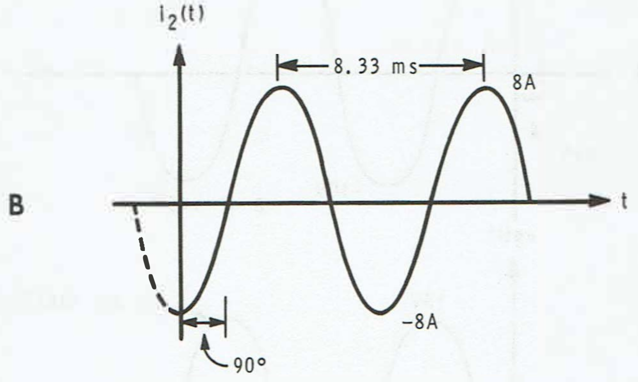


Figure 3-10

Three voltages that differ only in phase
A θ = 0°
B θ = +45°
C θ = −45°



i_1 VERSES TIME



i_2 VERSES TIME

Figure 3-11

Waveforms for Example 3-4
A i_1 verses time
B i_2 verses time

Example 3-4

Write equations that describe the currents in Figure 3-11A and B respectively.

Note that the period of each current is 8.33ms. Thus,

$$f = \frac{1}{T} = \frac{1}{8.33\text{ms}} = 120\text{ Hz}$$

The radian frequency (ω) is therefore:

$$\omega = 2\pi f = 2\pi(120) = 754 \text{ rad/s}$$

In Figure 3-11A, $I_m = 10A$, and $\theta = 90°$. Thus:

$$i_1(t) = I_m \sin(\omega t + \theta)$$
$$i_1(t) = 10 \sin(754t + 90°)$$

In Figure 3-11B, $I_m = 8A$, and $\theta = -90°$. Thus:

$$i_2(t) = I_m \sin(\omega t + \theta)$$
$$i_2(t) = 8 \sin(754t - 90°)$$

Incidentally, $i_1(t)$ may be described by:

$$i_1(t) = 10 \cos 754t$$

This is valid because the only difference between a sine and cosine wave is that a cosine wave starts 90° earlier than a sine wave. In this course, we will express all sinusoids as sine functions. However, if you needed to convert from sine notation to cosine notation or vice versa, you could use the following "trigonometric identities".

$$\cos(\omega t + \theta) = \sin(\omega t + \theta + 90°) \quad \text{(Eq. 3-10)}$$
$$\sin(\omega t + \theta) = \cos(\omega t + \theta - 90°) \quad \text{(Eq. 3-11)}$$

Thus, to convert from cosine to sine notation, you add 90°, and to convert from sine to cosine notation, you subtract 90°.

Example 3-5

(a) Convert $20 \cos(100t - 30°)$ to sine notation
(b) Convert $16 \sin(50t + 60°)$ to cosine notation

(a) $20 \cos(100t - 30°) = 20 \sin(100t - 30° + 90°)$
$= 20 \sin(100t + 60°)$

(b) $16 \sin(50t + 60°) = 16 \cos(50t + 60° - 90°)$
$= 16 \cos(50t - 30°)$

Frequency Domain Notation- Phasors

Expressions like $i(t) = I_m \sin(\omega t + \theta)$, and $\nu(t) = V_m \sin(\omega t + \theta)$ are called **time domain** expressions because it is obvious that the values of i(t) and $\nu(t)$ vary with time. In the simple generator of Figure 3-5, you saw that voltage values at a particular instant in time corresponded to specific positions of the armature — as the armature rotated (at ω rad/s) through the magnetic field. This suggests that, for **a constant frequency**:

A sinusoid may be described by providing its amplitude and phase angle.

Specifically the amplitude of a sinusoid is expressed as its **rms value**, and its phase angle is given in **degrees**.

Thus, the time domain expressions:

$$V_m \sin(\omega t + \theta) \text{ and } I_m \sin(\omega t + \theta)$$

are equivalent to

$$V_{rms} \angle \theta \quad \text{and} \quad I_{rms} \angle \theta$$

Expressions like $V_{rms} \angle \theta$, and $I_{rms} \angle \theta$ are called **frequency domain** expressions because they contain only amplitude and phase information. In addition $V_{rms} \angle \theta$, and $I_{rms} \angle \theta$ are called **phasors**. From your knowledge of complex numbers, you can see that a phasor voltage or current is described by a polar complex number. This fact will be very useful when you analyze AC circuits. The following examples illustrate how to convert time domain expressions to frequency domain expressions and vice versa.

Example 3-6

Convert the following time domain expressions to frequency domain expressions:

(a) $v_1(t) = 10 \sin 1885t$
(b) $v_2(t) = 10 \sin(1885t + 45°)$
(c) $v_3(t) = 10 \sin(1885t - 45°)$

In each case, $V_m = 10V$. Thus:

$$V_{rms} = 0.707\,V_m = 0.707\,(10V) = 7.07V$$

Therefore:

(a) $V_1 = 7.07V\angle 0°$
(b) $V_2 = 7.07V\angle 45°$
(c) $V_3 = 7.07V\angle -45°$

Note that capital letters are used to indicate phasor quantities. Table 3-1 summarizes the time to frequency domain conversions.

Time Domain	Frequency Domain
$v_1(t) = 10 \sin 1885t$	$V_1 = 7.07\angle 0°$
$v_2(t) = 10 \sin(1885t + 45°)$	$V_2 = 7.07\angle 45°$
$v_3(t) = 10 \sin(1885t - 45°)$	$V_3 = 7.07\angle -45°$

Table 3-1

Time to frequency domain conversions for Example 3-6

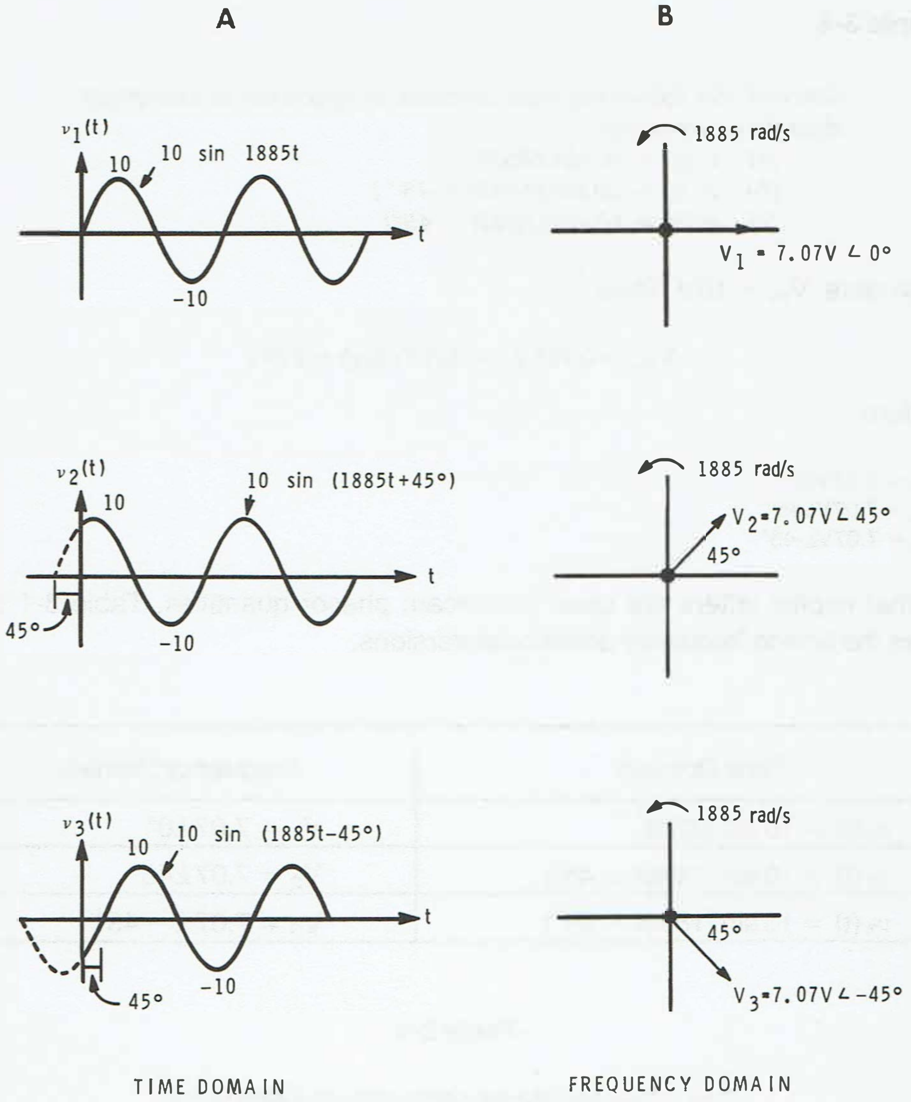


Figure 3-12

Time and frequency domain sketches for Example 3-6.
A Time domain
B Frequency domain

Figure 3-12 illustrates the time and frequency domain sketches of the voltages in Example 3-6. Since they illustrate phasor quantities, frequency domain sketches are called **phasor diagrams**. Notice in Figure 3-12B that the phasors rotate counterclockwise about the origin at ω rad/s to remind you of the relationship that exists between cyclic and radian frequency. Phasor diagrams are especially useful when you wish to compare the phase relationships of two or more sinusoids.

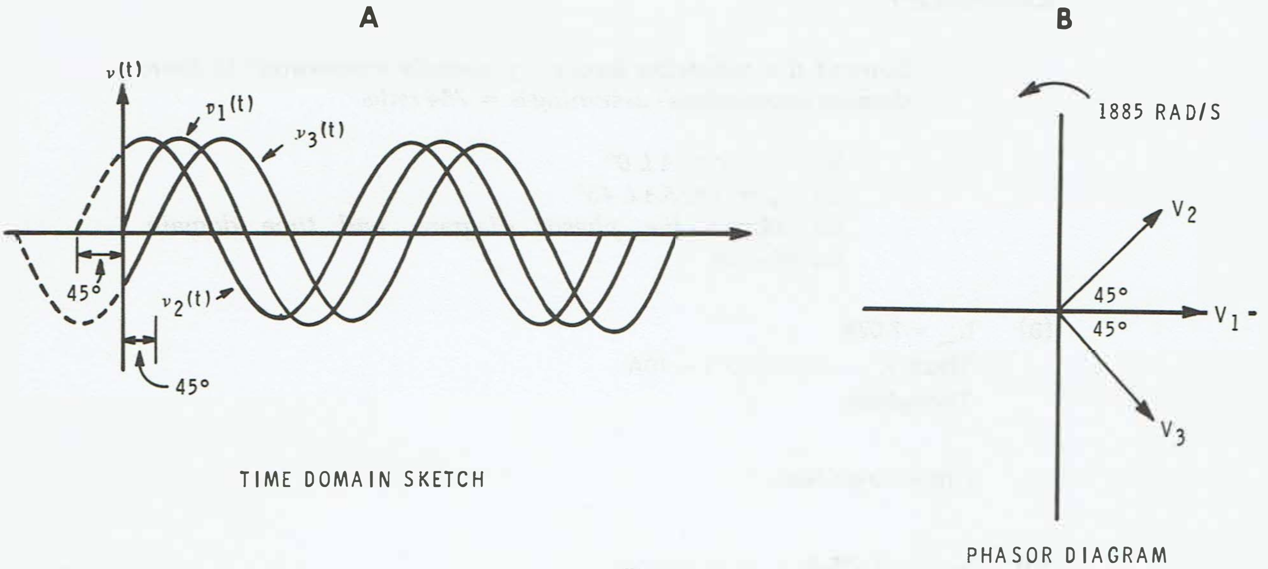


Figure 3-13

Combined time and frequency domain sketches for Example 3-6
It is much easier to compare phase relationships with phasor
diagrams than with time domain sketches.

Figure 3-13A illustrates the three time domain voltages of Example 3-6 sketched on the same scale. Naturally, this is the way the voltages would appear on an oscilloscope (assuming, of course, that the oscilloscope had three channels). If you wish to compare the phase relationships of each voltage with the others, it is much easier to sketch the phasor diagram shown in Figure 3-13B. As the phasors in Figure 3-13B rotate about the origin, the angles between them do not change. Thus, it is relatively easy to see that:

V_1 lags V_2 by 45°
V_1 leads V_3 by 45°
V_2 leads V_1 by 45°
V_2 leads V_3 by 90°
V_3 lags V_1 by 45°
V_3 lags V_2 by 90°

You can obtain this same information from the time domain sketches in Figure 3-13A; but, as you can see, it is more apparent in the phasor diagram of Figure 3-13B. Since you will only consider circuits where the frequency of the sinusoidal sources are constant and equal, it is understood that the phasors rotate counterclockwise about the origin at ω rad/s - even if this is not specifically indicated on the phasor diagrams. Naturally, the value of ω is necessary to convert a frequency domain expression to a time domain expression.

Example 3-7

Convert the following frequency domain expressions to time domain expressions - assuming $\omega = 754$ rad/s

(a) $I_1 = 7.07A \angle 0°$
(b) $I_2 = 3.535A \angle 45°$
(c) Sketch the phasor diagram, and time domain waveforms.

(a) $I_{1_{rms}} = 7.07A$
Thus, $I_{1_m} = 1.414\,(7.07) = 10A$
Therefore:

$i_1\,(t) = 10 \sin 754\,t$

(b) $I_{2rms} = 3.535$ A
Thus, $I_{2_m} = 1.414\,(3.535) = 5A$
Therefore:

$i_2\,(t) = 5 \sin (754t + 45°)$

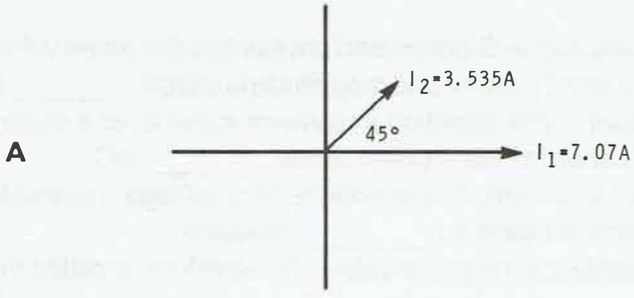
Table 3-2 summarizes the frequency-to-time-domain conversions.

Frequency Domain	Time Domain
$I_1 = 7.07A\angle 0°$	$i_1(t) = 10 \sin 754t$
$I_2 = 3.535\,A\angle 45°$	$i_2(t) = 5 \sin (754t + 45°)$

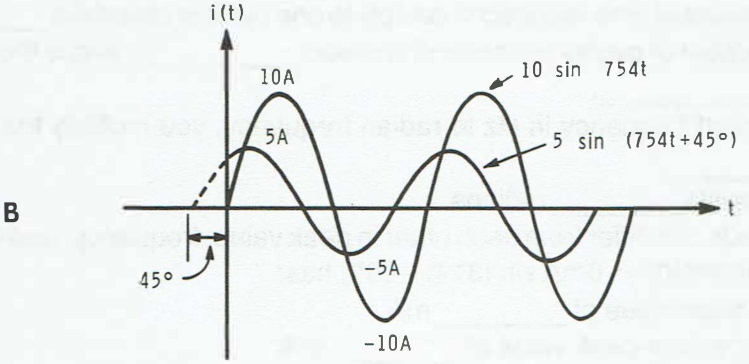
Table 3-2

Frequency to time domain conversions for Example 3-7. In this case, ω is 754 rad/s.

(c) Figure 3-14A, and Figure 3-14B illustrate the phasor diagram and time domain waveforms



PHASOR DIAGRAM



TIME DOMAIN WAVEFORMS

Figure 3-14

Phasor diagram and time domain waveforms for Example 3-7

A Phasor diagram

B Time domain waveforms

Self-Test Review

1. A current that flows in one direction and always has the same value, or a voltage that has a constant polarity and magnitude is called ________ DC.
2. A current that flows in one direction and varies in value, or a voltage that has one polarity and varies in magnitude is called ________ DC.
3. If the direction of a current, or the polarity of a voltage, reverses each half cycle, the waveform is called an ________ waveform.
4. The maximum positive or negative value of a waveform is called the ________ value.
5. A sine wave has a peak-to-peak value equal to ________ times its peak value.
6. The effective or rms value of a sine wave equals ________ times its peak value.
7. The peak value of a sine wave equals ________ times its rms value.
8. The true average value of a sinusoid is ________.
9. The amount of time required to complete one cycle is called the ________.
10. The number of cycles per second is called __________, and is the reciprocal of the __________.
11. To convert frequency in Hz to radian frequency, you multiply the frequency by ________.
12. 360° equals ________ radians.
13. Sinusoids can differ from each other in peak value, frequency, and/or ______.
14. The current $i(t) = 6\text{mA} \sin(314t - 60^\circ)$ has:
    (a) A peak value of ________ mA.
    (b) A peak-to-peak value of ________ mA.
    (c) An rms value of ________ mA.
    (d) A true average value of ________ mA.
    (e) A radian frequency of ________ rad/s.
    (f) A frequency of ________ Hz.
    (g) A period of ________ ms.
    (h) A phase angle of ________ degrees.
    (i) A frequency domain representation of __________ mA at ________ degrees.
15. If $I = 16\text{A}\ L\ -30^\circ$ then $i(t) =$ __________ assuming $\omega = 300$ rad/s.

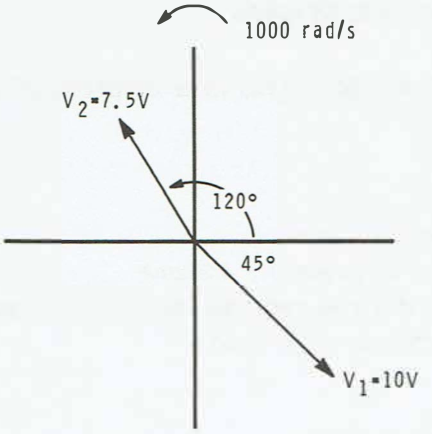


Figure 3-15

Phasor diagram for question 16

Refer to Figure 3-15 for the following questions.

16. V_1 has a phase angle of________.
17. V_2 has a phase angle of________.
18. V_1_______ V_2 by________.

Answers

1. pure
2. pulsating
3. AC
4. peak
5. two
6. 0.707
7. 1.414
8. zero
9. period
10. frequency, period
11. 2π
12. 2π
13. phase
14. (a) 6mA
    (b) 12mA
    (c) 4.242mA
    (d) 0
    (e) 314
    (f) 50
    (g) 20
    (h) −60°
    (i) 4.242mA at − 60°
15. 22.624 sin (300t − 30°)
16. −45°
17. 120°
18. lags, 165°

The solutions to questions 14 to 18 follow.

14. $i(t) = 6\text{mA} \sin(314t - 60°)$ This is a sinusoid of the form $I_m \sin(\omega t + \theta)$. Therefore:

    (a) $I_p = I_m = 6\text{mA}$
    (b) $I_{pp} = 2\,I_m = 2\,(6\text{mA}) = 12\text{mA}$
    (c) $I_{rms} = 0.707\,(I_m) = 0.707\,(6\text{mA}) = 4.242\text{mA}$
    (d) $V_{avg} = 0$ since the true average value of a sinusoid is zero.
    (e) From the equation $\omega = 314$ rad/s
    (f) $f = \dfrac{\omega}{2\pi} = \dfrac{314}{2\pi} = 50\text{ Hz}$
    (g) $T = \dfrac{1}{f} = \dfrac{1}{\underline{50\text{Hz}}} = 20\text{ms}$
    (h) From the equation $\theta = -60°$
    (i) $I = I_{rms} \angle \theta = 4.242\text{ mA} \angle -60°$

15. $I = 16\text{A} \angle -30°,\ \omega = 300$ rad/s
    $I_m = 1.414\,I_{rms} = 1.414\,(16\text{A}) = 22.624\text{A}$
    $i(t) = I_m \sin(\omega t + \theta)$
    $i(t) = 22.624 \sin(300t - 30°)$

16. In Figure 3-15, the angle between V_1 and the horizontal axis is $-45°$. Thus, the phase angle is $-45°$.

17. In Figure 3-15, the angle between V_2 and the horizontal axis is $120°$. Thus, the phase angle is $120°$.

18. In Figure 3-15, V_1 lags V_2. Since the angle **between** V_1 and V_2 is $120° + 45°$ or $165°$. This is the amount by which V_1 lags V_2.

REACTANCE AND IMPEDANCE

In Unit 2, you learned that capacitors oppose voltage changes, and inductors oppose current changes. For this reason, the effects of capacitance and inductance in DC circuits were significant only during the time the voltages and currents were changing from one value to another. In AC circuits, the currents and voltages are continually changing in a sinusoidal manner. For this reason, both capacitors and inductors oppose the flow of AC current. The opposition to the flow of AC current by a capacitor or inductor is called **reactance**.

As you will see, reactances are described by imaginary numbers. In circuits containing both resistance and reactance, the total opposition to the flow of AC current is called **impedance**. Since resistance is represented by a real number, and reactance by an imaginary number, impedance is represented by a complex number. Also since resistance, reactance, and impedance all oppose the flow of AC current, each quantity is expressed in units of ohms.

Capacitive Reactance

The opposition to the flow of AC current by a capacitor is called **capacitive reactance**. By using calculus, we can show that the capacitive reactance of a capacitor is given by:

$$X_c = \frac{-j}{\omega C} = \frac{-j}{2\pi f C} \qquad \text{(Eq.3-12)}$$

where X_c = capacitive reactance in ohms
ω = radian frequency in rad/s
C = capacitance in Farads.

Notice that capacitive reactance is described by an imaginary number.

Example 3-8

Calculate the reactance of a 2μF capacitor at a frequency of 1kHz.

$$X_C = \frac{-j}{\omega C} = \frac{-j}{2\pi(10^3)(2\times10^{-6})} = \frac{-j}{12.56\times10^{-3}} = -j79.6\Omega$$

In a purely capacitive circuit, a modified form of Ohm's law is used to calculate voltages and currents, specifically:

$$I_c = \frac{V_c}{X_c} \text{ and } V_c = I_cX_c \qquad \text{(Eq. 3-13)}$$

Normally, the values of I and V used in Equation 3-13 are rms values as illustrated by the following example.

Example 3-9

A 2μF capacitor is driven from an AC current source i(t) where i(t) = 178.2mA sin 6280t as shown in Figure 3-16A. Calculate the voltage across the capacitor. Also sketch the time and frequency domain responses.

This example introduces the general method you will use to analyze AC circuits. This method includes the following steps:

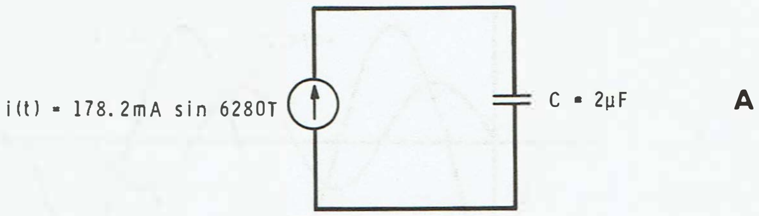
1. Convert the given problem from the time domain to the frequency domain. To do this, you express all sources as phasors and, using j notation, calculate the values of all reactances in the circuit.
2. Solve the frequency domain circuit for the desired responses.
3. Convert the frequency domain responses obtained in step 2 to time domain expressions.

Applying this procedure to the problem at hand:

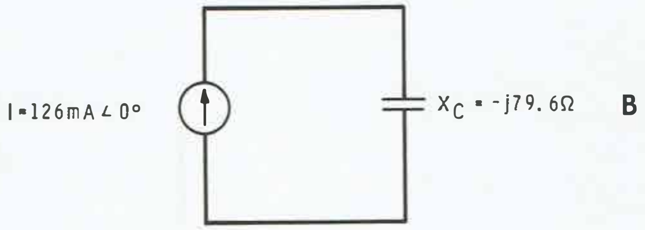
1. $i(t) = 178.2\text{mA} \sin 6280t$
   $I_{rms} = 0.707\,(178.2\text{mA})\angle 0°$
   $I_{rms} = 126\text{mA}\angle 0°$

$$X_c = \frac{-j}{\omega C} = \frac{-j}{6280(2\times10^{-6})} = -j79.6\Omega$$

Figure 3-16B illustrates the frequency domain circuit.



TIME DOMAIN CIRCUIT



FREQUENCY DOMAIN CIRCUIT

Figure 3-16

The time and frequency domain circuits for Example 3-9
A Time domain circuit
B Frequency domain circuit

2. In Figure 3-16B:

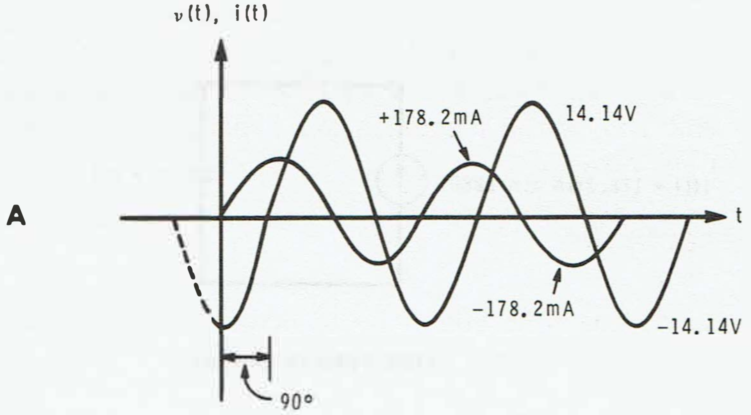
$V_c = I\,X_c$
$V_c = 126mA\angle 0^\circ\,(-j\,79.6\Omega)$
$V_c = 126mA\angle 0^\circ\,(79.6\Omega\angle -90^\circ)$
$V_c = 10V\ rms\angle -90^\circ$

3. $\nu_c(t) = V_m \sin(\omega t + \theta)$

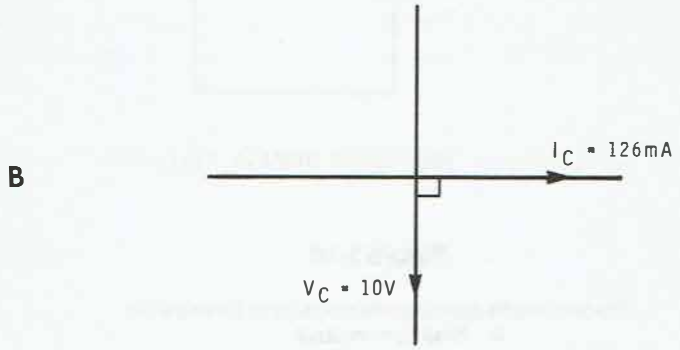
Since the voltage calculated in step 2 is an rms voltage, you must multiply the calculated value (10V) by 1.414 to obtain the peak value (V_m).

$V_m = 1.414.\,(10V) = 14.14V$

Therefore $\nu_c(t) = 14.14 \sin(6280t - 90^\circ)$



TIME DOMAIN RESPONSES



PHASOR DIAGRAM

Figure 3-17

Time and frequency domain responses for Example 3-9
Note that the current through the capacitor leads the voltage across the capacitor by 90°.
A Time domain responses
B Phasor diagram

Figure 3-17A illustrates the time domain responses. Similarly, the frequency domain responses are illustrated by the phasor diagram in Figure 3-17B. In both figures, notice that **the current through the capacitor leads the voltage across the capacitor by 90°**. You should memorize this important relationship between capacitor current and voltage.

Inductive Reactance

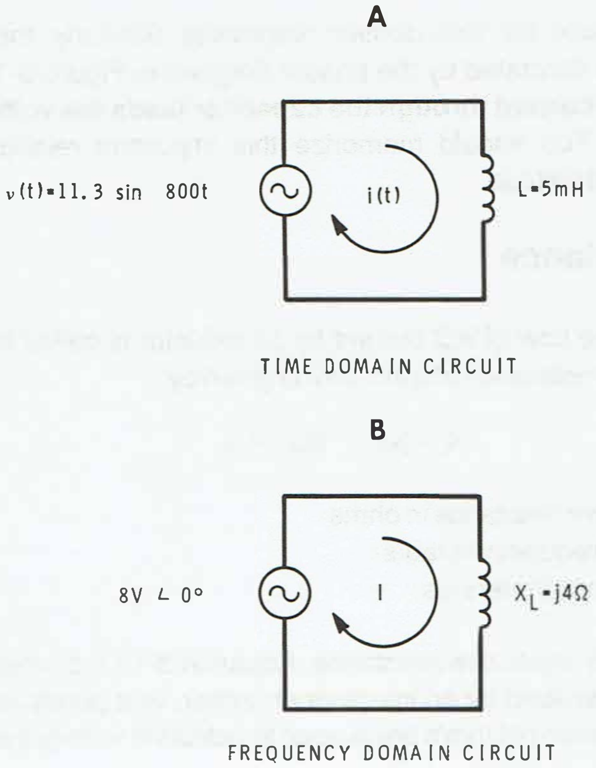
The opposition to the flow of AC current by an inductor is called **inductive reactance**. The inductive reactance of a inductor is given by:

$$X_L = j\omega L \qquad \text{(Eq. 3-13)}$$

where X_L = inductive reactance in ohms
ω = radian frequency in rad/s
L = inductance in Henries.

As was the case with capacitive reactance, Equation 3-13 indicates that inductive reactance is also described by an imaginary number. In a purely inductive circuit, the following modification of Ohm's law is used to calculate voltages and currents.

$$I_L = \frac{V_L}{X_L} \text{ and } V_L = I_L X_L$$



TIME DOMAIN CIRCUIT

FREQUENCY DOMAIN CIRCUIT

Figure 3-18

The time and frequency domain circuits for Example 3-10
A Time domain circuit
B Frequency domain circuit

Example 3-10

> *A voltage source $\nu(t) = 11.3 \sin 800t$ is connected across a 5mH inductor as illustrated in Figure 3-18A. Calculate the current flowing through the inductor. Also sketch the time and frequency domain responses.*

First, convert the problem to the frequency domain.

$$\nu(t) = 11.3 \sin 800t$$
$$V_{rms} = 0.707\,(11.3\text{V})\angle 0^\circ = 8\text{V}\angle 0^\circ$$

$$X_L = j\omega L = j\,(800)\,(5 \times 10^{-3}) = j4\Omega$$

The frequency domain circuit is illustrated in Figure 3-18B.

Next, solve for the frequency domain current.

$$I = \frac{V_L}{X_L} = \frac{8\text{V}\angle 0^\circ}{j4\Omega} = \frac{8\text{V}\angle 0^\circ}{4\Omega\angle 90^\circ} = 2\text{A}_{rms}\angle -90^\circ$$

Now convert the frequency domain current to a time domain expression.

$$i(t) = I_m \sin(\omega t + \theta)$$
$$i(t) = 2(1.414)\sin(800t - 90^\circ)$$
$$i(t) = 2.828 \sin(800t - 90^\circ)$$

Figure 3-18A and Figure 3-18B illustrate the time and frequency domain responses. **Note that the voltage across the inductor leads the current through the inductor by 90°**. This fact is important; you should memorize it promptly.

Impedance

When both resistive and reactive components are present in a circuit, each opposes the flow of AC current. The total opposition to the flow of AC current is called **impedance**. In its most general form then, Ohm's law may be modified as follows:

$$I = \frac{V}{Z},\ V = IZ,\ Z = \frac{V}{I} \qquad \text{(Eq. 3-14)}$$

As was the case with DC circuits, Ohm's law applies to the entire circuit, as well as to a portion of the circuit. As a matter of fact, virtually all of the analysis techniques used in DC circuits — voltage and current division, superposition, Thevenin's theorem, etc, — apply to AC circuits with one significant difference.

> *In general the currents, voltages, and impedances in AC circuits are described by complex numbers. In DC circuits these quantities are described by real numbers.*

This is the reason complex numbers were discussed in detail in Unit 2. The following examples illustrate how to extend some of the techniques of DC circuit analysis to AC circuits.

Example 3-11

> *Figure 3-19A illustrates a series RC circuit. Calculate the current through, and voltage across, the resistor and capacitor. Also sketch the time domain responses and a phasor diagram.*

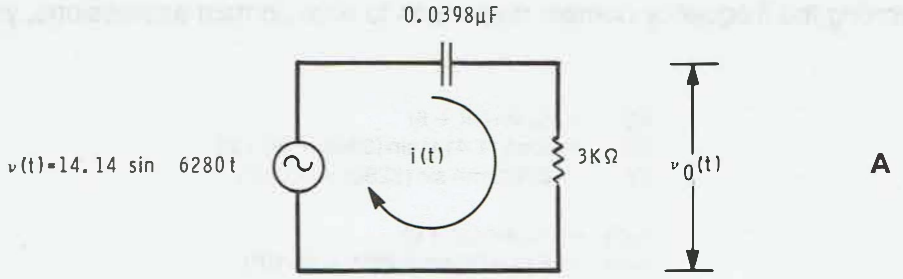
Converting to frequency domain notation, you obtain:

$$v(t) = 14.14 \sin 6280t$$
$$V = 14.14\,(0.707)\angle 0^\circ$$
$$V = 10V\angle 0^\circ$$

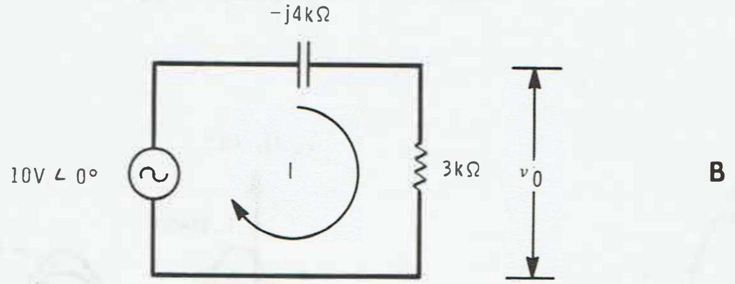
$$X_c = \frac{-j}{\omega C} = \frac{-j}{6280(0.0398)10^{-6}} = -j4k\Omega$$

Figure 3-19B illustrates the frequency domain circuit. The total impedance, Z_T, as seen from the source is:

$$Z_T = R - jX_c$$
$$Z_T = (3k - j4k)\ \Omega$$



TIME DOMAIN CIRCUIT



FREQUENCY DOMAIN CIRCUIT

Figure 3-19

Time and frequency domain circuits for Example 3-11

A Time domain circuit

B Frequency domain circuit

Converting to polar notation:

$$Z_T = \sqrt{(3k)^2 + (4k)^2} - \angle \arctan \frac{4k\Omega}{3k\Omega}$$

$$Z_T = 5k\Omega \angle -53.13^\circ$$

Thus,

$$I = \frac{V}{Z_T} = \frac{10V\angle 0^\circ}{5k\Omega\angle -53.13^\circ} = 2mA \angle 53.13^\circ$$

$$V_R = IR = 2mA \angle 53.13^\circ (3k\Omega) = 6V \angle 53.13^\circ$$

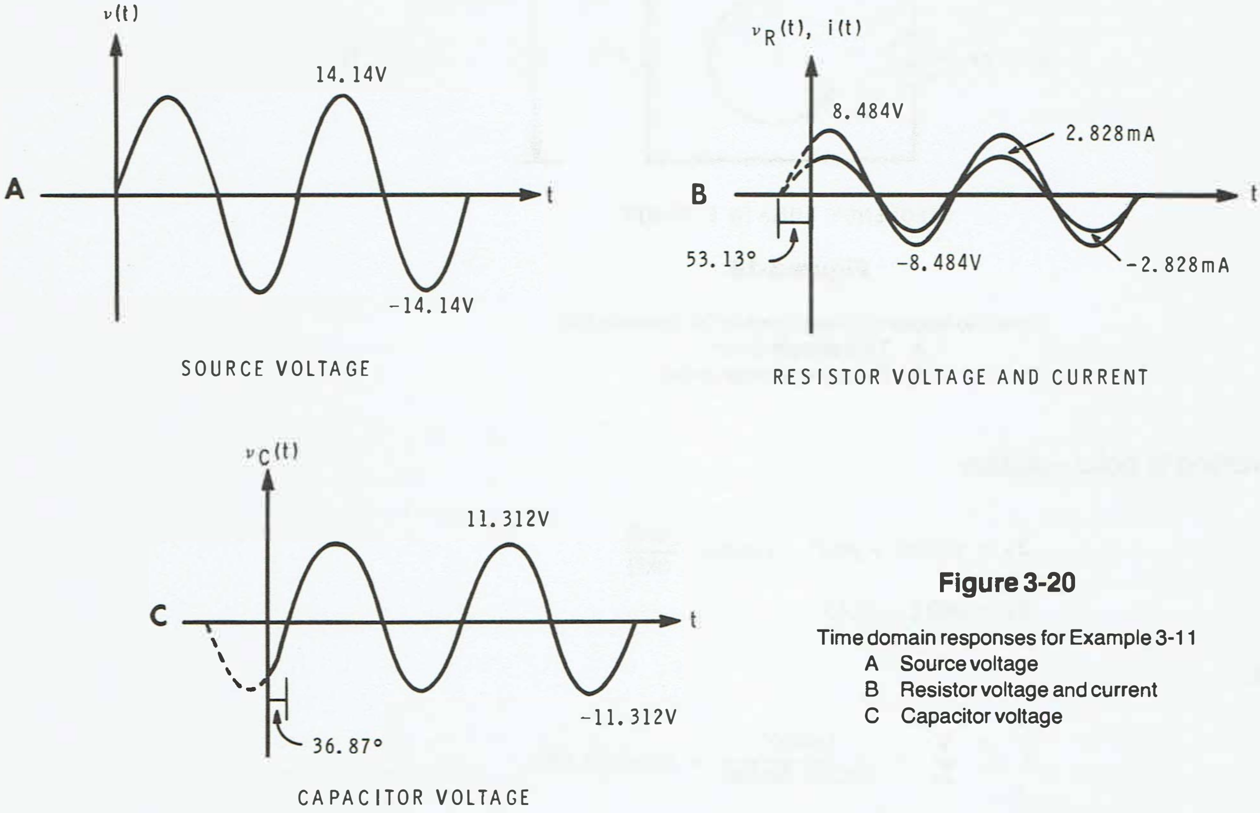
$$\begin{aligned} V_C = IX_C &= 2mA \angle 53.13^\circ (-j4k\Omega) \\ &= 2mA \angle 53.13^\circ (4k\Omega \angle -90^\circ) \\ &= 8V \angle -36.87^\circ \end{aligned}$$

Converting the frequency domain responses to time domain expressions, you obtain:

$$
\begin{aligned}
i(t) &= I_m \sin(\omega t + \theta) \\
i(t) &= 2\text{mA}\,(1.414)\sin(6280 + 53.13°) \\
i(t) &= 2.828\,\text{mA}\sin(6280t + 53.13°)
\end{aligned}
$$

$$
\begin{aligned}
\nu_R(t) &= V_m \sin(\omega t + \theta) \\
\nu_R(t) &= 6\,(1.414)\sin(6280t + 53.13°) \\
\nu_R(t) &= 8.484\sin(6280t + 53.13°)
\end{aligned}
$$

$$
\begin{aligned}
\nu_c(t) &= V_m \sin(\omega t + \theta) \\
\nu_c(t) &= 8\,(1.414)\sin(6280t - 36.87°) \\
\nu_c(t) &= 11.312\sin(6280t - 36.87°)
\end{aligned}
$$



SOURCE VOLTAGE

RESISTOR VOLTAGE AND CURRENT

CAPACITOR VOLTAGE

Figure 3-20

Time domain responses for Example 3-11
A Source voltage
B Resistor voltage and current
C Capacitor voltage

The time domain responses are illustrated in Figure 3-20. Similarly, Figure 3-21 illustrates the phasor diagram. In both Figure 3-20 and Figure 3-21, you should note the following:

1. The voltage across the resistor is in phase with the current flowing through the resistor. In AC circuits, resistor voltage and current are always in phase with each other.

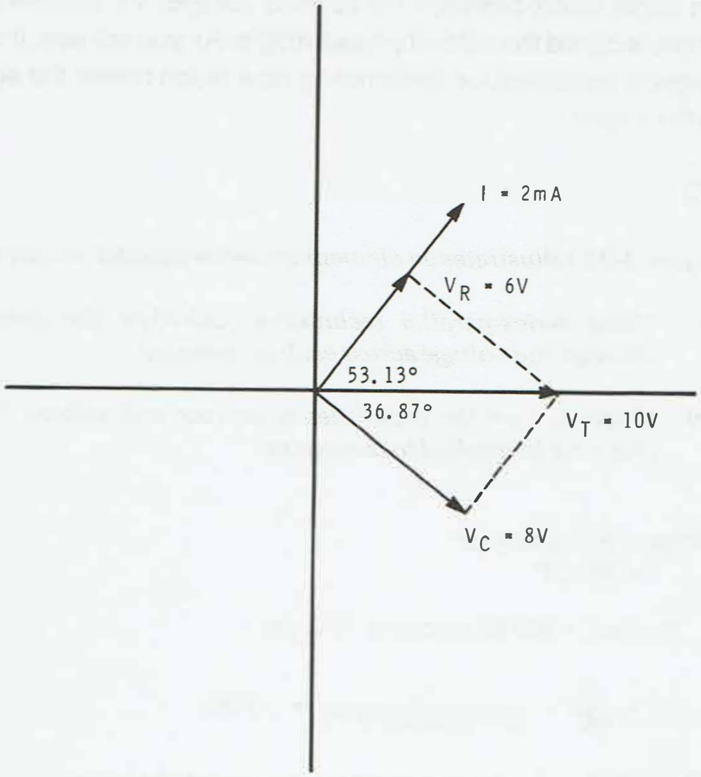


Figure 3-21

Phasor diagram for Example 3-11

2. The current flowing through the capacitor leads the voltage across the capacitor by 90°. Since the resistor voltage is in phase with the current, the voltage across the resistor also leads the capacitor voltage by 90°.

3. The **phasor sum** of the resistor and capacitor voltages equals the source voltage. This is illustrated graphically in Figure 3-21 by the fact that V_T is the diagonal of the rectangle formed by V_R and V_C. You can also demonstrate this mathematically by employing the rules provided in Unit 2 to add complex numbers, since phasor quantities are described by complex numbers. Specifically:

$$V_R + V_c = ?$$
$$6V \angle 53.13^\circ + 8V \angle -36.87^\circ = ?$$

Converting to rectangular notation, you obtain:

$$(6 \cos 53.13^\circ + j6 \cos 53.13^\circ) + (8 \cos -36.87^\circ + j8 \cos -36.87^\circ) = ?$$
$$(3.6 + j4.8) + (6.4 - j4.8) = ?$$
$$(3.6 + 6.4) + j(4.8 - 4.8) = ?$$
$$10 + j0 = 10V \angle 0^\circ \text{ which equals } V_T!$$

4. An angle exists between the applied voltage, V_T, and the current, I. This angle is called the **circuit phase angle**. As you will see, the circuit phase angle is important for determining how much power the source supplies to the circuit.

Example 3-12

Figure 3-22A illustrates an elementary series parallel AC circuit.

(a) Using series-parallel techniques, calculate the current through and voltage across each component.

(b) Illustrate how the principles of current and voltage division may be applied to the circuit.

(a) $$70.7 \sin 3770t = 70.7(0.707) \angle 0^\circ = 50\text{V} \angle 0^\circ$$

$$X_L = j\omega L = j(3770)(1.33)(10^{-3}) = j5\Omega$$

$$X_C = \frac{-j}{\omega C} = \frac{-j}{(3770)(26.5)(10^{-6})} = -j10\Omega$$

Figure 3-22B illustrates the frequency domain circuit. To simplify your notation you can define each branch impedance as follows:

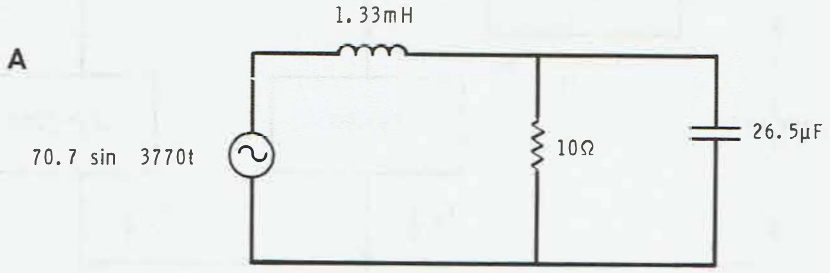
$$Z_1 = j5\Omega,\ Z_2 = 10\Omega,\ \text{and}\ Z_3 = -j10\Omega$$

Note that Z_2 is in parallel with Z_3. Thus:

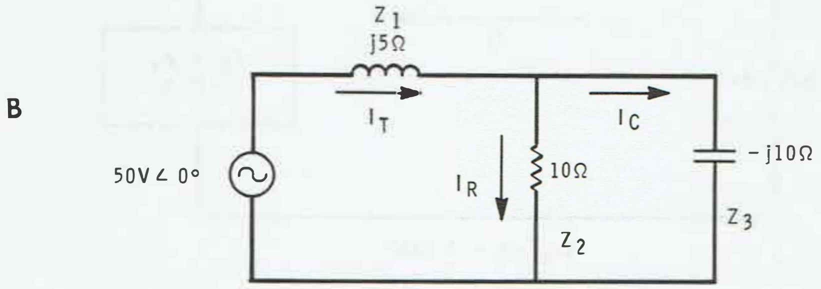
$$\begin{aligned} Z_2 \| Z_3 &= \frac{10(-j10)}{10-j10} = \frac{(-j100)}{(10-j10)} \cdot \frac{(10+j10)}{(10+j10)} \\ &= \frac{-j^2 1000 - j1000}{10^2+10^2} = \frac{1000 - j1000}{200} \\ &= (5-j5)\,\Omega \end{aligned}$$

In polar form:

$$Z_2 \| Z_3 = \sqrt{5^2+5^2} - \angle \arctan 5/5 = 7.07\Omega \angle -45^\circ$$



TIME DOMAIN CIRCUIT



FREQUENCY DOMAIN CIRCUIT

Figure 3-22

Time and frequency domain circuits for Example 3-12
A Time domain circuit
B Frequency domain circuit

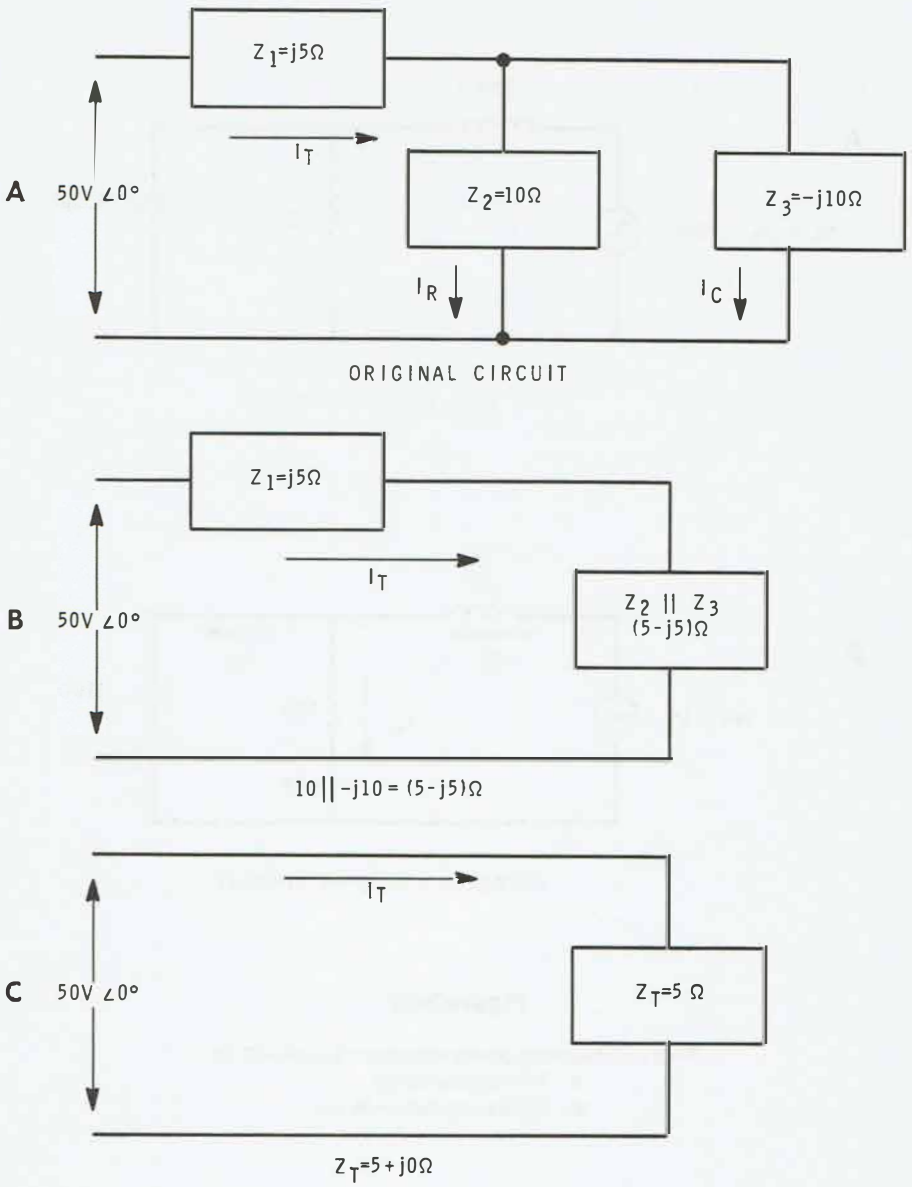


Figure 3-23

Reducing the circuit in Figure 3-22B by series-parallel combinations

A Original circuit
B $10 || -j10 = (5-j5)\Omega$
C $Z_T = 5 + j0\Omega$

Z_1 is in series with the parallel combination of Z_2 and Z_3 as shown in Figure 3-23. Thus:

$$Z_T = Z_1 + Z_2 || Z_3$$
$$Z_T = j5 + (5 - j5) = 5 + j0 = 5\Omega$$

In Figure 3-23C you can see that:

$$I_T = \frac{V}{Z_T} = \frac{50V \angle 0°}{5\Omega \angle 0°} = 10A \angle 0°$$

In Figure 3-23B, it is obvious that I_T flows through Z_1 and the equivalent impedance of $Z_2 \,||\, Z_3$. Therefore:

$$\begin{aligned} V_{Z_1} = I_T Z_1 &= 10A \angle 0° (j5\Omega) \\ &= 10A \angle 0° (5\Omega \angle 90°) \\ &= 50V \angle 90° \end{aligned}$$

Similarly, the voltage across the $(5 - j5)\Omega$ equivalent impedance is:

$$\begin{aligned} V_{Z_2||Z_3} &= 10A \angle 0°(5 - j5)\Omega \\ &= 10A \angle 0°(7.07\Omega \angle -45°) \\ &= 70.7V \angle -45° \end{aligned}$$

The voltage across the $(5 - j5)\Omega$ equivalent impedance in Figure 3-23B is the voltage across Z_2 and Z_3 in the original circuit. Therefore,

$$V_{Z_2} = V_{Z_3} = 70.7V \angle -45°$$

$$I_{Z_2} = \frac{70.7V \angle -45°}{10\Omega \angle 0°} = 7.07A \angle -45°$$

$$I_{Z_3} = \frac{70.7V \angle -45°}{-j10\Omega} = \frac{70.7V \angle -45°}{10\Omega \angle -90°} = 7.07A \angle 45°$$

Converting the calculated responses to time domain expressions, you obtain:

$$\begin{aligned} i_L(t) &= 10\,(1.414) \sin(3770t + 0°) \\ &= 14.14 \sin 3770t \end{aligned}$$

$$\begin{aligned} \nu_L(t) &= 50\,(1.414) \sin(3770t + 90°) \\ &= 70.7 \sin(3770t + 90°) \end{aligned}$$

$$\begin{aligned} i_R(t) &= 7.07(1.414) \sin(3770t - 45°) \\ &= 10 \sin(3770t - 45°) \end{aligned}$$

$$\begin{aligned} \nu_R(t) &= 70.7\,(1.414) \sin(3770t - 45°) \\ &= 100 \sin(3770t - 45°) \end{aligned}$$

$$\begin{aligned} i_C(t) &= 7.07\,(1.414) \sin(3770t + 45°) \\ &= 10 \sin(3770t + 45°) \end{aligned}$$

$$\nu_C(t) = \nu_R(t) = 100 \sin(3770t - 45°)$$

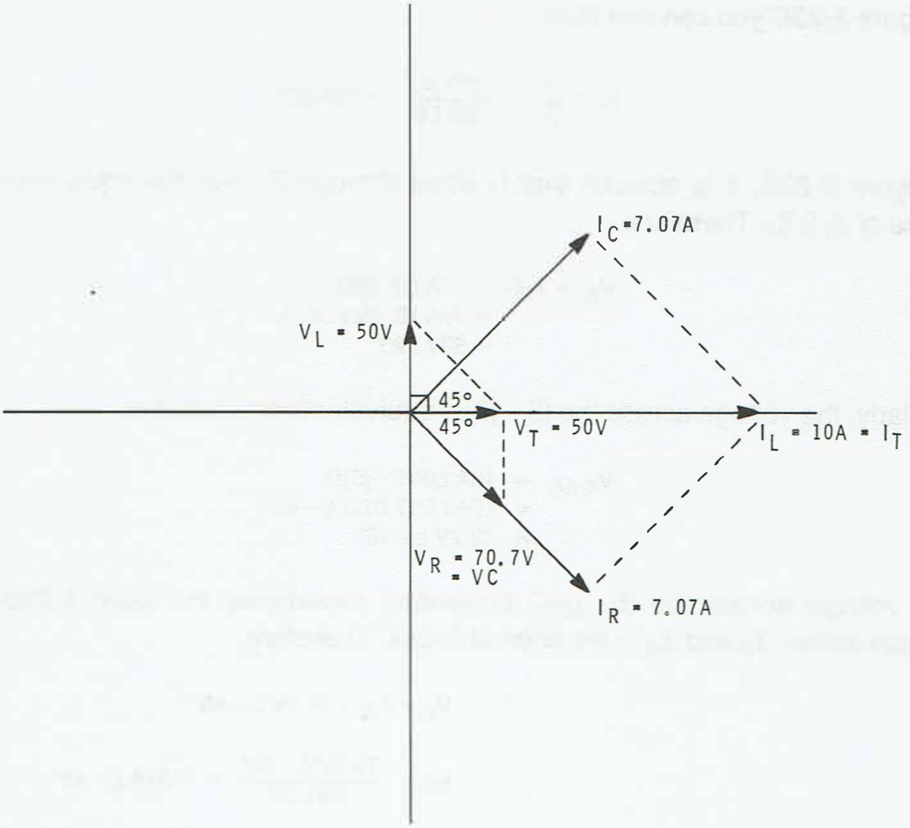


Figure 3-24

Phasor diagram for Example 3-12

The phasor diagram for the circuit is easy to sketch from the frequency domain calculations, and is illustrated in Figure 3-24. Note that the phasor sum of the inductor voltage and resistor voltage, and the phasor sum of the inductor voltage and the capacitor voltage equals the source voltage. Also note that the phasor sum of the resistor current and capacitor current equals the inductor current.

(b) To illustrate the current division principle, refer to Figure 3-23A.

$$I_R = \frac{I_T \cdot Z_3}{Z_2 + Z_3}$$

From part (a) you know $I_T = 10\text{A}\angle 0°$, $Z_2 = 10\Omega$, and $Z_3 = -j10\Omega$. Therefore:

$$I_R = \frac{10\text{A}\angle 0° \cdot (-j10\Omega)}{(10 - j10)\Omega} = \frac{10\text{A}\angle 0°(10\angle -90°)}{\sqrt{10^2 + 10^2} - \angle \arctan 10/10}$$

$$I_R = \frac{10\text{A}\angle 0°(10\angle -90°)}{14.14\angle -45°} = 7.07\text{A}\angle -45°$$

Similarly for I_c

$$I_c = \frac{I_T \cdot Z_2}{Z_2 + Z_3} = \frac{10\text{A}\angle 0°(10\angle 0°)}{14.14\angle -45°} = 7.07\text{A}\angle 45°$$

To illustrate the voltage division principle, refer to Figure 3-23B where $V = 50\text{V}\angle 0°$, $Z_1 = j5\Omega$, and $Z_2 \| Z_3 = 5 - j5\Omega$. Thus:

$$V_{Z_1} = \frac{V(Z_1)}{Z_1 + (Z_2 \| Z_3)} = \frac{50\text{V}\angle 0° \cdot j5\Omega}{j5\Omega + (5 - j5)\Omega}$$

$$V_{Z_1} = \frac{50\text{V}\angle 0°(5\angle 90°)}{5\angle 0°} = 50\text{V}\angle 90° = V_L$$

Similarly for $(Z_2 \| Z_3)$

$$V_{Z_2 \| Z_3} = \frac{V(Z_2 \| Z_3)}{Z_1 + (Z_2 \| Z_3)} = \frac{50\text{V}\angle 0°(5 - j5)\Omega}{j5\Omega + (5 - j5)\Omega}$$

$$= \frac{50\text{V}\angle 0° \sqrt{5^2 + 5^2} - \angle \arctan 5/5}{5\angle 0°}$$

$$= \frac{50\text{V}\angle 0°(7.07\angle -45°)}{5\angle 0°} = 70.7\text{V}\angle -45° = V_R = V_C$$

Example 3-13

Use Thevenin's theorem to calculate the capacitor current in Example 3-12.

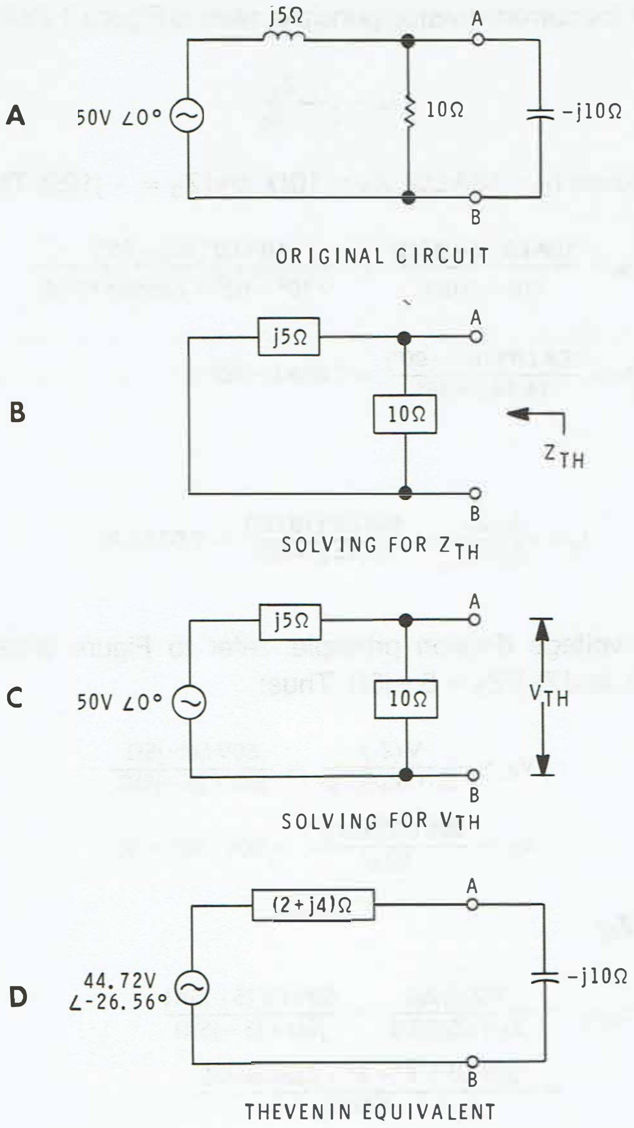


Figure 3-25

Using Thevenin's theorem to calculate the capacitor current for Example 3-13

A Original circuit.
B Solving for Z_{TH}
C Solving for V_{TH}
D Thevenin equivalent

To calculate Z_{TH}, you remove the load and reduce the voltage source to zero as shown in Figure 3-25B, Thus:

$$Z_{TH} = 10\Omega \,||\, j5\Omega = \frac{10(j5)}{10+j5} = \frac{j50}{10+j5} \cdot \frac{10-j5}{10-j5}$$

$$Z_{TH} = \frac{-j^2 250 + j500}{10^2+5^2} = \frac{250+j500}{125} = (2+j4)\Omega$$

To calculate V_{TH}, you remove the load and determine the open circuit voltage as shown in Figure 3-25C. Applying the voltage division principle, you obtain:

$$V_{TH} = \frac{50V\angle 0^\circ (10\Omega)}{(10+j5)\Omega} = \frac{500\angle 0^\circ}{\sqrt{10^2+5^2}\,\angle \arctan 5/10}$$

$$V_{TH} = \frac{500\angle 0^\circ}{11.18\angle 26.56^\circ} = 44.72V\angle -26.56^\circ$$

Figure 3-25D illustrates the Thevenin equivalent circuit with the load connected. Thus:

$$I_c = \frac{44.72V\angle -26.56^\circ}{(2+j4-j10)\Omega} = \frac{44.72V\angle -26.56^\circ}{(2-j6)\Omega}$$

$$I_c = \frac{44.72V\angle -26.56^\circ}{\sqrt{2^2+6^2} - \angle \arctan 6/2} = \frac{44.72V\angle -26.56^\circ}{6.32\Omega\angle -71.56^\circ}$$

$$I_c = 7.07A\angle 45^\circ$$

AC Power

The basic formulas for computing power in DC circuits can be used in AC circuits if the values of the currents and voltages are rms values. Thus:

$$P = I^2R = \frac{V^2}{R} = IV \qquad \text{(Eq.3-15)}$$

where P = power in watts
I = rms current in amperes
V = rms voltage in volts
R = resistance in ohms.

In AC circuits, both current and voltage are continually changing in a sinusoidal manner. The instantaneous AC power is the product of the current and voltage at each instant in time. Figure 3-26 illustrates how instantaneous AC power changes with time in a purely resistive, capacitive, and inductive circuit. In each case, the instantaneous power varies sinusoidally at a frequency twice that of the current and voltage. In a purely resistive circuit, the current and voltage are in phase with each other.

To obtain the instantaneous power curve in Figure 3-26A, you simply multiply the instantaneous current and voltage values and plot the resulting products. When a positive quantity (current) is multiplied by another positive quantity (voltage) the product is positive. Also, whenever two negative values are multiplied, the product is positive. For these reasons, the instantaneous power curve in a purely resistive circuit consists of all positive values.

The average power will also be positive as shown in Figure 3-26A. This average power equals the product of the rms values of current and voltage in the circuit. In purely capacitive and inductive circuits, the current and voltage are 90° out of phase with each other. At certain instants, the current will be positive and the voltage negative and vice versa. At other instants, both the current and voltage are positive, or negative.

For these reasons, the instantaneous power curves in purely capacitive and inductive circuits consist of both positive and negative values as shown in Figure 3-26B, and Figure 3-26C respectively. Note that the positive and negative portions of the instantaneous power curves in a purely capacitive and inductive circuit are equal. Thus, the average power dissipated by an ideal capacitor or inductor is zero.

The point to remember here is that only resistance dissipates power. When you are computing the power in an AC circuit containing capacitors or inductors, simply determine the circuit resistance and either the current through that resistance or voltage across it, and then use the power formulas given in Equation 3-15.

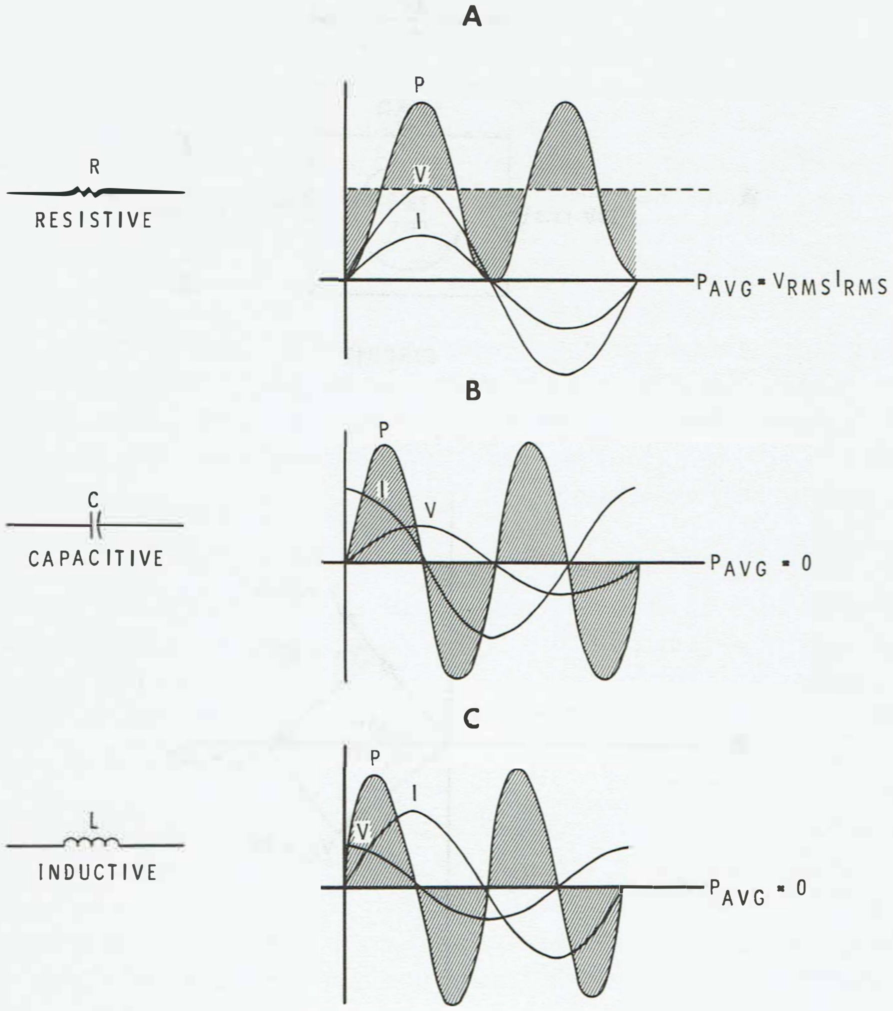


Figure 3-26

Power dissipation in a purely resistive, capacitive, and inductive circuit.

A Resistive
B Capacitive
C Inductive

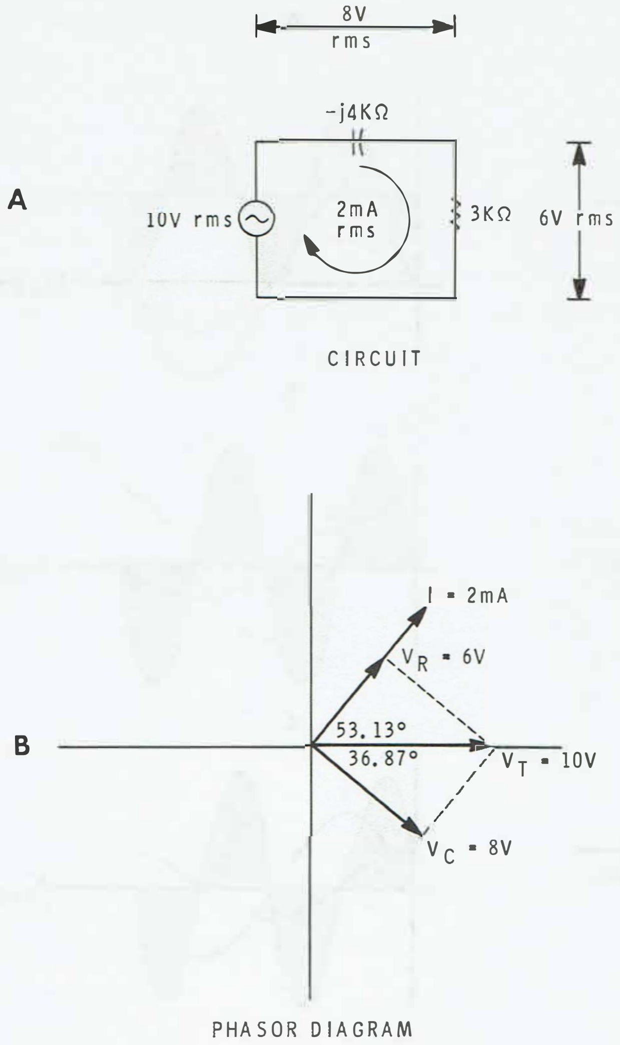


Figure 3-27

The circuit and phasor diagram from Example 3-11. See the text for a discussion of AC power calculations.
A Circuit
B Phasor diagram.

Example 3-14

Figure 3-27 summarizes the results of Example 3-11. Calculate the total power delivered by the source to the circuit.

From the discussion of AC power given previously, you know that only the resistance in Figure 3-27A dissipates power. Thus, you can use the power formulas given in Equation 3-15 to compute the power supplied by the source to the circuit.

$$P = I^2R = (2mA)^2\,(3k\Omega) = 12mW$$

$$P = \frac{V^2}{R} = \frac{(6V)^2}{3k\Omega} = 12mW$$

$$P = IV = 2mA(6V) = 12mW$$

As you can see, each formula predicts the same amount of power. In circuits containing two or more resistors, the total power supplied by the source to the circuit equals the sum of the powers dissipated by each resistor. It is also possible, and often easier, to calculate the power delivered by a source to a circuit from the formula:

$$P = V_TI_T\cos\theta \qquad \text{(Eq. 3-16)}$$

where $P =$ power in watts
$V_T =$ rms source voltage in volts
$I_T =$ rms source current in amperes
$\cos\theta =$ cosine of the circuit phase angle

Recall that the circuit phase angle is the angle between the source voltage and current. The cosine of the circuit phase angle is called the **power factor**.

Example 3-15

Use Equation 3-16 to calculate the total power delivered by the source to the circuit in Figure 3-27.

In Figure 3-27B you can see that the circuit phase angle is 53.13°. Thus:

$$P = V_T\,I_T\cos\theta$$
$$P = 10V\,(2mA)\cos 53.13° = 12mW$$

Since only resistance dissipates power, this is also the power dissipated by the 3kΩ resistor in Figure 3-27A.

In Unit 1, you learned that a DC circuit, or source, delivers maximum power to a load when the load resistance equals the Thevenin resistance of the circuit, or source. In AC circuits, the condition for maximum power transfer is complicated by the fact that most AC sources have Thevenin equivalent impedances rather than resistances. With calculus, it is possible to prove the following:

1. For a purely resistive load, an AC source delivers maximum load power when the load resistance equals the absolute value of the Thevenin impedance of the source. Thus, $R_L = \sqrt{R_{TH}^2 + X_{TH}^2}$ for maximum power transfer.

2. If the load is an impedance maximum, power transfer occurs when the load impedance is the conjugate of the Thevenin impedance.

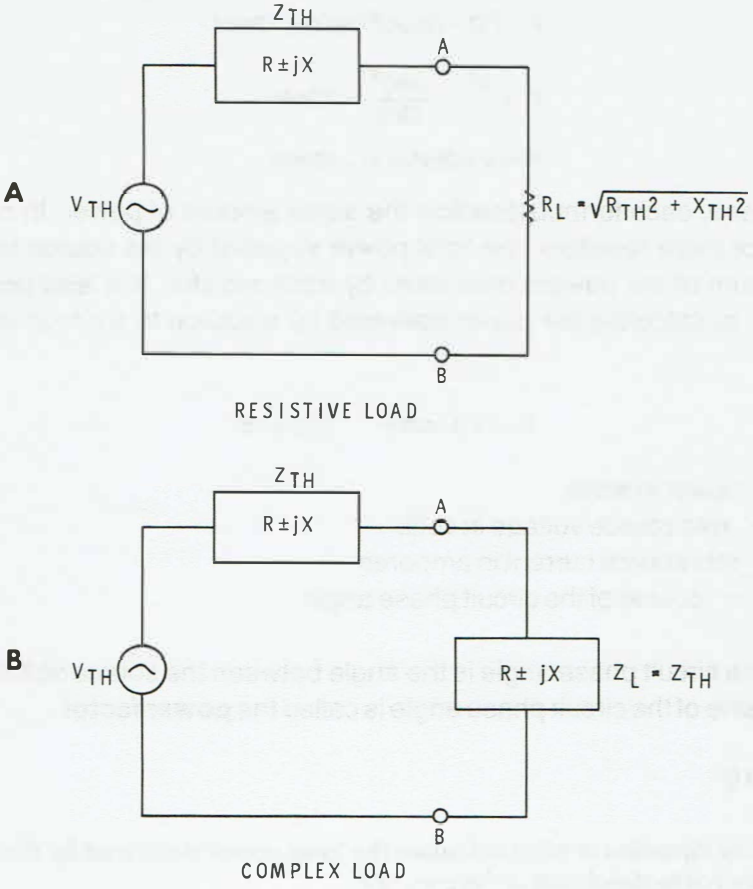


Figure 3-28

Maximum power transfer theorem for AC circuits
A Resistive load
B Complex load

Figure 3-28 illustrates the maximum power transfer theorem for AC circuits.

Example 3-16

> *An AC source has a Thevenin impedance of $(3+j4)\Omega$. What value of load impedance is required for maximum power transfer for:*
>
> *(a) A purely resistive load?*
>
> *(b) A complex load? That is, a load that has both resistance and reactance.*

(a) $R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = \sqrt{(3)^2 + (4)^2} = 5\Omega$

(b) $Z_L = Z_{TH}$ Since $Z_{TH} = (3+j4)\Omega$ $Z_L = (3-j4)\Omega$

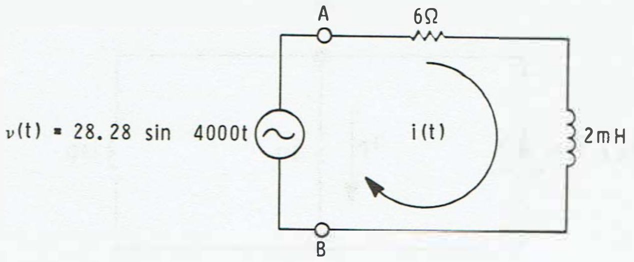


Figure 3-29

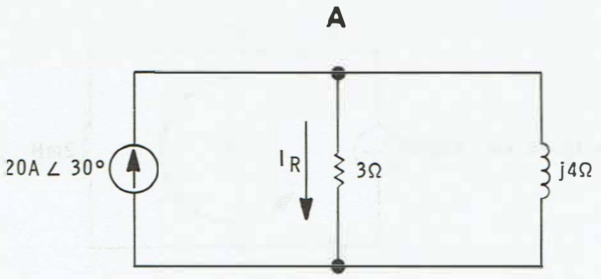
Circuit for questions 19 t0 31

Self-Test Review

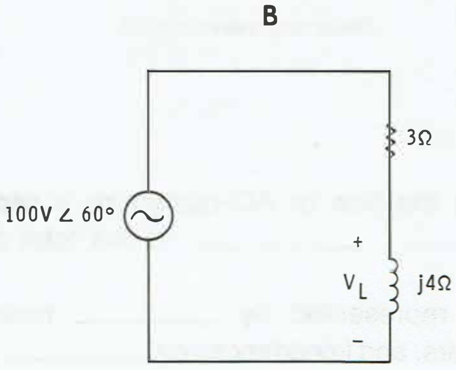
19. The opposition to the flow of AC current by a capacitor, or inductor is called ________ . ________ is the total opposition to the flow of AC current.
20. Resistances are represented by ________ numbers, reactances by ________ numbers, and impedances by ________ numbers.

Refer to Figure 3-29 for questions 19 through 31.

21. The frequency domain representation of the source voltage is ________ V, *L* ________ degrees.
22. The inductive reactance of the 2mH coil is ________ ohms.
23. In rectangular form, the impedance Z_{AB} equals ________ ohms.
24. In polar form, the impedance Z_{AB} is ________, *L* ________ degrees.
25. The phasor current is ________ A, *L* ________ degrees.
26. The resistor voltage is ________ V, *L* ________ degrees.
27. The inductor voltage is ________ V, *L* ________ degrees.
28. The power dissipated by the 6Ω resistor is ________ W.
29. The circuit power factor is ________ .
30. The total power delivered by the source to the circuit is ________ W.
31. The time domain expressions for the current, resistor voltage, and inductor voltage are:
    $i(t) =$ ________
    $\nu_R(t) =$ ________
    $\nu_L(t) =$ ________
32. A voltage source has a Thevenin impedance of (2 − j7)Ω. For maximum power transfer, the load impedance equals ________ ohms. In the case of a purely resistive load, the load resistance should equal ________ ohms for maximum power transfer.



QUESTION 33



QUESTION 34

Figure 3-30

Circuits for questions 33 and 34
A Question 33
B Question 34

33. Refer to Figure 3-30A. The current through the 3Ω resistance is ______ A, ∠ ________ degrees.
34. Refer to Figure 3-30B. The voltage across the j4Ω reactance is ______ V, ∠ ________ degrees.

Answers

19. reactance, impedance
20. real, imaginary, complex
21. 20V, ∠0°
22. j8Ω
23. (6 + j8)Ω
24. 10Ω, ∠53.13°
25. 2A, ∠ − 53.13°
26. 12V, ∠ − 53.13°
27. 16V, ∠36.9°
28. 24W
29. 0.6
30. 24W
31. $i(t) = 2.828 \sin(4000t - 53.13°)$
$v_R(t) = 16.97 \sin(4000t - 53.13°)$
$v_C(t) = 22.62 \sin(4000t + 36.87°)$
32. (2 + j7)Ω, 7.28Ω
33. 16A ∠66.87°
34. 80V ∠96.87°

The solutions to questions 21 through 34 follow:

21. $V = 0.707(28.28)\angle 0° = 20V\angle 0°$

22. $X_L = j\omega L = j(4000)(2\times 10^{-3}) = j8\Omega$

23. $Z_{AB} = R + jX_L = (6 + j8)\Omega$

24. $Z_{AB} = \sqrt{6^2+8^2}\angle \arctan 8/6 = 10\Omega\angle 53.13°$

25. $I = \dfrac{V}{Z_{AB}} = \dfrac{20V\angle 0°}{10\Omega\angle 53.13°} = 2A\angle -53.13°$

26. $V_R = IR = 2A\angle -53.13°(6\Omega) = 12V\angle -53.13°$

27. $V_L = IX_L = 2A\angle -53.13°(j8\Omega)$
$= 2\angle -53.13°(8\angle 90°)$
$= 16V\angle 36.9°$

28. $P = I_{rms}{}^2 R = (2A)^2(6\Omega) = 24W$

29. $pf = \cos\theta = \cos -53.13° = 0.6$

30. Only the 6Ω resistor dissipates power. Therefore, the source supplies 24W to the circuit. If you prefer, you can use Equation 3-16 to calculate the power supplied to the circuit. Thus:
$P_T = V_T I_T \cos\theta = 20V(2A)(0.6) = 24W$

31. Since the currents and voltages calculated previously are rms values, you must convert them to peak values, and then employ the general equation for a sinusoid. Therefore:
$i(t) = 1.414(2)\sin 4000t = 2.828\sin(4000t - 53.13°)$
$\nu_R(t) = 1.414(12)\sin 4000t = 16.97\sin(4000t - 53.13°)$
$\nu_L(t) = 1.414(16)\sin(4000t + 36.9°) = 22.62\sin(4000t + 36.9°)$

To help you visualize the responses, Figure 3-31 illustrates the time and frequency domain sketches.

32. $Z_{TH} = (2 - j7)\Omega$. For maximum power transfer $Z_L = Z_{TH} = (2 + j7)\ \Omega$. For a purely resistive load $R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = \sqrt{(2)^2 + (7)^2} = 7.28\ \Omega$

33. Employing the current division principle:

$$I_R = 20A\angle 30^\circ \cdot \frac{j4\Omega}{(3 + j4)\Omega}$$

$$I_R = 20A\angle 30^\circ \ \frac{(4\angle 90^\circ)}{5\angle 53.13^\circ}$$

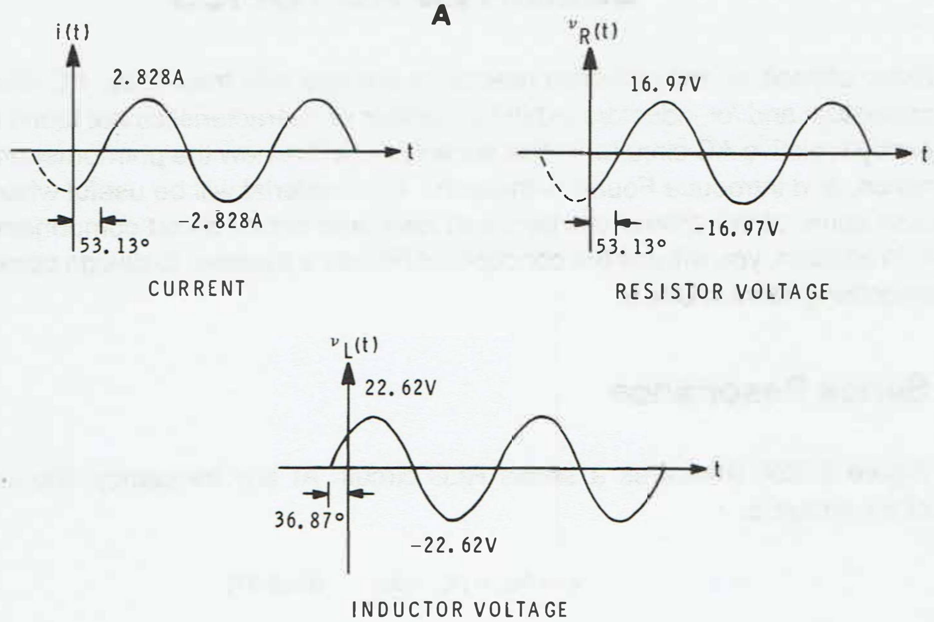
$$I_R = \frac{80\angle 120^\circ}{5\angle 53.13^\circ} = 16A\angle 66.87^\circ$$

34. Employing the voltage division principle:

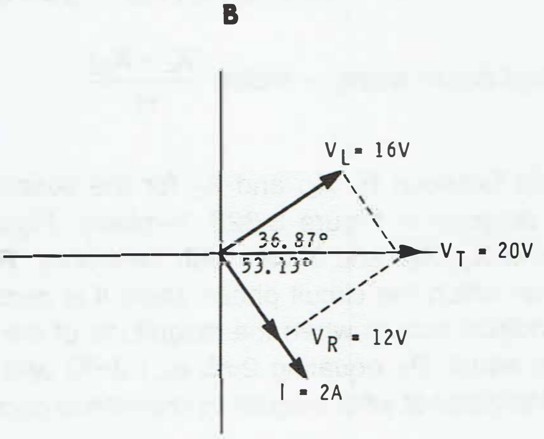
$$V_L = 100V\angle 60^\circ \ \frac{j4\Omega}{(3 + j4)\Omega}$$

$$V_L = 100V\angle 60^\circ \ \frac{(4\angle 90^\circ)}{5\angle 53.13^\circ}$$

$$V_L = \frac{400\angle 150^\circ}{5\angle 53.13^\circ} = 80V\angle 96.87^\circ$$



TIME DOMAIN RESPONSES



PHASOR DIAGRAM

Figure 3-31

Time and frequency domain sketches for the circuit in Figure 3-29
A Time domain responses
B Phasor diagram

SELECTED AC TOPICS

Since capacitive and inductive reactance change with frequency, AC circuits with capacitors and /or inductors exhibit a number of characteristics not found in DC, or purely resistive AC circuits. In this section we will review the phenomenon of resonance, and introduce Fourier's theorem. This material will be useful when we discuss some of the differences between ideal and actual circuit components in Unit 4. In addition, you will use the concepts of Fourier's theorem to design power supply smoothing filters in Unit 5.

Series Resonance

Figure 3-32A illustrates a series RLC circuit. At any frequency, the impedance of the circuit is:

$$Z = R_S + jX_L - jX_C \qquad \text{(Eq.3-17)}$$

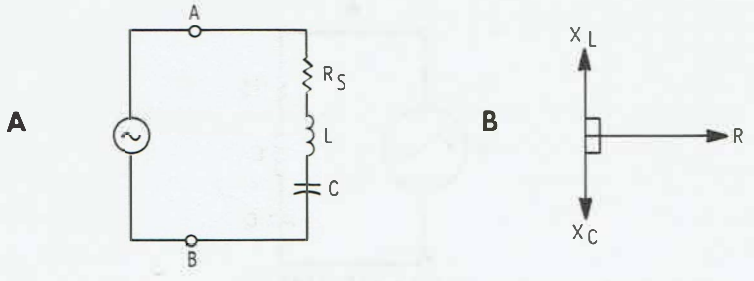
In polar form:

$$Z = \sqrt{R_S^2 + (X_L - X_C)^2}\ \angle\theta \qquad \text{(Eq.3-18)}$$

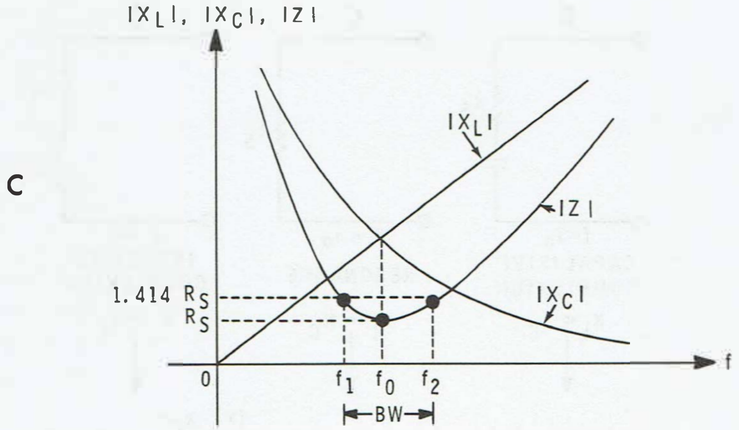
where θ = circuit phase angle = $\arctan \dfrac{(X_L - X_C)}{R}$

The relationship between R, X_L, and X_C for the series RLC circuit is indicated by the vector diagram in Figure 3-32B. Similarly, Figure 3-32C illustrates how the magnitude of X_L, X_C, and Z vary with frequency. **Resonance** is defined as the frequency at which the circuit phase angle θ is zero degrees. For the series circuit, this condition occurs when the magnitude of the inductive and capacitive reactances are equal. By equating $2\pi fL$ to $1/2\pi fC$ and solving for f, you obtain a formula that indicates at what frequency resonance occurs. Specifically:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{0.159}{\sqrt{LC}} \qquad \text{(Eq.3-19)}$$



SERIES RLC CIRCUIT VECTOR DIAGRAM



IMPEDANCE VERSES FREQUENCY

Figure 3-32

The series RLC circuit
A Series RLC circuit
B Vector diagram
C Impedance verses frequency

The subscript "$_0$" simply indicates this is the resonant frequency. Equation 3-19 can be rearranged to indicate the value of capacitance required to resonate with a particular value of inductance, and vice versa. Thus:

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{0.02533}{f_0^2 C} \qquad \text{(Eq.3-20)}$$

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{0.02533}{f_0^2 L} \qquad \text{(Eq.3-21)}$$

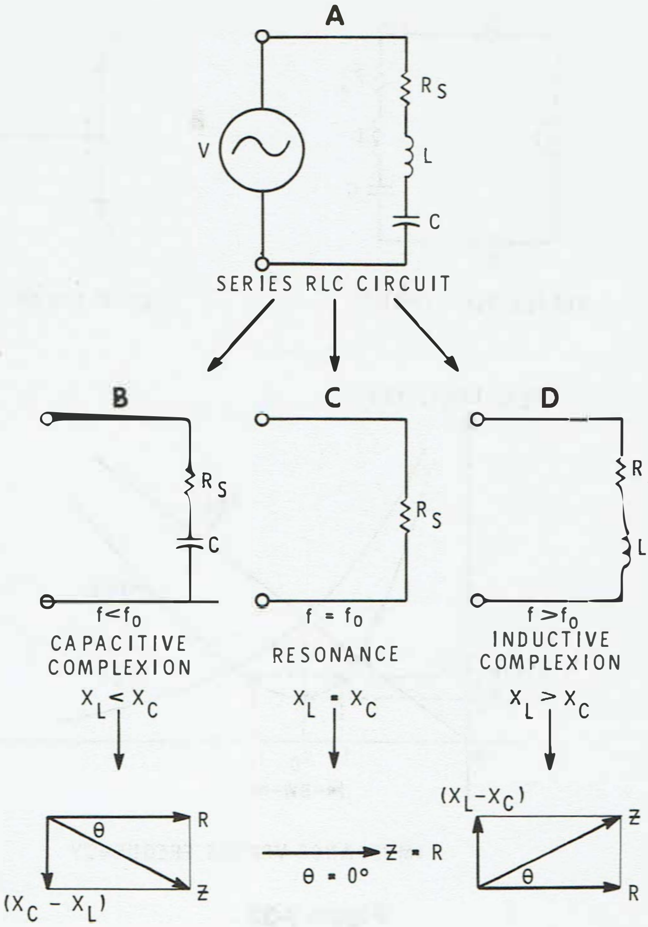


Figure 3-33

Equivalent circuits for the series RLC circuit for frequencies below, at, and above the resonant frequency

A Series RLC circuit
B Capacitive complexion
C Resonance
D Inductive complexion

Figure 3-32C contains a great deal of information. By carefully examining Figure 3-32C, you can deduce the following:

1. For frequencies less than f_0, $X_C > X_L$. Thus, the net reactance is capacitive. For this reason, the circuit appears to the source to be a series RC circuit as illustrated in Figure 3-33B. This condition is described by saying the circuit exhibits a capacitive complexion.

2. For frequencies greater than f_0, $X_L > X_C$, resulting in a net inductive reactance as illustrated in Figure 3-33D.

3. When the source frequency equals f_0, $X_L = X_C$. Consequently the net reactance is zero, making the impedance purely resistive, and equal to R_s as shown in Figure 3-33C.

An important "figure of merit" that determines the characteristics of a resonant circuit is the quality factor Q_0, which is defined as follows:

$$Q_0 = \frac{|X_L|}{R_S} \quad \text{(series circuit)} \qquad \text{(Eq. 3-22)}$$

Note in Figure 3-32C that, for frequencies on either side of f_0, the impedance increases rapidly. Thus the circuit responds well only to the narrow band of frequencies between the lower cutoff frequency f_1, and the upper cutoff frequency f_2. The band of frequencies between f_2 and f_1 is called the circuit **bandwidth**, BW. Note that at either f_1 or f_2, the impedance has a magnitude of 1.414 R_s. This indicates the current flowing in the circuit at f_1 or f_2, has a magnitude of $V/1.414\ R_s$ or 70.7% of the maximum value of V/R_s. In addition, the power supplied to the circuit at f_1 or f_2 is 50% of the maximum value at resonance. An interesting relationship exists between the bandwidth, resonant frequency, and quality factor. Specifically:

$$BW = \frac{f_0}{Q_0} \qquad \text{(Eq. 3-23)}$$

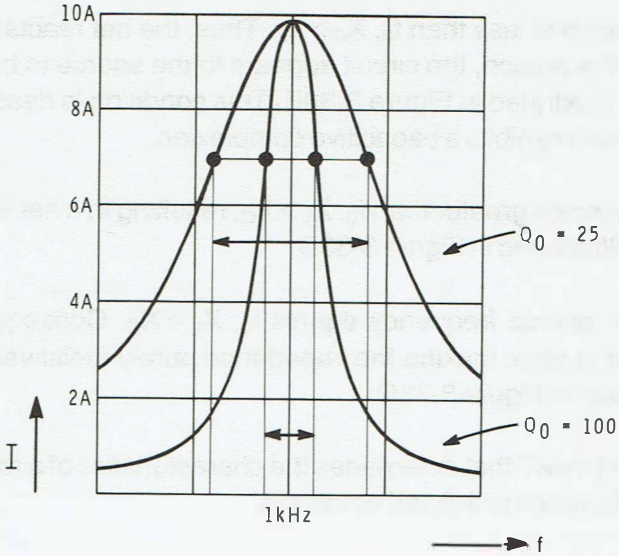


Figure 3-34

Bandwidth decreases as Q_0 increases

Thus, for a given resonant frequency the larger Q_0 is, the smaller the BW, and vice versa. In a series resonant circuit, Q_0 also indicates the factor by which the source voltage may be multiplied to obtain the magnitude of the inductor and capacitor voltages at resonance. Since at resonance:

$$|V_L| = |V_C| = Q_0V \qquad \text{(Eq. 3-24)}$$

Exact equations for f_1 and f_2 are rather complicated. In most circuits, the following approximate formulas are adequate:

$$f_1 \approx f_0 - \frac{BW}{2} \qquad \text{(Eq. 3-25)}$$

$$f_2 \approx f_0 + \frac{BW}{2} \qquad \text{(Eq. 3-26)}$$

Example 3-17

> *Figure 3-34 illustrates current-verses-frequency curves for two RLC series circuits. Note that each curve has the same resonant frequency and current value at resonance. Calculate the bandwidth for the circuits represented by each curve. Also, estimate the cutoff frequencies.*

For $Q_0 = 25$

$$BW = \frac{f_0}{Q_0} = \frac{1\,kHz}{25} = 40\,Hz$$

$$f_1 \approx f_0 - \frac{BW}{2} = 1\,kHz - 20\,Hz = 980\,Hz$$

$$f_2 \approx f_0 + \frac{BW}{2} = 1\,kHz + 20\,Hz = 1020\,Hz$$

Similarly for $Q_0 = 100$

$$BW = \frac{1\,kHz}{100} = 10\,Hz$$

$f_1 \approx 1\,kHz - 5\,Hz = 995\,Hz$
$f_2 \approx 1\,kHz + 5\,Hz = 1005\,Hz$

Example 3-18

Calculate the value of C required for resonance in Figure 3-35A. Also calculate the current and voltages across the various components.

L = 2.5mH and f_0 = 31.8kHz. Thus:

$$C = \frac{0.02533}{f_0^2 L} = \frac{0.02553}{(31.8 \times 10^3)^2 (2.5 \times 10^{-3})} \approx 0.01 \mu F$$

Since the circuit is resonant, $Z = R_s = 50\Omega$. Thus:

$$I = \frac{V}{R_S} = \frac{10V}{50\Omega} = 0.2A$$

$$X_L = j\omega L = j(2\pi)(31.8 \times 10^3)(2.5 \times 10^{-3}) = j500\Omega$$

At resonance the magnitude of X_L and X_C are equal. Thus X_C must equal $-j500\Omega$. Since I = 0.2A we have:

$$\begin{aligned} V_R &= 0.2A \angle 0^\circ (50\Omega) = 10V \\ V_L &= 0.2A \angle 0^\circ (j500\Omega) = 0.2A \angle 0^\circ (500\,\Omega \angle 90^\circ) = 100V \angle 90^\circ \\ V_C &= 0.2A \angle 0^\circ (-j500\Omega) = 0.2A \angle 0^\circ (500\Omega \angle -90^\circ) = 100V \angle -90^\circ \end{aligned}$$

If you prefer, V_L and V_C may be calculated as follows:

$$Q_0 = \frac{|X_L|}{R_S} = \frac{500\Omega}{50\Omega} = 10$$

$$|V_L| = |V_C| = Q_0 V = 10(10V) = 100V.$$

Naturally V_L leads I by 90°, and V_C lags I by 90°. For this reason, the voltage across the series combination of L and C is zero. Figure 3-35B and Figure 3-35C summarize the various responses. In summary, the series resonant circuit has the following important characteristics:

1. The impedance is purely resistive, **minimum**, and equals the series resistance, R_S. At resonance, the source voltage and current are in phase.
2. Current is maximum, and equals V/R_S.
3. The source voltage is dropped across the series resistance, R_S.
4. V_L and V_C are equal in magnitude, and 180° out of phase with each other.
5. The magnitude of the inductor and capacitor voltages can be greater than the source voltage.

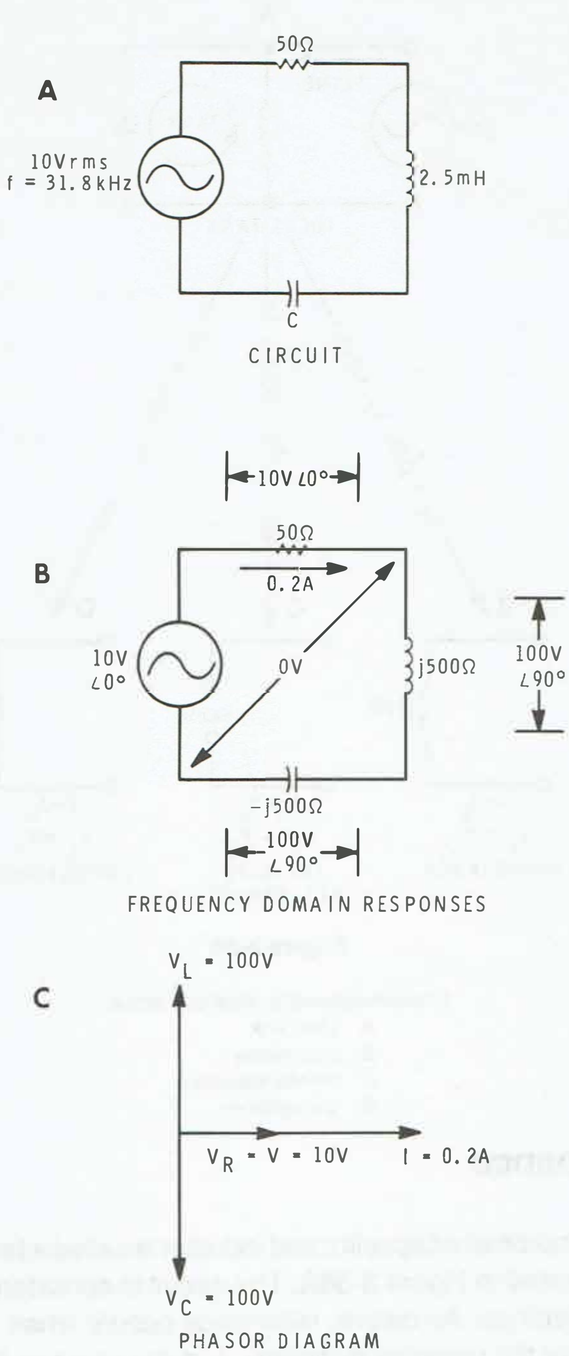


Figure 3-35

Circuit and frequency domain responses for Example 3-18

- A Circuit
- B Frequency domain responses
- C Phasor diagram

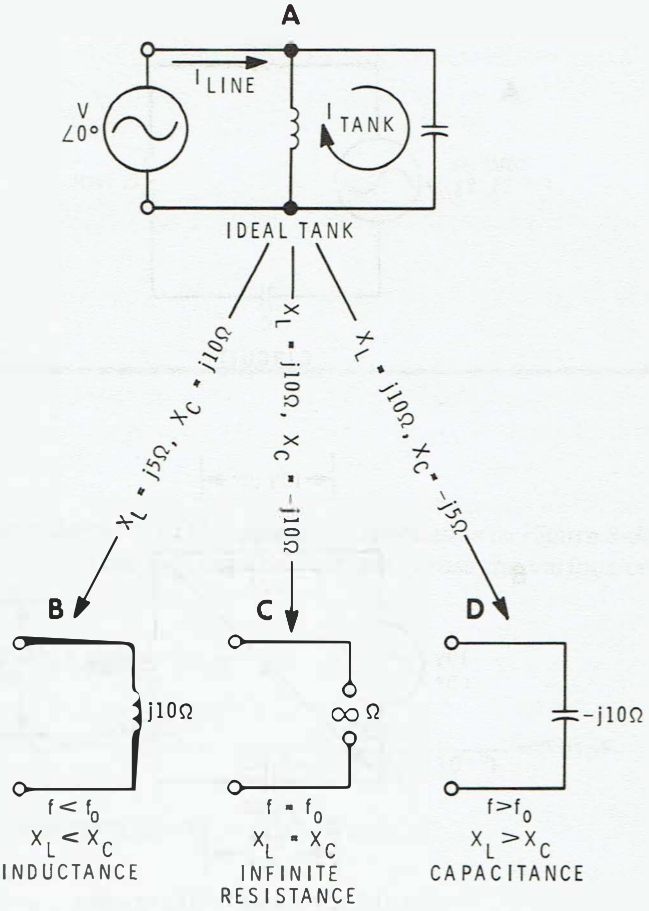


Figure 3-36

Characteristics of an ideal tank circuit
A Ideal tank
B Inductance
C Infinite resistance
D Capacitance.

Parallel Resonance

The parallel combination of a capacitor and inductor is called a **tank circuit**. An ideal tank circuit is illustrated in Figure 3-36A. The circuit is considered ideal because it does not have resistance. As before, resonance occurs when $|X_L| = |X_C|$. Thus, the formula for the resonant frequency f_0 is the same as it was for the series RLC circuit. At any frequency, the impedance of the circuit is:

$$Z = jX_L \,||\, -jX_C = \frac{jX_L(-jX_C)}{jX_L - jX_C} \qquad \text{(Eq. 3-27)}$$

For frequencies less than f_0, $X_C > X_L$. Consequently, the circuit exhibits an **inductive complexion**, which is just the opposite of the series circuit. To understand why this happens, assume that at some frequency less than f_0, $X_C = -j10\Omega$ and $X_L = j5\Omega$. Thus:

$$Z = \frac{j5\Omega(-j10\Omega)}{j5\Omega - j10\Omega} = \frac{-j^2 50}{-j5} = j10\Omega$$

As you can see, the circuit appears to the source to be an inductive reactance of j10Ω.

Similarly for frequencies greater than f_0, $X_L > X_C$, causing the circuit to exhibit a **capacitive complexion**. For example, if $X_L = j10\Omega$ and $X_C = -j5\Omega$:

$$Z = \frac{j10\Omega(-j5\Omega)}{j10\Omega - j5\Omega} = \frac{-j^2 50}{j5} = -j10\Omega$$

As the source frequency approaches the resonant frequency of the tank, $|X_L|$ approaches the value of $|X_C|$. Thus, the denominator of Equation 3-27 approaches zero. This indicates that the impedance of the ideal tank is **infinite** at the resonant frequency, f_0. As far as the source is concerned, it sees an open circuit. For this reason, the **line current** in Figure 3-36A is zero at the resonant frequency, f_0. This **does not** mean that the inductor and capacitor currents are zero — since the source voltage is still connected across each component. At resonance then:

$$Z_{TANK} = \infty\,\Omega$$
$$I_{LINE} = 0$$
$$|X_L| = |X_C| = X$$

$$I_L = \frac{V}{jX} = \frac{V}{X} \angle -90°$$

$$I_C = \frac{V}{-jX} = \frac{V}{X} \angle 90°$$

Since I_L lags V by 90° and I_C leads V by 90°, I_L and I_C are 180° out of phase with each other. This means that when the current is flowing in one direction through L, an equal current is flowing in the opposite direction through C. Thus, the phasor sum of I_L and I_C equal zero at resonance — which of course equals the line current. Incidentally, the current flowing through L and C is called the tank current. Figure 3-36 summarizes the action of the ideal tank circuit.

By shunting the ideal tank circuit with a parallel resistance, R_P, you obtain the parallel RLC circuit shown in Figure 3-37A. Once again, the formula for calculating the resonant frequency, f_0, remains unchanged. The quality factor, Q_0, is defined as follows:

$$Q_0 = \frac{R_P}{|X_L|} \text{ (parallel circuit)} \qquad \text{(Eq. 3-28)}$$

Note that Q_0 is calculated differently for the parallel RLC circuit than it was for the series RLC circuit.

Based upon our discussion of the ideal tank circuit, you can visualize the behavior of the parallel RLC circuit as follows:

1. When $f<f_0$, $X_L<X_C$, and the circuit acts like a parallel RL circuit as shown in Figure 3-37B.
2. When $f>f_0$, $X_L>X_C$, and the circuit acts like a parallel RC circuit as shown in Figure 3-37D.
3. When $f=f_0$, $|X_L|=|X_C|$. Assuming an ideal inductor and capacitor, the parallel combination of L and C has infinite resistance and acts like an open circuit. Since R_P is in parallel with this open circuit the impedance of the circuit equals R_P as shown in Figure 3-37C.

At resonance, a parallel RLC circuit exhibits the following characteristics:

1. The impedance is **maximum**, and equals the shunt resistance R_P.
2. Line current is **minimum**, and equals V/R_P.
3. The entire source current flows through R_P.
4. I_L and I_C are equal in magnitude, and 180° out of phase with each other.

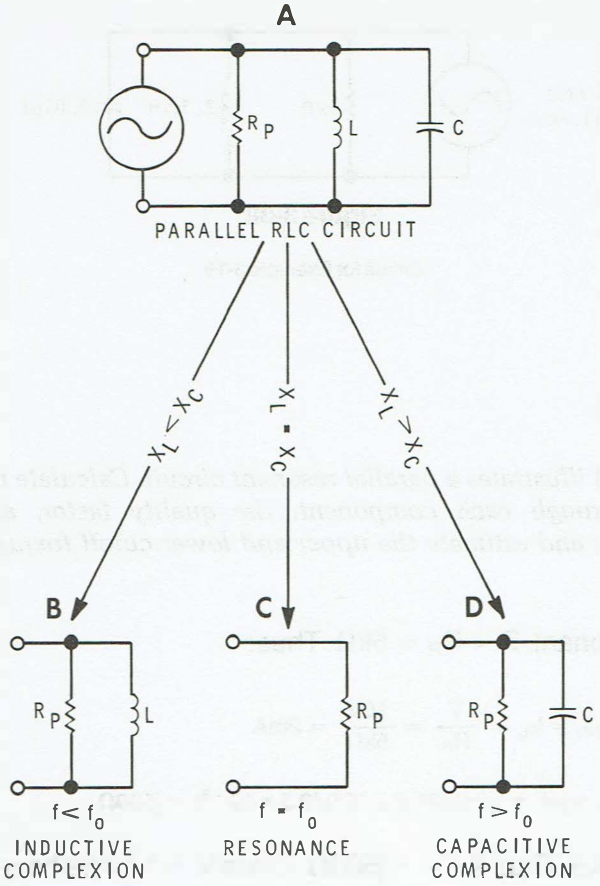


Figure 3-37

The parallel RLC circuit
A Parallel RLC circuit
B Inductive complexion
C Resonance
D Capacitive complexion

5. The magnitude of the inductor and capacitor currents can be greater than the line current, since at resonance:

$$|I_L| = |I_C| = Q_0 I_{LINE}$$

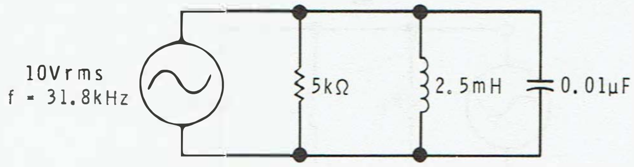


Figure 3-38

Circuit for Example 3-19

Example 3-19

Figure 3-38 illustrates a parallel resonant circuit. Calculate the current through each component, the quality factor, and bandwidth; and estimate the upper and lower cutoff frequencies.

Since the circuit is resonant, $Z = R_P = 5k\Omega$. Thus:

$$I_{LINE} = I_{R_P} = \frac{V}{R_P} = \frac{10V}{5k\Omega} = 2mA$$

$$X_L = j\omega L = j(2\pi)(31.8 \times 10^3)(2.5 \times 10^{-3}) = j500\Omega$$

At resonance $|X_L| = |X_C|$. Thus $X_C = -j500\Omega$. Since $V = 10V$ we have:

$$I_L = \frac{10V \angle 0^\circ}{j500\Omega} = 20mA \angle -90^\circ$$

$$I_C = \frac{10V \angle 0^\circ}{-j500\Omega} = 20mA \angle 90^\circ$$

$$Q_0 = \frac{R_P}{|X_L|} = \frac{5k\Omega}{500\Omega} = 10$$

If you prefer, I_L and I_C may be calculated as follows:

$$|I_C| = |I_L| = Q_0 I = 10\,(2mA) = 20mA$$

Naturally, I_L lags V by 90°and I_C leads V by 90°.

$$BW = \frac{f_0}{Q_0} = \frac{31.8kHz}{10} = 3.18kHz$$

$$f_1 \approx f_0 - \frac{BW}{2} = 31.8kHz - 1.59kHz = 30.21kHz$$

$$f_2 \approx f_0 + \frac{BW}{2} = 31.8kHz + 1.59kHz = 33.39kHz$$

Real inductors have resistance. The effects of such resistance will be considered in Unit 4.

Fourier's Theorem

In the early 1800's, the French mathematician Jean-Baptiste Joseph Fourier discovered that complex periodic waves can be represented by the sum of a constant value and a harmonic series of sinusoids. Stated mathematically:

$$\nu(t) = B_0 + B_1 \cos \omega t + B_2 \cos 2\omega t + \text{---} Bn \cos n\,\omega t + A_1 \sin \omega t + A_2 \sin 2\omega t + \text{---} An \sin n\,\omega t \qquad \text{(Eq. 3-29)}$$

where B_0 = average (DC) value of the complex wave

$B_1, B_2, \text{--} B_n$ = peak values of the cosine terms

$A_1, A_2, \text{--} A_n$ = peak values of the sine terms

The term **complex wave** simply means that the waveform is not a sinusoid. Thus, square, sawtooth, and triangular waveforms are all examples of complex waves. The **fundamental frequency** is the frequency of the complex wave. Similary, the term **harmonic series** means that the frequencies of the sine and cosine terms are whole-number multiples of the fundamental frequency. For example, if a complex wave had a frequency of 1kHz, its Fourier representation might contain frequencies of 1kHz, 2kHz, 3kHz, and so forth. Frequencies like 500 Hz, 750 kHz, and 2.5 kHz could not be present, since they are not harmonics (whole-number multiples) of the fundamental frequency, 1kHz.

A given complex wave does not necessarily contain every possible harmonic. Some complex waves contain only even harmonics, others only odd harmonics, etc. In addition, some complex waves contain only sine terms, while others contain only cosine terms. The point to be emphasized is that each complex wave is unique and, therefore, has a unique Fourier representation.

The calculation of the various coefficients in Equation 3-29 is quite tedious and requires a sound knowledge of calculus. Fortunately, the Fourier series for many of the waveforms encountered in electronics are provided in most of the engineering reference books available at most libraries. Fourier's theorem may be stated more compactly with the following notation:

$$\nu(t) = B_0 + \sum_{n=1}^{\infty} (A_n \sin n\,\omega t + B_n \cos n\,\omega t) \qquad \text{(Eq.3-30)}$$

The symbol Σ means summation. Thus, by substituting $n = 1, 2, 3$, and so forth into Equation 3-30, you obtain Equation 3-29.

Example 3-20

Figure 3-39A illustrates the sawtooth waveform. For such a waveform:

$$B_0 = \frac{A}{2},\ B_n = 0, \text{and } A_n = \frac{A}{n\pi}$$

Similarly, the Fourier series is given by:

$$f(t) = B_0 - A_n \sum_{n=1}^{\infty} \sin n\,\omega\, t$$

Figure 3-39B illustrates a sawtooth voltage. Work out the peak values of the first five AC terms, the average voltage, and write an equation that describes the waveform.

Since $B_n = 0$, the waveform contains only sine terms.

$$A_n = \frac{A}{n\pi}$$

Therefore:

$$\begin{aligned} A_1 &= 31.4\text{V}/1(\pi) = 10\text{V} \\ A_2 &= 31.4\text{V}/2(\pi) = 5\text{V} \\ A_3 &= 31.4\text{V}/3(\pi) = 3.33\text{V} \\ A_4 &= 31.4\text{V}/4(\pi) = 2.5\text{V} \\ A_5 &= 31.4\text{V}/5(\pi) = 2\text{V} \end{aligned}$$

$$B_0 = \frac{A}{2} = \frac{31.4\text{V}}{2} = 15.9\text{V}$$

This is the average (DC) voltage.

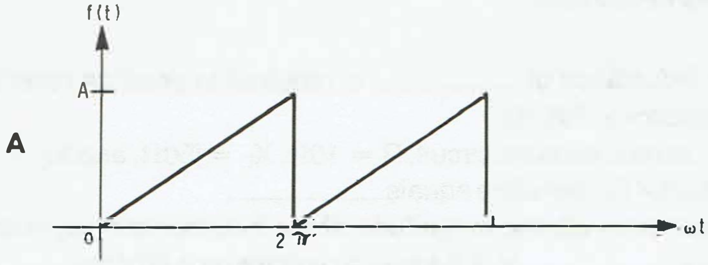
To write an equation that describes the waveform, you need to calculate the radian frequencies of each AC term. In Figure 3-39B, you can see that the period of the complex wave is 62.8ms. Thus:

$$f = \frac{1}{T} = \frac{1}{62.8\text{ms}} = 15.9\text{Hz}$$

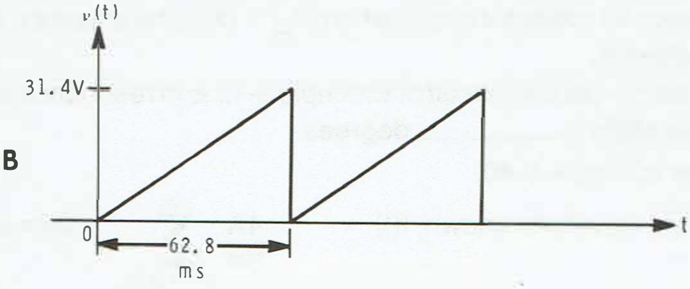
$$\omega = 2\pi f = 2\pi(15.9) = 100 \text{ rad/s}$$

Thus the fundamental radian frequency is 100 rad/s, the second harmonic 200 rad/s, and so forth. Substituting the calculated values into Equation 3-29 yields:

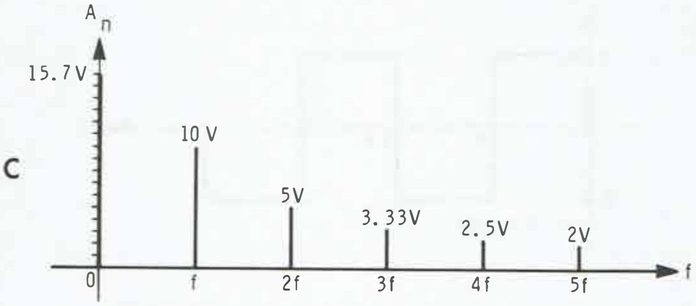
$$\begin{aligned} v(t) = 15.9 &- 10\sin 100t - 5\sin 200t - 3.33\sin 300t \\ &- 2.5\sin 400t - 2\sin 500t \text{ ---} \end{aligned}$$



THE SAWTOOTH WAVEFORM



A 31.4V PEAK SAWTOOTH VOLTAGE



FREQUENCY SPECTRUM FOR THE VOLTAGE IN B

Figure 3-39

The sawtooth frequency spectrum
A The sawtooth waveform
B A 31.4V peak sawtooth voltage
C Frequency spectrum for the voltage in B

Figure 3-39C illustrates a convenient way of summarizing the characteristics of a complex waveform. The frequency spectrum of a complex wave tells you at a glance what frequencies are contained in a complex wave, and the amplitude of each component.

Self-Test Review

35. An inductance of ________ is required to produce resonance with a $40\mu F$ capacitor at 795 Hz.
36. In a series resonant circuit, $R = 10\Omega$, $X_L = j50\Omega$, and $X_C = -j50\Omega$. The quality factor Q_0 therefore equals ________.
37. In question 36, the magnitude of the inductor and capacitor voltages would equal ________ V, if the source voltage was 20V rms.
38. A parallel resonant circuit has $R_P = 100k\Omega$, and $X_L = j2k\Omega$. The circuit bandwidth equals ________ Hz assuming the resonant frequency is 1kHz.
39. In question 38 $f_1 \approx$ ________ Hz, and $f_2 \approx$ ________ Hz.
40. In a parallel resonant circuit when $|X_L| > |X_C|$, the circuit exhibits a ________ complexion.
41. The series-parallel circuit of Example 3-12 is in resonance because the circuit phase angle is ________ degrees.
42. Refer to Figure 3-40.

For the waveform shown, $f(t) = \frac{4A}{n\pi} \cdot \sum_{n=1,3,5,\ldots}^{\infty}$ $\sin n\,\omega t$, and $B_0 = 0$

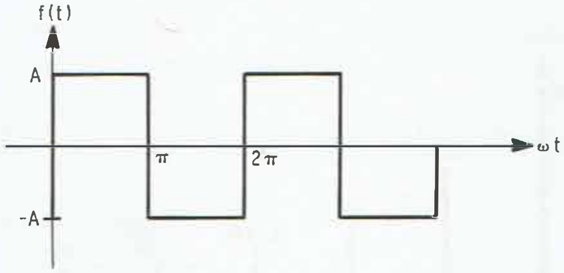


Figure 3-40

Waveform for question 42

Using this information, sketch a frequency spectrum for a square wave whose peak voltage is 100V, and whose period is 0.318 seconds. Include the first eight harmonics.

Answers

35. 1mH
36. 5
37. 100V
38. 20Hz
39. 990, 1010
40. capacitive
41. zero
42. See Figure 3-41.

The solutions follow.

35. $C = 40\mu F$, $f_0 = 795Hz$ Thus:

$$L = \frac{0.02533}{f_0^2C} = \frac{0.02533}{(795)^2(40\times10^{-6})} = 1mH$$

36. $Q_0 = |X_L|/R_S = 50\Omega/10\Omega = 5$

37. $|V_L| = |V_C| = Q_0V = 5(20V) = 100V_{rms}$

38. First calculate Q_0.
$Q_0 = R_P/|X_L| = 100k\Omega/2k\Omega = 50$
$BW = f_0/Q_0 = 1kHz/50 = 20Hz$

39. $f_1 \approx f_0 - BW/2 = 1kHz - 10Hz = 990Hz$
$f_2 \approx f_0 + BW/2 = 1kHz + 10Hz = 1010Hz$

40. Since $X_L > X_C$, the circuit exhibits a capacitive complexion, which is just the opposite of a series circuit.

42. First, calculate the fundamental frequency.

$$f = \frac{1}{T} = \frac{1}{0.318s} = 3.14\,\text{Hz}$$

Next, calculate the peak values of the first eight sine terms:

$$A_1 = \frac{4(100)}{1(\pi)} = 127.2\text{V} \quad f = 3.14\text{Hz}$$

$$A_3 = \frac{4(100)}{3(\pi)} = 42.4\text{V} \quad 3f = 9.42\text{Hz}$$

$$A_5 = \frac{4(100)}{5(\pi)} = 25.44\text{V} \quad 5f = 15.7\text{Hz}$$

$$A_7 = \frac{4(100)}{7(\pi)} = 18.17\text{V} \quad 7f = 21.98\text{Hz}$$

A_2, A_4, and A_8 are zero, since the waveform does not contain even harmonics. Figure 3-41 illustrates the resulting frequency spectrum.

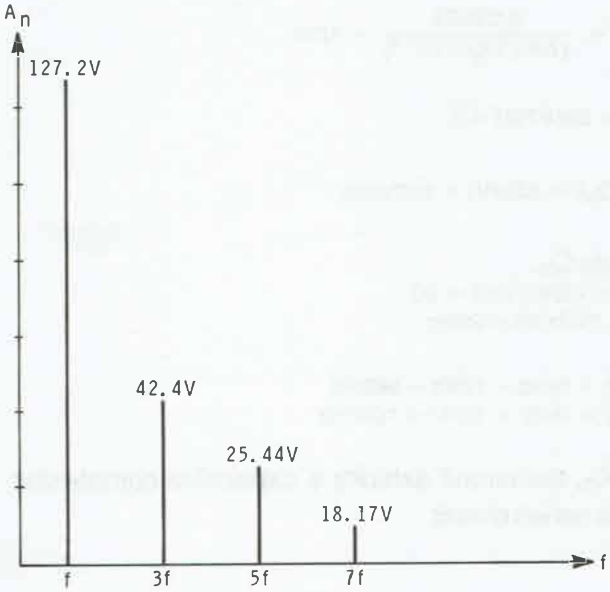


Figure 3-41

Frequency spectrum for question 42

SUMMARY

The following points summarize the key ideas of this unit.

Periodic signals repeat their basic patterns after fixed intervals of time. Examples of periodic signals include pulsating DC and AC waveforms. The sinusoid is the most important periodic signal in electronics. For this reason, definitions were provided for the following terms: peak value, peak-to-peak value, average value, rms value, period, cyclic frequency, radian frequency, and phase angle. The basic equation for a sinusoid, $\nu(t) = V_m \sin(\omega t + \theta)$, is called a time domain expression. The equivalent frequency domain expression, $V = V_{rms} \angle\theta$, is used to construct phasor diagrams. Such diagrams are useful when you wish to compare the phase angles of two or more sinusoids.

Reactance is the opposition to the flow of AC current by a capacitor, or inductor. Reactances are described by imaginary numbers. Impedance is the total opposition to the flow of AC current, and is described by a complex number. The methods used to analyze DC circuits can be extended to include AC circuits by following the rules of complex arithmetic provided in Unit 2.

When calculating power in AC circuits, you must use the rms values of current and voltage. Ideal inductors and capacitors do not dissipate power. Thus the amount of power delivered by a source to the circuit is $V_T I_T \cos\theta$. The term $\cos\theta$ is called the power factor, where θ is the circuit phase angle. In order for an AC source to deliver maximum power to a load, the load impedance should equal the conjugate of the Thevenin impedance of the source. If the load is purely resistive, maximum power transfer occurs when $R_L = \sqrt{R_{TH}^2 + X_{TH}^2}$.

AC circuits with capacitors and/or inductors exhibit characteristics that change with frequency. Resonance occurs at the frequency where the circuit phase angle is zero degrees. At resonance, the impedance of the circuit is purely resistive. Series and parallel resonant circuits have characteristics that are the opposite of each other. Thus, the quality factor of a series resonant circuit is defined as $Q_0 = |X_L|/R$, while in a parallel resonant circuit, $Q_0 = R_P/|X_P|$.

In both series and parallel resonant circuits the value of the resonant frequency, f_0, and the quality factor, Q_0, determine the bandwidth since $BW = f_0/Q_0$. Fourier's theorem states that a complex wave is composed of an average value and a harmonic series of sinusoids. Given the Fourier series for a particular waveform, you can construct the frequency spectrum of the waveform. Such a diagram shows you what frequencies make up a complex wave, and their relative amplitudes.

UNIT EXAMINATION

The following multiple choice examination is designed to test your understanding of material presented in this unit. Place a check beside the multiple choice answer (A,B,C, or D) that you feel is most correct. When you have completed the examination, compare your answers with the correct ones that appear after the exam.

1. Sinusoids can differ from each other in:
   A. Amplitude.
   B. Frequency.
   C. Phase.
   D. Amplitude, frequency, and/or phase.

2. AC current changes in
   A. Value only.
   B. Value and direction.
   C. Frequency and value.
   D. Direction only.

3. $I = 10A \angle 30°$ and $\omega = 100$ rad/s. Therefore i(t) equals:
   A. $14.14 \sin(100t + 30°)$
   B. $14.14 \sin 100t$
   C. $14.14 \sin(100t - 30°)$
   D. $10 \sin(100t + 30°)$

4. $\nu(t) = 6 \sin(628t - 60°)$. In phasor notation:
   A. $V = 6 \angle 0°$
   B. $V = 6 \angle 60°$
   C. $V = 6 \angle -60°$
   D. $V = 4.242 \angle -60°$

5. A reactance of $-j15\Omega$ is:
   A. Inductive.
   B. Capacitive.
   C. Resistive.
   D. Not possible.

6. $Z_{TH} = (3 - j7.)\Omega$. For maximum power transfer, Z_L equals:
   A. $(3 + j7)\Omega$.
   B. $(-3 - j7)\Omega$.
   C. $(-3 + j7)\Omega$.
   D. $(3 - j7)\Omega$.

Refer to Figure 3-42 for questions 7 to 12.

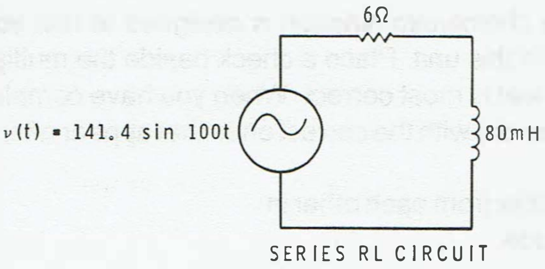


Figure 3-42

Circuit for questions 7 to 12

7. The impedance of the circuit is:
   - A. $13\Omega \angle 0^\circ$.
   - B. $10\Omega \angle 53.13^\circ$.
   - C. $10\Omega \angle -53.13^\circ$.
   - D. $13\Omega \angle 36.87^\circ$.

8. The rms voltage across the inductor is:
   - A. 60V.
   - B. 100V.
   - C. 80V.
   - D. 65V.

9. The circuit power factor is:
   - A. 0.8.
   - B. 0.6.
   - C. Unity.
   - D. 0.

10. The power delivered by the source to the circuit is:
    - A. 1 kw.
    - B. 0.6 kw.
    - C. 1.2 kw.
    - D. 0.567 kw.

11. What size capacitor should be connected in series with the inductor to produce resonance?
    - A. 1252.4 μF.
    - B. 12.524 μF.
    - C. 125.24 μF.
    - D. 80 μF.

12. Assuming the capacitor in question 11 was in the circuit, the impedance of the circuit would equal:
    - A. $j8\Omega$.
    - B. $6 + j8\Omega$.
    - C. $6\Omega \angle 53.13°$.
    - D. $6\Omega \angle 0°$.

13. A series circuit has $R_S = 25\Omega$, $X_L = j1k\Omega$, and $X_C = -j1k\Omega$. Q_0 is, therefore:
    - A. 0.025.
    - B. 625.
    - C. 25.
    - D. 40.

14. Assuming $f_0 = 800Hz$, the bandwidth in question 13 is:
    - A. 32 Hz.
    - B. 400 Hz.
    - C. 20 Hz.
    - D. 800 Hz.

15. The formula for calculating Q_0 in a parallel resonant circuit is:
    - A. $|X_L|/R_P$.
    - B. $R_P/|X_L|$.
    - C. $|X_C|/R_P$.
    - D. $|X_L|/|X_C|$.

16. Which is not a complex wave?
    - A. A square wave.
    - B. A sawtooth wave.
    - C. A triangular wave.
    - D. A sine wave.

17. A square wave has a frequency of 100Hz. The frequency of the seventh harmonic is:
    - A. 700Hz.
    - B. 100Hz.
    - C. Not present.
    - D. A function of B_0.

18. If $f(t) = 10 + \sum_{n=1}^{\infty} \cos n\omega t$, the waveform:
    - A. Contains only odd sine terms.
    - B. Contains only one cosine term.
    - C. Is a sinusoid.
    - D. Contains all cosine terms.

EXAMINATION ANSWERS

1. D — Sinusoids can differ in either amplitude, frequency, and/or phase.

2. B — AC currents change in both value and direction.

3. A — $i(t) = I_m \sin(\omega t + \theta)$. Since $I_{rms} = 10A$ $I_m = 1.414\,(10) = 14.14A$. The phase angle θ is given as 30°, and ω as 100 rad/s. Thus, $i(t) = 14.14 \sin(100t + 30°)$.

4. D — $V = V_{rms} \angle\theta$. Since $V_m = 6V$ $V_{rms} = 0.707(6)$ or 4.242V. θ is given as $-60°$ so $V = 4.242V \angle -60°$.

5. B — Capacitive reactance is prefixed by a minus j term. Thus $-j15\Omega$ represents a capacitive reactance.

6. A — For maximum power transfer, $Z_L = Z_{TH}$. Since $Z_{TH} = (3-j7)\Omega$ $Z_L = (3+j7)\Omega$.

7. B — First, calculate the inductive reactance. $X_L = j\omega L = j(100)\,(80 \times 10^{-3}) = j8\Omega$ Thus, $Z = R + jX_L = 6 + j8\Omega$. Converting to polar form, you obtain:
   $Z = \sqrt{6^2+8^2} \angle \arctan 8/6 = 10\Omega \angle 53.13°$.

8. C — Converting the source voltage from peak to rms, you obtain $V = 0.707\,(141.4) = 100V$. $I = V/Z = 100V \angle 0° / 10\Omega \angle 53.13° = 10A \angle -53.13°$.
   Thus $V_L = I X_L = 10A \angle -53.13°\,(j8\Omega)$
   $V_L = 10 \angle -53.13°\,(8 \angle 90°) = 80V \angle 36.87°$.
   Therefore, the rms inductor voltage is 80V.

9. B — The power factor is the cosine of the circuit phase angle. In this case, $\theta = -53.13°$.
   Thus $pf = \cos -53.13° = 0.6$.

10. B — Only the resistor dissipates power. Using rms values, you obtain $P = I^2R = (10)^2(6) = 0.6kw$.

11. A — Since $\omega = 100$ rad/s, the frequency of the source is:

$$f = \frac{\omega}{2\pi} = \frac{100}{2\pi} = 15.9\text{Hz}.$$

$$\text{Thus, } C = \frac{0.02533}{f_0^2\,L} = \frac{0.02533}{(15.9)^2(80 \times 10^{-3})}$$

$$C = 1252.4\mu F$$

12. D — Since the circuit is in resonance, $Z = R$ which equals $6\Omega \angle 0°$.

13. D — For a series resonant circuit, $Q_0 = |X_L|/R$ which equals $1k\Omega/25\Omega$ or 40.

14. C — $BW = f_0/Q_0 = 800Hz/40 = 20Hz$.

15. B — In a parallel resonant circuit, $Q_0 = R_{P/|X_L|}$.

16. D — A complex wave is periodic but is not a sine wave. Complex waves are composed of an infinite number of sinusoids of various amplitudes and frequencies.

17. A — A harmonic is a whole-number multiple of the fundamental frequency. Thus $f_7 = 7f_0 = 7(100Hz) = 700Hz$.

18. D — The summation symbol $\sum_{n=1}^{\infty}$ indicates all harmonics are present. Since only cos n ωt appears in the equation, no sine terms are present in the series.

Unit 4

REAL CIRCUIT COMPONENTS

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INTRODUCTION

All circuit components exhibit the properties of resistance, capacitance, and inductance. Specific components called resistors, capacitors, and inductors are designed to optimize one property while minimizing the other two. Given the large variety of components available, it is not always simple to select a specific type of resistor, capacitor, or inductor.

In general, the choice of a particular type component should be based on the component's characteristics and those of the circuit in which it will be used. Consequently, cost, tolerance, availability, and reliability, are some of the factors that should be considered before a final choice is made.

UNIT OBJECTIVES

When you have completed this unit, you should be able to:

1. Calculate the resistance of a conductor at 20°C if the resistivity and dimensions of the conductor are given.
2. Calculate the resistance of a conductor at any temperature if the conductor's temperature coefficient and resistance at one other temperature are known.
3. Define the following terms: tolerance, wattage, maximum working voltage, temperature coefficient, voltage coefficient, noise, preferred values, and hot-spot temperature.
4. Compare the characteristics of carbon composition, wire-wound, and film resistors.
5. Calculate the capacitance of a parallel-plate capacitor with various dielectric materials.
6. Define the following terms: leakage current, insulation resistance, equivalent series resistance, quality factor, and dissipation factor.
7. Compare the characteristics of mica, paper, glass, ceramic, plastic film, and electrolytic capacitors.
8. State why equalizing resistors are used with series-connected electrolytic capacitors.
9. Calculate the values of coupling and bypass capacitors.
10. Calculate the inductance of a single-turn, air-core inductor.
11. Calculate the inductance of a coil for various core materials.
12. Distinguish between hard and soft magnetic materials.
13. Explain how to minimize hysteresis, eddy-current, and flux-leakage losses.
14. Convert the series equivalent circuit of an inductor to its parallel equivalent circuit.
15. Calculate the resonant impedance of a practical tank circuit.

UNIT ACTIVITY GUIDE

☐ Read section on “Resistors.”

☐ Answer Self-Test Review questions 1-15.

☐ Read section on “Capacitors.”

☐ Answer Self-Test Review questions 16-29.

☐ Perform Experiment 4 in Unit 8.

☐ Read section on “Inductors.”

☐ Answer Self-Test Review questions 30-40.

☐ Study Summary.

☐ Complete Unit Examination.

☐ Check Examination Answers.

RESISTORS

Resistors are designed to introduce a **specific amount** of resistance into a circuit. Generally speaking, resistors are classified as either fixed, or variable. Additional classification is usually based on the manufacturing process used to make the resistor. Consequently, most resistors are classified as **composition, wire-wound,** or **film** resistors. Naturally each type has its unique strengths and weaknesses.

In this section, we discuss some of the important characteristics of the most popular resistor types. This discussion should help you in selecting an appropriate resistor type for a specific application.

Resistor Basics

The resistance of an ideal resistor would be constant. Changes in temperature, humidity, voltage, time, and so forth, would not change the resistance value at all. In addition, the ideal resistor would only exhibit the electrical property of resistance; capacitive and inductive effects would be nonexistent.

Real resistors **always** contain some capacitance and inductance as illustrated by the equivalent circuit in Figure 4-1. At high frequencies, the reactances of the series inductance and shunt capacitance are significant enough that they can no longer be neglected. For this reason alone, the effective AC resistance of a resistor may be very different from its DC value. Interestingly enough, the equivalent circuit in Figure 4-1 could also be used to represent a real inductor, or real capacitor. Naturally, the component values would be different depending upon whether or not the equivalent circuit represented a real resistor, capacitor, or inductor.

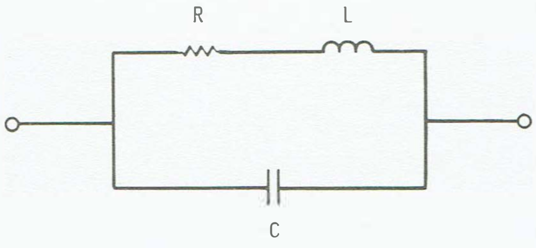


Figure 4-1

Simplified equivalent circuit for a physical resistor.

In the United States, wire diameters are measured in units of a thousandth of an inch called a mil. Similarly, a circular mil, CM, represents the cross-sectional area of a circular conductor whose diameter is one mil. Wire tables normally express the cross-sectional area of conductors in units of circular mils.

The **resistivity** of a material is defined as the resistance of a sample of the material whose length is one foot, whose cross-sectional area is one circular mil, and whose temperature is 20°C. Thus, if you know the resistivity, length, and cross-sectional area of a conductor, you can calculate its DC resistance at 20°C from the following formula.

$$R = \frac{PL}{A} \qquad \text{(Eq. 4-1)}$$

where R = resistance at 20°C in Ohms (Ω)
L = length if feet (ft)
A = cross-sectional area in circular mils (CM)
P = resistivity in Ohm circular mils/foot ($\frac{\Omega CM}{ft}$)

With real resistors, changes in temperature produce changes in resistance. For a given material, the amount of resistance change due to a change in temperature is indicated by the materials **temperature coefficient** of resistance, TC. Most materials have positive temperature coefficients — this indicates that resistance increases when temperature increases. Carbon is an example of a material that has a negative temperature coefficient.

The resistance of a material whose temperature coefficient is negative will decrease when temperature increases. Table 4-1 lists the resistivities and temperature coefficients for selected materials. Consequently, you can use Equation 4-1 to calculate the resistance of a conductor at 20°C. For other temperatures, you can use the following formula.

$$R_2 = R_1 [1 + \alpha (t_2 - t_1)] \qquad \text{(Eq. 4-2)}$$

where R_2 = resistance in ohms at the higher temperature t_2.
R_1 = resistance in ohms at the lower temperature t_1.
$(t_2 - t_1)$ = temperature difference — both t_1 and t_2 are in °C.
α = the material's temperature coefficient of resistance.

Material	Resistivity Ω CM/ft	Temperature Coefficient
Silver	9.9	0.0038
Copper	10.37	0.00393
Aluminum	17.0	0.00391
Nichrome	660	0.00017
Carbon	21 000	−0.0005

Table 4-1

Material properties of selected substances.

Example 4-1

AWG # 22 wire has a cross-sectional area of 642.4 CM. Calculate the resistance of 100 feet of copper wire at 20°C, 100°C, and –10°C.

$$\text{At } 20°\text{C} \qquad R = \frac{\rho L}{A} = \frac{10.37\,(100)}{642.4} = 1.614\Omega$$

Since 100°C > 20°C, $R_1 = 1.614\Omega$, $t_1 = 20°\text{C}$, and $t_2 = 100°\text{C}$. Then:

$$R_2 = R_1\,[1 + \alpha\,(t_2 - t_1)]$$
$$R_2 = 1.614\,[1 + 0.00393\,(100 - 20)]$$
$$R_2 = 1.614\,[1.3144)$$
$$R_2 = 2.121\Omega$$

Similarly, since 20°C > –10°C, $R_2 = 1.614\Omega$, $t_1 = -10°\text{C}$, and $t_2 = 20°\text{C}$. Thus:

$$1.614 = R_1\,[1 + 0.00393\,(20 - (-10))]$$
$$1.614 = R_1\,[1.1179]$$
$$R_1 = \frac{1.614}{1.1179} = 1.444\Omega$$

Definitions

When resistors, as well as other components, are discussed, you will often encounter the following terms.

Tolerance. The maximum allowable deviation from the nominal value. For example, a 100Ω resistor whose tolerance is ±10% may have an actual value anywhere between 90Ω and 110Ω.

Wattage. This refers to the maximum power a component can dissipate at a specified temperature without being destroyed, or changing its value. At temperatures above the specified temperature, the power rating must be decreased. Manufacturers usually provide derating curves for this purpose.

Maximum Working Voltage. This is the maximum voltage that can be placed across the component on a continuous basis.

Temperature Coefficient. This indicates the amount of change in the component value due to changes in temperature. The temperature coefficient, TC, is usually expressed as a percentage change in the nominal value per °C, or in parts per million (ppm) per °C.

Example 4-2

A resistor has a nominal value of 10kΩ at 25°C, and a TC of 1 200 ppm/°C. Calculate the resistance at 150°C.

$$1\,200 \text{ ppm} = 1\,200/1\,000\,000 = 0.0012$$

Thus:

$$1\,200 \text{ ppm/°C} = 0.0012\text{/°C}$$

The change in resistance, ΔR equals the TC times the change in temperature Δt, and the nominal resistance R_1. Thus:

$$\Delta R = TC\,(\Delta T)\,(R)$$
$$\Delta R = 0.0012\,(150\text{-}25)\,(10\text{k}\Omega)$$
$$\Delta R = 1.5\text{k}\Omega$$

The resistance at 150°C is, therefore:

$$R = R + \Delta R$$
$$R = 10\text{k}\Omega + 1.5\text{k}\Omega = 11.5\text{k}\Omega$$

Voltage Coefficient. This indicates the amount of change in the component value due to changes in voltage. The voltage coefficient, VC, is often expressed as a percentage change in the nominal value per volt.

Noise is any undesired signal. The random motion of charges within all components produces noise that appears at the terminals of the component as a voltage. This **Johnson**, or **white** noise contains all frequencies and can be calculated by:

$$V_J = \sqrt{4KTR\,(BW)} \qquad \text{(Eq. 4-3)}$$

where:
V_J = rms Johnson noise
K = Boltzmann's constant = 1.38×10^{-23}
T = temperature in degrees Kelvin (°K = °C + 273)
R = resistance of component (Ω)
BW = bandwidth

Equation 4-3 indicates that Johnson noise increases with temperature, resistance value, and bandwidth. Total noise includes Johnson noise plus all other sources of noise.

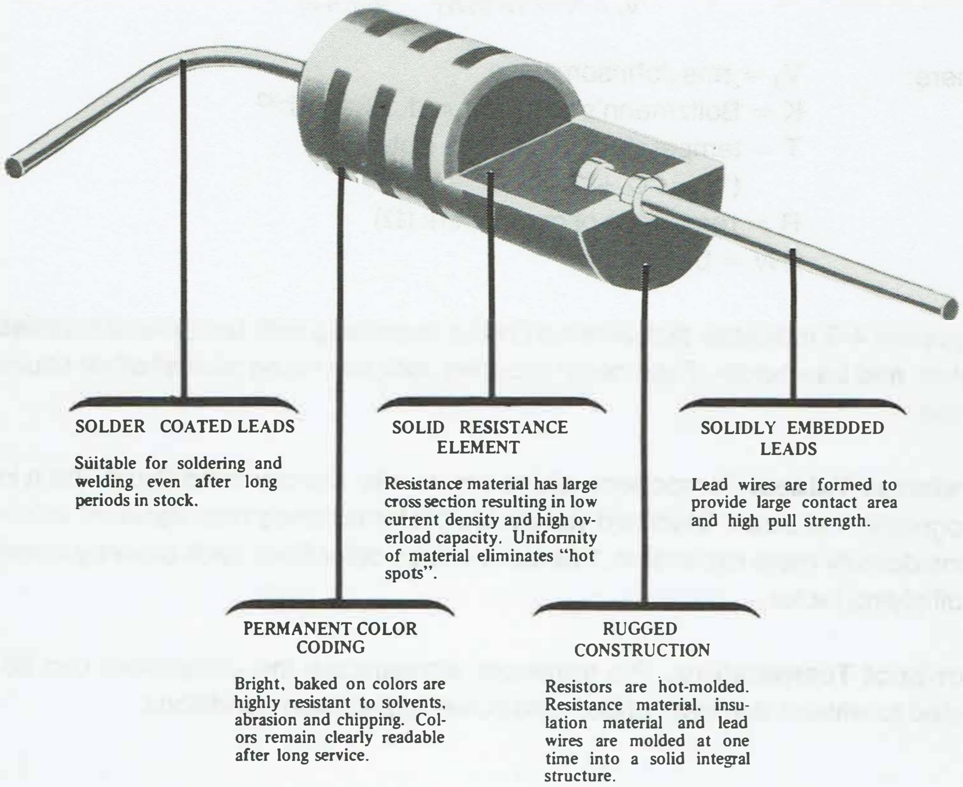
Preferred Values. Component values are usually standardized to provide a logical progression of sizes. Standard values are preferred since non-standard values are considerably more expensive. Standard values differ from each other by a constant multiplying factor.

Hot-Spot Temperature. The maximum temperature the component can be subjected to without damage, under zero power dissipation conditions.

Fixed Composition Resistors

Composition resistors are made by combining a conducting material, resinous binder, and filler to form the resistor body. The conducting material usually consists of carbon granules, although other materials may be used. Carbon composition resistors are inexpensive, available in a wide range of resistance values, and, until recently, were the most common type of resistor used in electronics.

Figure 4-2 illustrates the construction details of a typical carbon composition resistor. Similarly, Table 4-2 summarizes typical characteristics of carbon composition resistor. Note in Table 4-2 the following non-ideal characteristics:



SOLDER COATED LEADS

Suitable for soldering and welding even after long periods in stock.

SOLID RESISTANCE ELEMENT

Resistance material has large cross section resulting in low current density and high overload capacity. Uniformity of material eliminates "hot spots".

SOLIDLY EMBEDDED LEADS

Lead wires are formed to provide large contact area and high pull strength.

PERMANENT COLOR CODING

Bright, baked on colors are highly resistant to solvents, abrasion and chipping. Colors remain clearly readable after long service.

RUGGED CONSTRUCTION

Resistors are hot-molded. Resistance material, insulation material and lead wires are molded at one time into a solid integral structure.

Figure 4-2

The carbon composition resistor.

RANGE	1Ω to 22MΩ				
TOLERANCE (%)	±20 ±10 ±5				
WATTAGE (W) Specified at 40 or 70°C	1/8	1/4	1/2	1	2
MAXIMUM WORKING VOLTAGE (WV)	150	250	350	500	500
HOT SPOT TEMPERATURE (°C)	150	130	130	130	130
VC (% change in resistance/V)	0.05	0.035	0.035	0.02	0.02
TC	Varies with resistance value and temperature.				
NOISE	Significant Johnson noise for R ⩾ 1MΩ				
COST	Low				
FREQUENCY EFFECTS	Little effect for f < 100kHz. Above 100kHz, resistors ⩾ 300kΩ start to decrease in value. Above 1MHz, all resistance values decrease.				
APPLICATIONS	Low power, moderate voltage, low-medium frequency applications where precise resistance values are **not** required.				

Table 4-2

Typical characteristics of carbon composition resistors.

1. Resistance changes with temperature, voltage, and frequency.
2. The temperature coefficient of resistance also changes with temperature, and for a given temperature, it depends upon the resistance value.
3. Large resistance values tend to be noisy.

In addition, carbon composition resistors are a poor choice for humid environments, because moisture causes the carbon granules to move away from each other — thus increasing the resistance of the resistor. This effect is just the opposite of most other resistor types which tend to decrease in resistance when exposed to moisture.

Carbon composition resistors are also susceptible to resistance changes due to **cycling**. This means a permanent change in resistance can occur when the resistor is alternately operated between no dissipation and maximum dissipation. for these, and other reasons, carbon composition resistors are not suitable for applications requiring precise, stable resistance values. If changes in resistance can be tolerated and the environment is not harsh, then for moderate changes in voltage, frequency, and temperature, carbon composition resistors are a good choice.

Fixed Wire-Wound Resistors

Wire-wound resistors are made by winding wire on a core, and then covering the resistor with a protective coating. The characteristics of the resistor are determined by the type of wire used, the manner in which the wire is wound, the type of core material used, and the type of coating used to seal the resistor.

Figure 4-3 illustrates the construction details of a typical wire-wound resistor. Of the various types of resistors currently available, wire-wound resistors are the least standardized. Consequently, they are available in numerous shapes, sizes, and power ratings.

For a given physical size, the maximum resistance value is limited by the available space and the resistivity of the wire used to wind the resistor. Thus, for a given wattage, the range in available resistance values is considerably less than for composition resistors.

Since wire-wound resistors are essentially coils of wire, they have considerably more inductance than either composition, or film-type resistors. In an attempt to minimize inductance, pairs of identical coils are wound in opposite directions, and then connected in parallel. The disadvantage of these "noninductive" windings is that, for a given size resistor, the maximum resistance value that may be obtained is quite limited. Due to their construction, wire-wound resistors can be made with tolerances of ±1% or less.

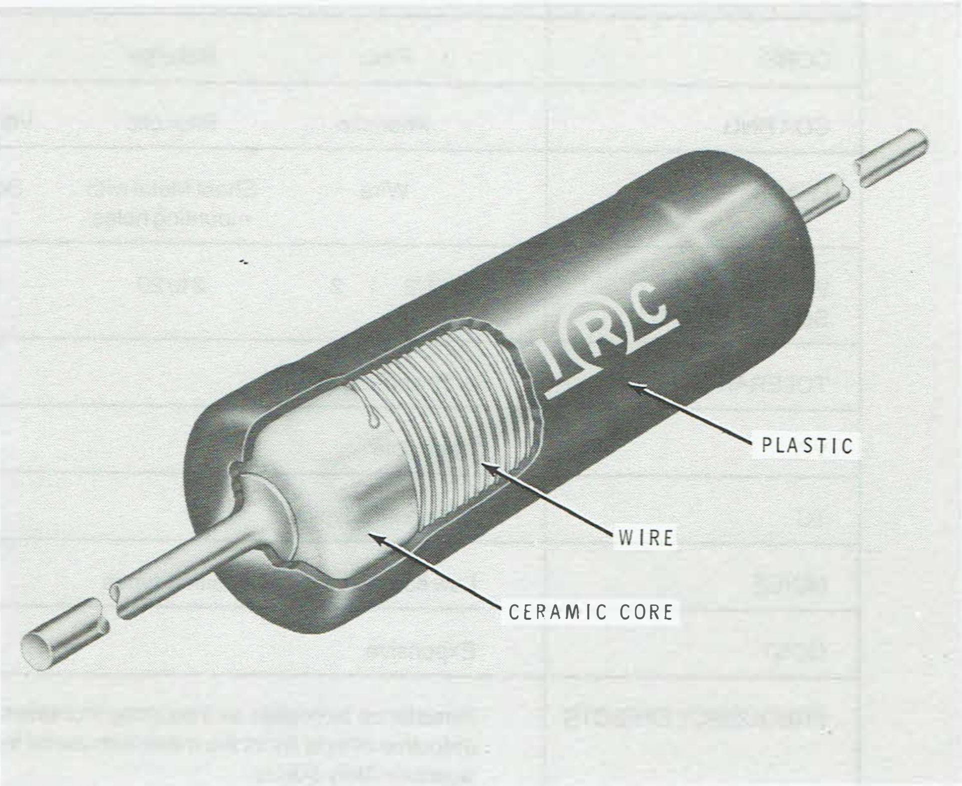


Figure 4-3

The wire-wound resistor.

In addition, numerous high-power, wire-wound resistors are readily available. Table 4-3 summarizes some typical characteristics of wire-wound resistors. Compared to composition resistors, wire-wound resistors have much more stable resistance values. The principle disadvantages of wire-wound resistors include their comparatively high cost, and inherent inductive nature. This latter characteristic makes wire-wound resistors unsuitable for high frequency applications.

	LOW POWER	MEDIUM POWER	HIGH POWER
CORE	Fiber	Bakelite	Ceramic
COATING	Phenolic	Phenolic	Vitreous enamel
LEADS	Wire	Sheet Metal with mounting holes	Soldering lugs
WATTAGE (W) Specified at 40°C	1/2 1 2	2 to 20	5 to 200 +
TOLERANCE (%)	± 10 and ± 5 typical		
VC	Negligible		
TC	250 ppm/°C typical		
NOISE	Low compared to composition-types		
COST	Expensive		
FREQUENCY EFFECTS	Resistance increases as frequency increases. Significant inductive effects limits the maximum useful frequency to approximately 50kHz.		
MISCELLANEOUS	Precision ± 1% or less resistors are available.		
APPLICATIONS	Circuits requiring precise, stable resistance values with low noise. Also high power applications. Not suitable for high frequency applications.		

Table 4-3

Typical Characteristics of Wire-Wound Resistors

Film Resistors

Solid carbon-composition resistors are inexpensive, but have resistance values that can change significantly with changes in temperature, voltage, cycling, and humidity. The stability of wire-wound resistors is much better. Unfortunately, wire-wound resistors are expensive, and exhibit significant amounts of inductance and capacitance, which limits their usefulness to relatively low frequencies.

Film resistors have characteristics that tend to lie between the extremes offered by solid carbon-composition and wire-wound resistors. For this reason, film resistors are becoming increasingly popular and, along with carbon composition resistors, are the most commonly used resistor type.

Film resistors are formed by depositing a resistance film on a ceramic or glass rod. For this reason, such resistors are often referred to as deposited-film resistors. If the thickness of the film is one millionth of an inch or less, the resistor is called a **thin-film** resistor. Similarly, if the film is thicker than one millionth of an inch, the resistor is considered a **thick-film** resistor.

For low resistance values, the resistance film is continuous from one end of the rod to the other. In such cases, the resistance is determined by the film's resistivity, thickness, and length. To obtain higher resistance values, a spiral groove is cut through the film, as shown in Figure 4-4. Note that the size of the groove determines the length and width of the ribbon of resistance film that remains. Thus, by varying the size of the groove cut through the film, it is possible to obtain a wide range of resistance values.

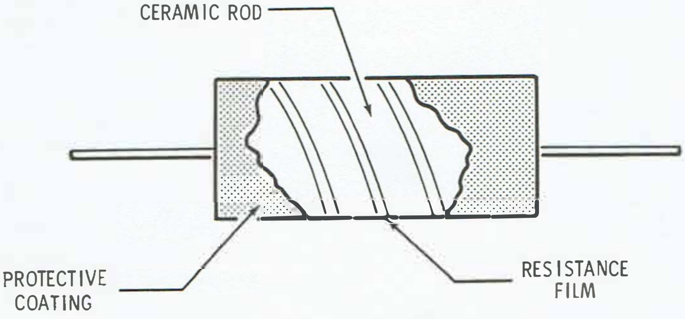


Figure 4-4
Simplified drawing of a film resistor.

The material used for the resistance film, and the manner in which it is deposited on the insulating rod, largely determine the characteristics of the resistor. Carbon film resistors, for example, are inexpensive and, generally speaking, are more stable than solid composition resistors.

Numerous metal-film types are also available. These resistors are made by depositing various metal alloys on glass, or ceramic rods. Precision metal film resistors are available that compare very favorably with the precision characteristics of wire-wound resistors. In addition, film resistors exhibit considerably less inductance and capacitance than wire-wound resistors. For this reason, film resistors can be used at frequencies of several megahertz or more, whereas wire-wound resistors cannot.

Cermet is a material that is a combination of a ceramic and metal. Cermet resistors are usually physically smaller than other resistor types. This makes them attractive for applications where space is limited. In addition, cermet resistors can withstand very high voltages and exhibit stable resistance values in extreme environments.

So-called bulk film resistors are made by etching a pure metal on a glass substrate. Bulk film resistors have the best high frequency characteristics of any resistor type currently available. In addition, their temperature coefficient, and other characteristics are unsurpassed. The disadvantages of bulk film resistors include their high cost, limited range of available resistance values, and limited power rating. Excluding these disadvantages bulk film resistors come closer to the characteristics of an ideal resistor than any other resistor type.

Table 4-4 summarizes some typical characteristics of film resistors.

	Carbon	Metal	Cermet	Bulk
RANGE	1Ω to 1MΩ	10Ω to several MΩ	10Ω to hundreds MΩ	30Ω to hundreds KΩ
TOLERANCE	±5%	1% typical	1% typical	<0.01%
WATTAGE	2W maximum	5W maximum	3W maximum	1W maximum
TC	200 ppm/°C	20-200 ppm/°C	100 ppm/°C	1 ppm/°C
COST	low	Expensive	Expensive	Moderate
MISCELLANEOUS	Improved performance over solid-composition types.	Numerous precision types available	Excellent choice for harsh environments can withstand very high voltages.	Best high frequency response. Approximates an ideal resistor.

Table 4-4
Typical characteristics of film resistors.

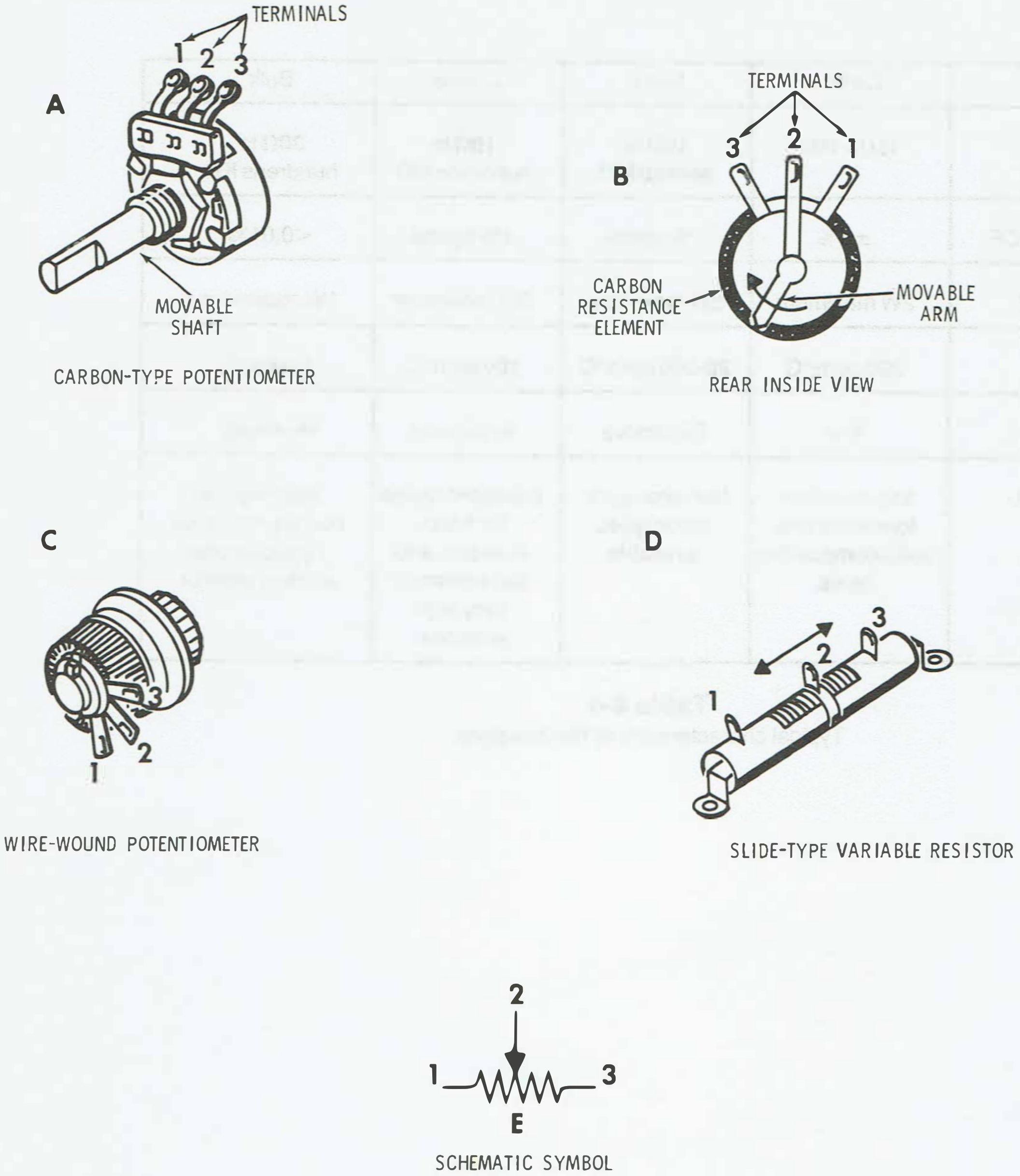


Figure 4-5

Variable resistors.

A. Carbon potentiometer.
B. Rear inside view.
C. Wire-wound potentiometer.
D. Slide-type variable resistor.
E. Schematic symbol.

Variable Resistors

Variable resistors that have carbon composition, wire-wound, or film-type resistive elements are widely available. Generally speaking, the characteristics discussed earlier for fixed resistors also apply to variable resistors. There are, however, several differences between fixed and variable resistors that you should be aware of. The wattage rating of a variable resistor, for example, applies to the maximum resistance value. Consequently, if only a portion of the maximum resistance is being used, the wattage rating should be reduced accordingly. Variable resistors are usually referred to as potentiometers or "pots."

Figure 4-5A and Figure 4-5B illustrate the construction details of a carbon pot. Similarly, Figure 4-5C illustrates a wire-wound pot, while a sliding-contact variable resistor is illustrated in figure 4-5D. Sliding-contact variable resistors are generally used in high-power applications where the resistance must be initially set, or occasionally reset.

Typically, the movable arm of a pot can move through an arc of approximately 300°. In addition, the change in resistance between the movable arm, which is called the wiper, and either end of the pot can vary in either a linear or nonlinear manner as shown in Figure 4-6. Logarithmic or "audio taper" pots used in volume controls are examples of nonlinear pots. For obvious reasons, pots used as voltage dividers change resistance in a linear manner as the shaft of the pot is turned.

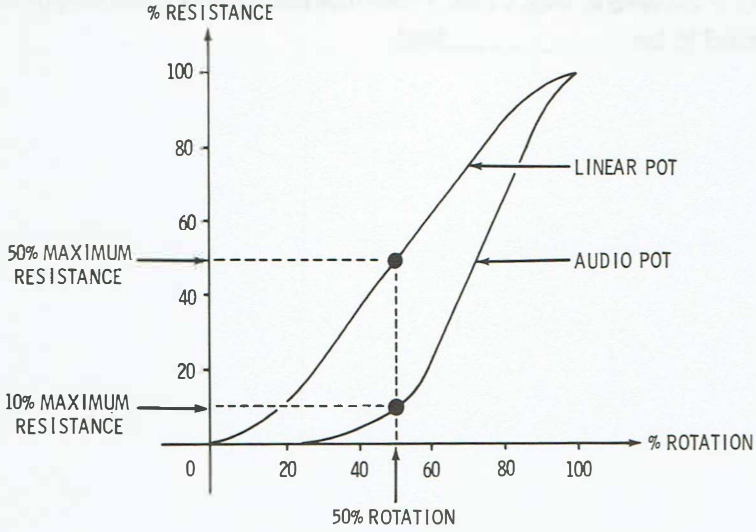


Figure 4-6

Typical resistance versus rotation characteristics of linear and nonlinear potentiometers.

Self-Test Review

1. Real resistors exhibit the properties of ____________________ and ____________________ in addition to resistance.
2. Most conductors have ____________________ temperature coefficients.
3. The DC resistance of a conductor depends upon its resistivity, length, cross-sectional area, and ____________.
4. Wire diameters are normally given in units of ____________ ________.
5. ______________ resistors are the least stable type.
6. __________ ______________ resistors exhibit the most noise.
7. ______________ resistors have the best high-frequency characteristics.
8. ________-____________ resistors have the poorest high-frequency characteristics
9. Precision ________ ____________ resistors have characteristics that rival wire-wound resistors.
10. ______________ film resistors can withstand very high voltages.
11. Overall, ____________ film resistors come closest to the characteristics of ideal resistors.
12. Audio potentiometers are examples of __________ pots.
13. Thick-film resistors have a film thickness greater than one __________ of an inch.;
14. A resistor has a nominal value of 100Ω at 25°C and a temperature coefficient of 1000 ppm/°C. Its resistance at 100°C is __________ ohms.
15. A roll of AWG # 22 silver wire has a resistance of 2Ω. The cross-sectional area of AWG # 22 wire is 642.4 CM. From this information, the length of the wire is calculated to be __________ feet.

Answers

1. inductance, capacitance
2. positive
3. temperature
4. circular mils
5. composition
6. Carbon composition
7. Bulk
8. Wire-wound
9. metal film
10. cermet
11. bulk
12. nonlinear
13. millionth
14. 107.5
15. 129.78

The solutions to questions 14, and 15 follow.

14.

$$1000 \text{ ppm} = 1000/1{,}000{,}000 = 0.001$$

Thus

$$1000 \text{ ppm}/°\text{C} = 0.001/°\text{C}$$

$$\Delta R = TC\,(\Delta T)\,(R_1)$$
$$\Delta R = 0.001\,(100–25)\,(100\Omega)$$
$$\Delta R = 7.5\Omega$$

The resistance at 100°C is, therefore:

$$R = R + \Delta R$$
$$R = 100\Omega + 7.5\Omega$$
$$R = 107.5\Omega$$

15.

$$R = 2\Omega$$
$$A = 642.4 \text{ CM}$$
$$P = 9.9\Omega \text{ CM/ft} \quad \text{from Table 4-1}$$

Since $R = \frac{Pl}{A}$, you can solve the equation for the length l. Thus.

$$l = \frac{RA}{P} = \frac{2\Omega\,(642.4)}{9.9} = 129.78 \text{ ft}$$

CAPACITORS

Next to resistors, capacitors are the most frequently used circuit components. Because of their widespread use, there are a number of different types of capacitors available. For this reason, the selection of a particular capacitor for a given circuit is often more involved than the selection of a suitable resistor. As with all components, numerous factors must be considered before a final choice is made.

Capacitor Basics

In its simplest form, a capacitor consists of two metal plates separated by an insulating material called the **dielectric** — as shown in Figure 4-7. The capacitance of this type capacitor is given by:

$$C = \frac{0.225\ KA}{d} \qquad \text{(Eq. 4-4)}$$

where

C = capacitance in picofarads (pf)
A = area of one plate in square inches (in^2)
d = distance between the plates in inches (in)
K = the relative dielectric constant of the material separating the plates.

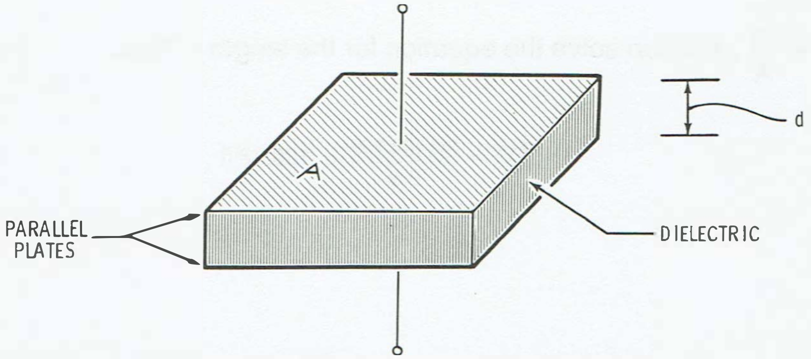


Figure 4-7
The parallel plate capacitor.

Table 4-5 lists typical values for the relative dielectric constants of commonly used materials.

Material	Relative Dielectric Constant K
Vacuum,	1
Air	1
Mica	7
Paper	4
Glass	6
Porcelain	6
Aluminum oxide	10
Tantalum oxide	11

Table 4-5
Relative dielectric constants for commonly used materials.

By definition, the dielectric constant of a vacuum is 1. The dielectric constant for air is only sightly larger, so we will assume it is also 1. The term "relative dielectric constant" indicates that the properties of a given material are being compared with those of a vacuum or air. Equation 4-4 indicates that capacitance is directly proportional to the value of the material's relative dielectric constant. Thus, by using different dielectric materials, different values of capacitance can be obtained for a given area and distance.

Example 4-3

Two parallel metal plates are separated by 0.01 in. The area of one plate is 0.5 in^2. Calculate the capacitance for an air dielectric, then for a dielectric of tantalum oxide.

For air K = 1. Thus,

$$C = \frac{0.225\,(1)\,(0.5)}{(0.01)} = 11.25\text{pF}$$

The relative dielectric constant of tantalum oxide is 11. Thus, if tantalum oxide is used for the dielectric material the resulting capacitance will be 11 times greater than if air was the dielectric material.

$$C = 11\,(11.25\text{pF}) = 123.75\text{pF}$$

Definitions

When discussing capacitors, a knowledge of the following terms will prove useful.

Leakage Current. Since real dielectrics are not perfect insulators, a small current will flow in a capacitor when a DC voltage is applied across the capacitor. This current is called the leakage current.

Insulation Resistance. This is the resistance of the dielectric material separating the plates of the capacitor. Typically insulation resistance decreases as temperature increases. Consequently, leakage current tends to increase as temperature increases.

Equivalent Series Resistance. This is the effective series resistance of the capacitor. The equivalent series resistance, ESR, tends to increase with frequency.

Quality Factor. This is a figure of merit and is defined as follows.

$$Q = \frac{X_C}{ESR} \qquad \text{(Eq. 4-5)}$$

Large values of Q indicate a high quality capacitor. That is, one whose losses are small.

Dissipation Factor. The dissipation factor, D, is the reciprocal of the quality factor, Q. Small values of D are desirable since they indicate high quality.

Ripple Voltage. If both DC and AC voltages are present, the maximum voltage is the sum of the DC and peak AC value. This maximum value must not exceed the capacitor's rated voltage. Some capacitor types tolerate ripple voltages better than other types.

Capacitor Types

Capacitors may be classified as either fixed or variable. Additional classification is usually based on the type of dielectric material used to make the capacitor. Figure 4-8 illustrates a variable capacitor that uses air as the dielectric material. In this case, the capacitance value can be changed by rotating the shaft, since this changes the effective plate area, and hence capacitance. Capacitors similar to the one shown in Figure 4-8 typically can change capacitance value over a 10:1 range. Such capacitors are used as tuning elements in radios, televisions, and so forth. Capacitance values range from a few pF to approximately 500 pF. In order to help you become familiar with the characteristics of the various capacitor types, a brief discussion of the more common varieties follows.

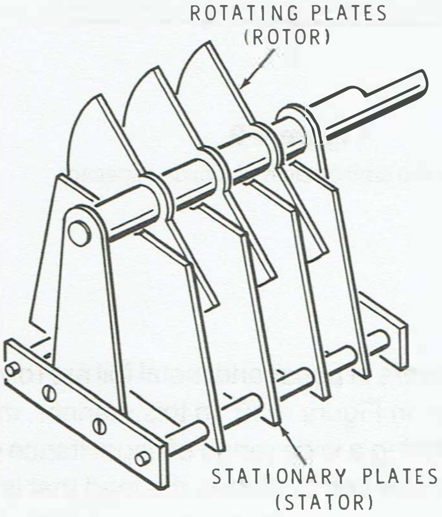


Figure 4-8
A variable capacitor that uses air for the dielectric material.

MICA

Mica is a chemically inert, naturally occurring mineral whose dielectric properties are relatively stable with frequency, temperature, humidity, and other environmental changes. For this reason, mica was one of the first materials used to make capacitors. Mica capacitors have a sandwich-like construction as shown in Figure 4-9. Usually, several such sandwiches are stacked together in a single unit in order to obtain increased capacitance. Silvered mica capacitors have a thin silver layer rather than tin-lead foil for the capacitor plates. This construction results in a capacitor that has more stable electrical characteristics, and greater mechanical strength.

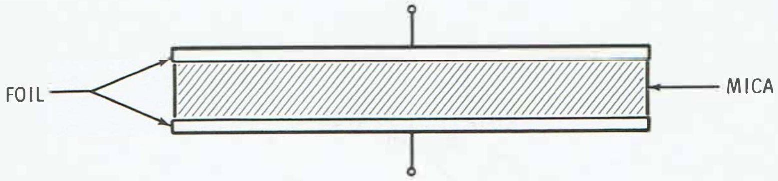


Figure 4-9
Sandwich-like construction of a mica capacitor.

PAPER

In paper capacitors, alternate layers of paper and metal foil are rolled together, forming a tubular structure as shown in Figure 4-10. In this manner, manufacturers can produce low cost capacitors offering a wide range of capacitance values. paper capacitors usually have a band at one end to identify the lead that is connected to the outside metal foil. Grounding this lead improves noise immunity, since the inner foil is effectively shielded from stray electric fields. The principle disadvantage of paper capacitors is their sensitivity to temperature changes.

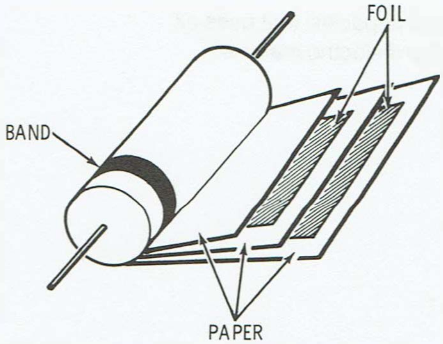


Figure 4-10
Construction of a typical paper capacitor.

GLASS

The construction of a glass capacitor is similar to that of a mica capacitor. Glass capacitors are very reliable and offer stable capacitance values. They are an excellent choice for humid environments. However, they are relatively expensive.

CERAMIC

Ceramic materials have a large range of dielectric characteristics. Consequently, ceramics are the most versatile dielectric materials currently available. So-called low-k ceramic dielectrics can be manufactured with a temperature coefficient that is essentially zero. This type of capacitor is usually marked with the letters "NPO" to indicate the zero temperature coefficient. Markings like P100 and N100 indicate positive and negative temperature coefficients, respectively, of 100 ppm/°C.

High-k ceramic capacitors have larger capacitance values than low-k capacitors. However, they are not nearly as stable with changes in temperature, voltage, and frequency.

PLASTIC FILM

In these capacitors, the dielectric consists of a thin film of plastic. Currently, the most popular plastic dielectrics are polystyrene, and polyester (trade name mylar). Polystyrene capacitors have excellent temperature characteristics but tend to be physically large. Polyester capacitors are usually smaller but their temperature characteristics are nowhere nearly as stable as polystyrene capacitors. Improved manufacturing techniques will probably increase the range and number of applications for plastic film capacitors.

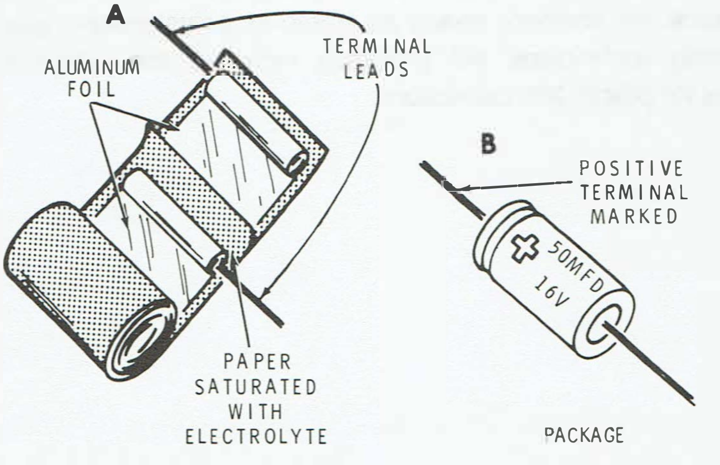
ELECTROLYTIC

Electrolytic capacitors have the largest amount of capacitance for a given physical size. The dielectric consists of a thin oxide layer which is formed by electrochemical action during the manufacturing process. Figure 4-11 illustrates a popular type of aluminum electrolytic capacitor. Sheets of aluminum foil are separated by paper that has been saturated with a conductive paste called the electrolyte. In Figure 4-11, you should note the following:

1. The foil with the aluminum oxide coating is the positive plate.
2. The electrolyte is the negative plate.
3. The dielectric is the very thin layer of aluminum oxide.

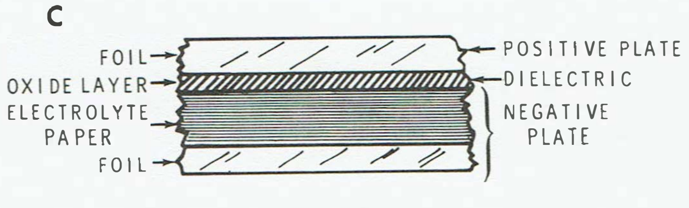
Because the aluminum oxide layer is very thin, and because the area of the aluminum sheets is large, the capacitance-to-volume ratio is also large. Unfortunately, the thin oxide layer limits the maximum voltage that can be applied across the capacitor to relatively low values. In addition, the thin oxide layer results in relatively low values of leakage resistance, and consequently, large leakage currents.

Note that the capacitor in Figure 4-11 is **polarized**. This means that the capacitor will work properly only if the correct polarity is observed when the capacitor is put into a circuit. For this reason, one lead of the capacitor is marked with a + sign as shown in Figure 4-11B. By forming oxide coating on both plates, it is possible to make nonpolarized electrolytic capacitors. By doing this, however, you significantly decrease the capacitance-to-volume ratio.



PACKAGE

CONSTRUCTION



STRUCTURE

Figure 4-11

A polarized aluminum electrolytic capcitor.
A. Construction.
B. Package.
C. Structure.

The insulation resistance of electrolytic capacitors varies greatly from one capacitor to another. In addition, the insulation resistance of a given capacitor tends to decrease with time. For these reasons, series-connected electrolytic capacitors often use **equalizing resistors** as shown in Figure 4-12. Without the equalizing resistors, the voltage across each capacitor would depend upon the value of the capacitor's insulation resistance. If one capacitor had an insulation resistance considerably larger than the others, the voltage developed across it could easily exceed its rated value. Usually, equalizing resistors are chosen with values between 22kΩ and 56kΩ.

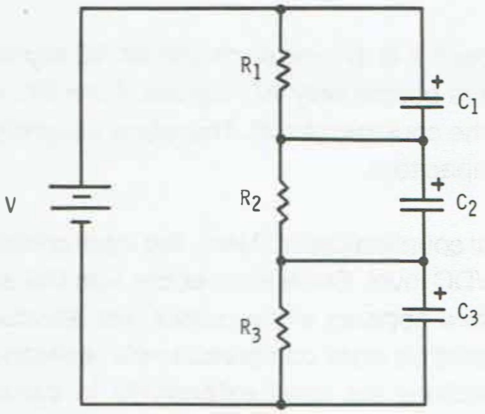


Figure 4-12

Using equalizing resistors for series-connected electrolytic capacitors.

Since the value of the equalizing resistor is small compared to the capacitor's insulation resistance, the parallel combination of an equalizing resistor and a capacitor's insulation resistance essentially equals the value of the equalizing resistor. In this way, the voltage developed across a capacitor depends upon the value of the equalizing resistor, and not on the capacitors insulation resistance.

Capacitors are often connected in series is to increase the effective voltage rating. Two 100μF, 250V series-connected capacitors, for example, are equivalent to a single 50μF, 500V capacitor. This technique is often used in power supply filter circuits, since two 100μF, 250V capacitors might be less expensive than a single 50μF, 500V capacitor.

When aluminum-oxide electrolytic capacitors are not used, their oxide layers tend to deteriorate. For this reason, such capacitors are said to have a short **shelf life**. Tantalum oxide capacitors were developed to overcome this, and other disadvantages.

Generally speaking, tantalum oxide capacitors have long shelf lives, larger leakage resistances, smaller leakage currents, and larger capacitance values than comparable aluminum oxide capacitors. The principle disadvantages of tantalum oxide capacitors compared to aluminum oxide capacitors are their higher cost, and lower working voltage rating. Currently, tantalum oxide capacitors are available in foil, wet, and solid types. Each name refers to the particular manufacturing process used to make the capacitor.

Capacitor Applications

Applications for capacitors include coupling, bypassing, tuned circuits, timing, and filters. Often, the particular application will dictate what type of capacitor is appropriate. For this reason, a brief description of some typical capacitor applications follows.

COUPLING

The function of a coupling capacitor is to pass or couple an AC signal from one point to another. Coupling capacitors couple only AC signals. Pure DC signals are prevented from passing through the coupling circuit. Therefore, coupling capacitors are also referred to as blocking capacitors.

Figure 4-13 illustrates a typical coupling circuit. Here, the input consists of a 1V peak sine wave that "rides on" a 9 VDC level. Since a capacitor acts like an open circuit to DC, only the 1V peak sine wave appears at the output. For effective coupling, the reactance of the capacitor should be small compared to the resistance in series with the capacitor. Coupling capacitors are used extensively in transistor circuits to couple signals from one stage to another. Depending upon the frequencies to be coupled, any number of capacitor types are appropriate. Following this section, a number of capacitor summary charts are provided to help you select a particular type of capacitor for a specific application.

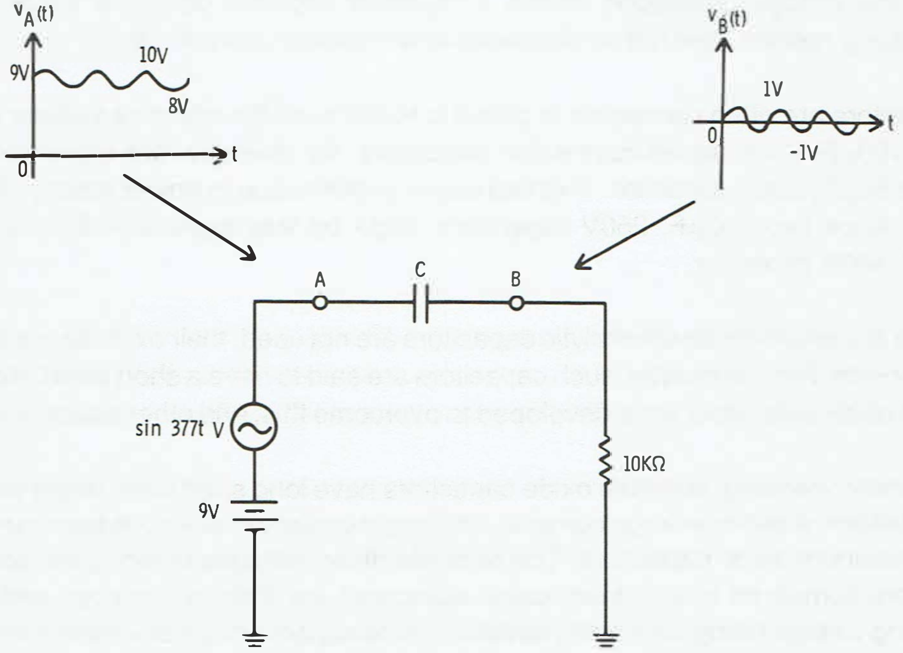


Figure 4-13

The coupling capacitor.

BYPASSING

The function of a bypass capacitor is just the opposite of a coupling capacitor. For this reason, bypass capacitors are also called decoupling capacitors. Figure 4-14 illustrates a typical bypassing circuit. Because the reactance of the capacitor is small compared to the series resistance, nearly all the AC voltage is dropped across the resistance. Thus, the AC output voltage is essentially zero. For this reason, point B is considered to be at AC ground potential. Since the capacitor acts like an open circuit for DC, whatever DC voltage is present at the input will also appear at the output.

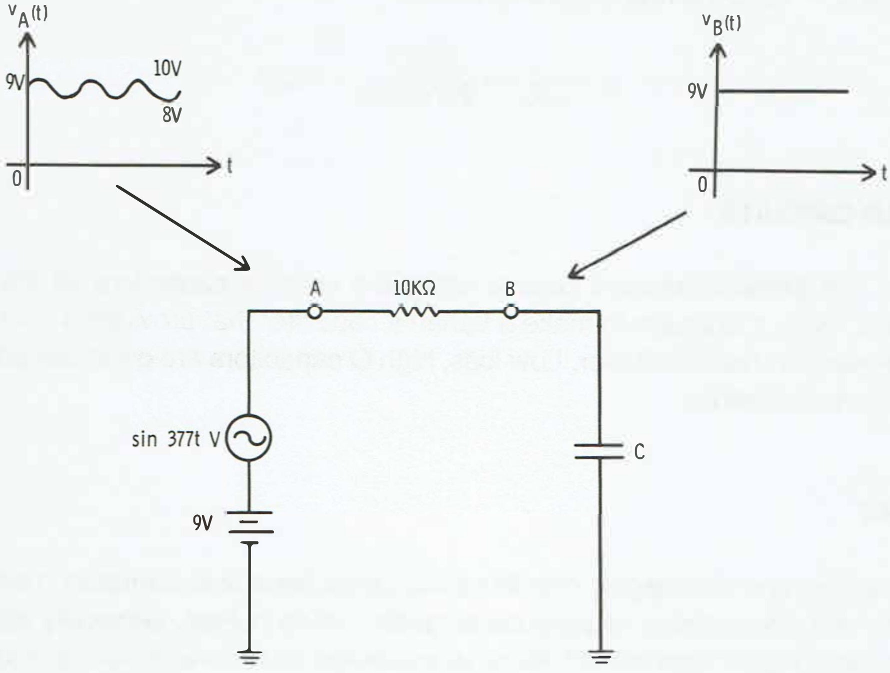


Figure 4-14

The bypass capacitor.

Bypass capacitors are also widely used in transistor circuits. In addition, bypass capacitors are often connected between the output of a power supply and ground to prevent transient voltage spikes from reaching sensitive equipment.

To calculate the value of a coupling or bypass capacitor for a given circuit you can use the following rule of thumb:

$$|X_C| = \frac{R_{TH}}{20} \quad \text{(Eq. 4-6)}$$

where: $|X_C|$ is the reactance of the capacitor at the **lowest** frequency to be coupled, or bypassed.

R_{TH} = Thevenin resistance as seen from the capacitor.

Example 4-4

Calculate the minimum size capacitor required to produce good coupling and bypassing in Figure 4-13, and Figure 4-14 respectively.

In each case, R = 10kΩ. Also the AC signal has a radian frequency of 377 rad/s, which corresponds to 60 Hz. Thus:

$$|X_C| = \frac{R}{20} = \frac{10k\Omega}{20\Omega} = 500\Omega$$

Since $|X_C| = 1/\omega C$ the required value of C is:

$$C = \frac{1}{\omega |X_C|} = \frac{1}{377\,(500\Omega)} = 5.3\mu F$$

TUNED CIRCUITS

Series and parallel resonant circuits often use variable capacitors as the tuning element, since it is easier to make a variable capacitor that provides a 10:1 tuning range than a variable inductor. Low loss, high Q capacitors are generally preferred for such appalications.

TIMING

RC charging and discharging circuits can be used alone or in conjunction with other circuits and components to produce accurate timing pulses. Generally speaking, capacitors for such applications must have leakage currents as small as possible. In addition, the capacitance values should not change significantly with time and temperature.

FILTERS

In power supplies, smoothing filters are used to smooth out the pulsating DC voltage produced by the power supply. Usually, large capacitance values are required for this purpose. Thus, electrolytic capacitors are generally preferred. For other types of filters, the choice of an appropriate capacitor type usually depends upon the frequencies the filter must process.

Capacitor Selection Guide

The following selection guide attempts to summarize some characteristics of the major capacitor types. Numerous varieties are usually available for a given type of capacitor. Paper capacitors, for example, may be impregnated with various oils, waxes, and jellies. Naturally, each variety has its unique characteristics. In addition, for a given type and variety, different manufacturers may specify different tolerances, ranges, and voltage ratings. Thus, the selection guide is intended for comparison purposes only. For more specific information, you should consult manufacturer's catalogs. The selection guide uses the following abbreviations:

Capacitance-to-volume ratio — C/V.
Low, medium, and high frequency — LF, MF, HF.
Maximum — Max.
Frequency — Freq.
Resistance — R.
Power factor — pf.

MICA

Range	1 pF to 0.1μF.
Tolerance (%)	±1, ±10.
Max Voltage	50V to 75kV.
Leakage R (MΩ)	1,000 to 100,000 +.
Freq Range	1kHz to 10GHz.
Cost	High.
Advantages	Low loss, stable values.
Disadvantages	Cost, low c/v ratio.

APPLICATIONS Coupling/bypassing HF, filters, tuned circuits.

PAPER

Range	100 pF to 100μF.
Tolerance (%)	±10, ±20.
Max Voltage	50V to 100 + kV.
Leakage R (MΩ)	100.
Freq Range	100Hz to 1MHz.
Cost	Low.
Advantages	Cost, wide range of values.
Disadvantages	Temperature sensitive.

APPLICATIONS Coupling/bypassing LF, LF filters, pf correction.

CERAMIC (low k)

Range	1 pF to 0.001 μF.
Tolerance (%)	±5, ±20.
Max Voltage	10kV.
Leakage R (MΩ)	1000.
Freq Range	1kHz to 10GHz.
Cost	Low.
Advantages	Zero temperature coefficient possible.
Disadvantages	Small capacitance values.

APPLICATIONS Temperature compensated circuits.

CERAMIC (high k)

Range	100 pF to 1 μF.
Tolerance (%)	+100, −20.
Max Voltage	100V
Leakage R (MΩ)	100.
Freq Range	1kHz to 10THz +.
Cost	Low.
Advantages	Large capacitance values compared to low k.
Disadvantages	Capacitance changes with temperature.

APPLICATIONS Coupling/bypassing HF.

GLASS

Range	1 pF to 10 μF.
Tolerance (%)	±1, ±10.
Max Voltage	20V to 6kV.
Leakage R (MΩ)	100,000 plus.
Freq Range	
Cost	High.
Advantages	Low loss, temperature stability.
Disadvantages	Cost, low c/v ratio.

APPLICATIONS Coupling/bypassing HF, tuned circuits.

PLASTIC FILM — Polystyrene

Range	500 pF to 20μF.
Tolerance (%)	±0.5.
Max Voltage	1kV.
Leakage R (MΩ)	10,000.
Freq Range	DC to 10GHz.
Cost	Medium.
Advantages	Temperature stability.
Disadvantages	Physical size, low c/v ratio.

APPLICATIONS Coupling/bypassing HF.

PLASTIC FILM — Polyester (Mylar)

Range	5000 pF to 20μF.
Tolerance (%)	±10, ±20.
Max Voltage	100V to 600V.
Leakage R (MΩ)	10,000.
Freq Range	100Hz to 1MHz.
Cost	Medium.
Advantages	Higher c/v ratio than polystyrene.
Disadvantages	Temperature sensitive.

APPLICATIONS Coupling/bypassing LF and MF.

ALUMINUM ELECTROLYTIC

Range	1μF to 10,000μF plus.
Tolerance (%)	+100 to –20.
Max Voltage	3V to 650V.
Leakage R (MΩ)	1MΩ or less.
Freq Range	15Hz to 10kHz.
Cost	Low/Medium.
Advantages	High c/v ratio.
Disadvantages	Large leakage current, short shelf life.

APPLICATIONS Coupling/bypassing LF, power supply smoothing filters.

TANTALUM ELECTROLYTIC

Range	0.1μ to 1,000μF.
Tolerance (%)	+100 to –20.
Max Voltage	250V.
Leakage R (MΩ)	10 plus.
Freq Range	DC to 1kHz.
Cost	Medium.
Advantages	Long shelf life, high c/v ratio compared to aluminum.
Disadvantages	Low voltage rating.

APPLICATIONS Coupling/bypassing LF, LF filters.

COMMENTS

Solid tantalum capacitors are the most commonly used types. Of the three varieties currently available, solid types have the best temperature characteristics.

Foil and wet types are available with higher voltage ratings than solid types — but are more temperature sensitive.

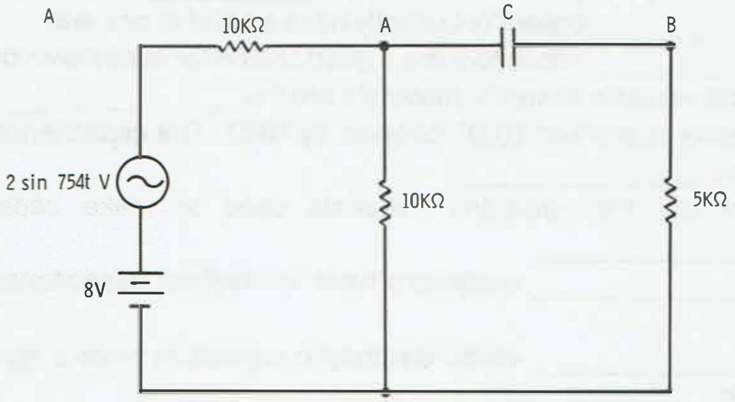
Self-Test Review

16. A parallel plate capacitor uses air as its dielectric material and has a capacitance of 2.2μF. If the dielectric were changed to porcelain, the capacitance would be __________ .
17. The capacitance of a parallel plate capacitor is __________ , assuming a plate area of 0.25 in^2, a distance between the plates of 0.001 in, and a mica dielectric.
18. The resistance of the dielectric material separating the plates of a capacitor is called the __________ resistance.
19. A low-loss capacitor would have a high __________ .
20. ______________ capacitors usually have a band at one end.
21. ______________ capacitors are a good choice for humid environments.
22. The most versatile dielectric materials are the _________________ .
23. A capacitor is marked 10μF followed by NPO. The capacitance at 65°C is therefore ______________ .
24. One of the first dielectric materials used to make capacitors was _________________ .
25. _________________ capacitors have the highest capacitance-to-volume ratio.
26. _________________ -oxide electrolytic capacitors have a relatively short shelf life.
27. Which plastic film capacitor; polystyrene or polyester, is more temperature sensitive? _________________ .

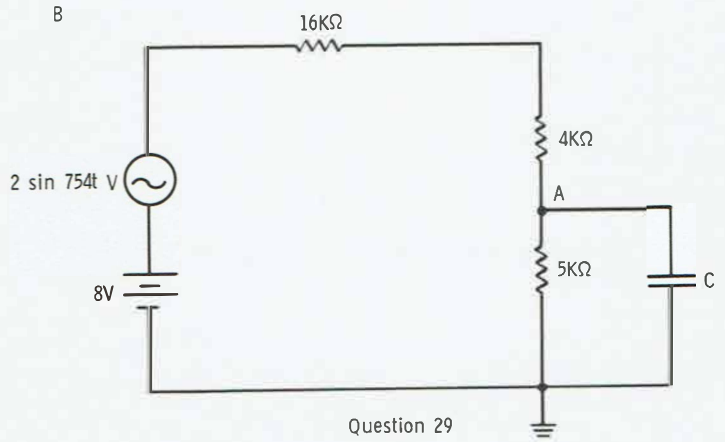
Refer to Figure 4-15.

28. In Figure 4-15A, the minimum size capacitor required for good coupling is ____________________.

29. In Figure 4-15B, the minimum size capacitor required for good bypassing is ____________________.



Question 28



Question 29

Figure 4-15

Circuits for questions 28 and 29.
A. Question 28.
B. Question 29.

Answers

16. $13.2\mu F$
17. 393.75 pF
18. insulation or leakage
19. Q
20. Paper
21. Glass
22. ceramics
23. $10\mu F$
24. mica
25. Electrolytic
26. Aluminum
27 polyester
28. $2.7\mu F$
29. $6.6\mu F$

The solutions to questions 16, 17, 23, 28, and 29 follow.

16. The capacitance with an air dielectric is given as $2.2\mu F$. Table 4-5 lists the relative dielectric constant of porcelain as 6. Therefore:

$$C = 6(2.2\mu F) = 13.2\mu F$$

17. $A = 0.25$ in^2, $d = 0.001$ in, and $k = 7$ from Table 4-5 Thus:

$$C = \frac{0.225kA}{d} = \frac{0.225(7)\,(0.25)}{0.001} = 393.75\text{ pF}$$

23. The designation NPO means the temperature coefficient is zero. Therefore, the value of C does not change with temperature, so $C = 10\mu F$.

28. Removing the capacitor in Figure 4-15A, and reducing both sources to zero lets you calculate the Thevenin resistance as viewed from the capacitor. Thus:

$$R_{TH} = 10k\Omega \| 10k\Omega + 5k\Omega = 10k\Omega$$

For good coupling $|X_C| = \frac{R_{TH}}{20} = \frac{10k\Omega}{20} = 500\Omega$

The radian frequency of the AC source is 754 rad/s. Thus, the minimum value of C required for good coupling is:

$$C = \frac{1}{\omega|X_C|} = \frac{1}{754(500\Omega)} = 2.7\mu F$$

29. Bypass capacitors are calculated in a manner similar to coupling capacitors. In figure 4-15B, once the capacitor is removed and the sources have been reduced to zero, you can see that:

$$R_{TH} = (16k\Omega + 4k\Omega) \parallel 5k\Omega$$

$$R_{TH} = 20k\Omega \parallel 5k\Omega = 4k\Omega$$

Thus: $$|X_C| = \frac{4k\Omega}{20} = 200\Omega$$

$$C = \frac{1}{\omega|X_C|} = \frac{1}{754(200)} = 6.6\mu F$$

INDUCTORS

Of the three basic circuit components (resistors, capacitors, and inductors), inductors are the least ideal. For this reason, RC circuits are used more extensively than RL circuits. In certain applications, however, inductors are absolutely necessary. In this section, you will become familiar with some of the non-ideal properties of real inductors, and the steps that can be taken to minimize them.

Inductor Basics

Figure 4-16 illustrates one of the simplest possible inductors. Here, a single layer of wire has been wound on a hollow cardboard, or some other, nonmagnetic core. In this type of inductor, the core material is air, since the cardboard form simply supports the numerous turns of wire. Consequently the inductor is referred to as an air-core inductor or an air-core coil. To a reasonable approximation, the inductance of such a coil is given by:

$$L_o = \frac{(rN)^2}{9r + 10L} \qquad \text{(Eq. 4-7)}$$

where L_o = inductance in microhenries (μH)
r = mean core radius in inches (in)
L = core length in inches (in)
N = number of turns

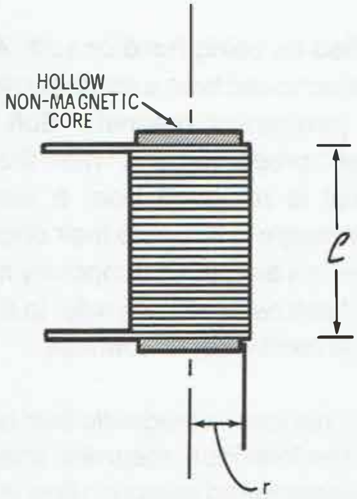


Figure 4-16

A single-layer air-core inductor.

Equation 4-7 indicates that the inductance of the coil depends upon the coil's geometry, and the number of turns of wire used to wind the coil. Formulas for computing the inductance of coils with more complex geometries and winding patterns are available in engineering handbooks.

Core Materials

One of the factors that can have a significant affect on the inductance of a coil is the type of material used for the core. Generally speaking, materials may be classified as magnetic or nonmagnetic. Magnetic materials become magnetized when placed in a magnetic field, while nonmagnetic materials do not. The atoms and molecules that make up all matter exhibit magnetic properties.

In magnetic materials, these "molecular magnets" tend to align themselves in the same direction when the material is placed in an external magnetic field. The magnetic fields of molecular magnets aligned in the same direction reinforce each other. Thus, when enough molecular magnets are aligned in the same direction, the material exhibits magnetic properties.

In unmagnetized materials, the directions of the various molecular magnets are random. Thus, some molecular magnetic fields reinforce each other, while an equal number cancel each other — producing a net magnetic field that is zero.

The molecular magnets in nonmagnetic materials do not have the ability to align themselves in the same direction when the material is placed in an external magnetic field. For this reason, such materials cannot be magnetized.

Magnetic materials are classified as being **hard** or **soft**. A hard magnetic material retains its magnetism when it is removed from a magnetic field. Thus, hard magnetic materials are used to make permanent magnets. Soft magnetic materials are different — they exhibit magnetic properties only when they are in a magnetic field. Once a soft magnetic material is removed from a magnetic field, it loses its magnetism, since its molecular magnets return to their original random position. For this reason, soft magnetic materials are called temporary magnets. It is important to realize that the terms hard, and soft as used here refer to the magnetic properties of a material and not to its physical hardness, or softness.

When current flows through a conductor, a magnetic field is created. If the conductor is wound in the form of a coil, the individual magnetic lines of force reinforce each other. Consequently, a more concentrated magnetic field is produced within the core of the coil, which increases the inductance substantially.

Flux is another name for lines of force. The term **flux density** refers to the number of lines of magnetic force per unit area. Thus flux density indicates the intensity or strength of a magnetic field. Stated mathematically:

$$B = \frac{\varnothing}{A} \qquad \text{(Eq. 4-8)}$$

where B = flux density.
$\varnothing$ = number of lines of magnetic force.
A = cross-sectional area through which the magnetic lines of force pass.

The units of flux, flux density, and other magnetic quantities depend upon the system of measurement being used. Table 4-6 summarizes typical magnetic units for the English, cgs, and mks measurement systems.

TERM	DESCRIPTION	SYMBOL	ENGLISH UNITS	CGS UNITS	MKS UNITS
FLUX	TOTAL LINES OF FORCE	$\varnothing$	LINES OR KILOLINES	MAXWELL 1 MAXWELL = 1 LINE	WEBER 1 WEBER = 100,000,000 LINES
FLUX DENSITY	LINES PER UNIT AREA	B	$\frac{\text{KILOLINES}}{\text{IN}^2}$	GAUSS $1 \text{ GAUSS} = \frac{1 \text{ MAXWELL}}{\text{CM}^2}$	$\frac{\text{WEBER}}{\text{M}^2}$
MAGNETOMOTIVE FORCE	TOTAL FORCE THAT PRODUCES FLUX	MMF	AMPERE-TURN	GILBERT 1 GILBERT = 0.796 AMP TURN 1 AMP-TURN = 1.25 GILBERT	AMPERE-TURN
FIELD INTENSITY, OR MAGNETIZING FORCE	FORCE PER UNIT LENGTH OF FLUX PATH	H	$\frac{\text{AMPERE-TURN}}{\text{IN}}$	OERSTED $1 \text{ OERSTED} = \frac{1 \text{ GILBERT}}{\text{CM}}$	$\frac{\text{AMPERE-TURN}}{\text{M}}$

Table 4-6

Comparison of magnetic units.

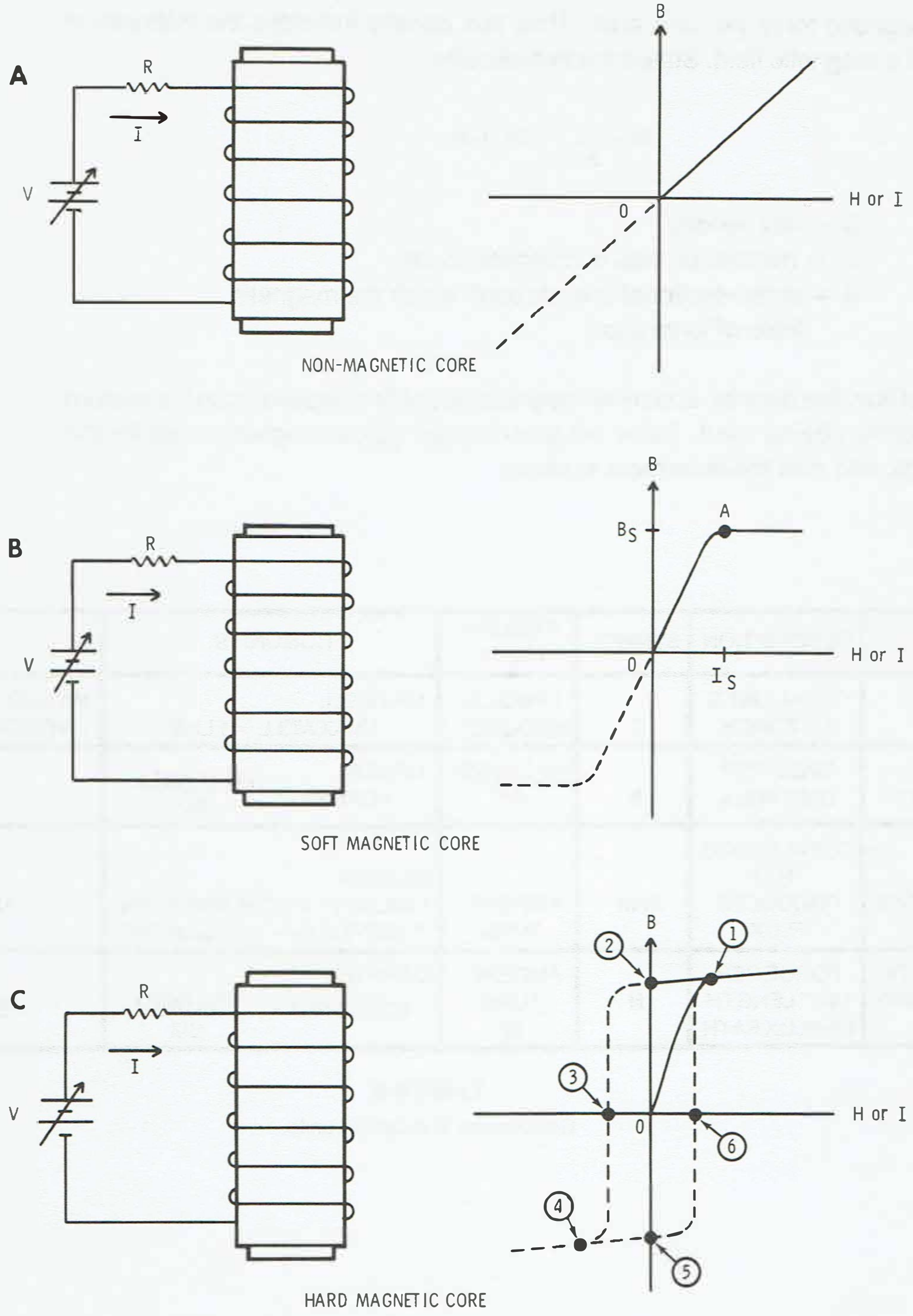


Figure 4-17

Flux density, B, E versus magnetizing force, H,
for these types of core materials.
A. Nonmagnetic core.
B. Soft magnetic core.
C. Hard magnetic core.

Flux density can be measured with an instrument called a flux meter. Figure 4-17 illustrates an experiment to determine the effects of different core materials for a simple coil. In Figure 4-17A, the coil is wound on a form that contains a nonmagnetic core. As the source voltage, which is initially zero volts, is gradually increased, current flows, thus creating a magnetic field. Since the length and number of turns of the coil are fixed in this experiment, the current, I, is a valid measure of the **magnetizing force**, H. Clearly, as the current increases so does the flux density. Thus, a nonmagnetic core is characterized by a linear relationship between flux density, B, and magnetizing force, H.

If the coil current is decreased to zero, the flux density will also decrease to zero. If the polarity of the voltage source is reversed and the experiment repeated, the only difference will be in the direction of current flow through the coil and the resulting polarity of the magnetic field. Thus, if the top of the coil was a north magnetic pole, and the bottom a south magnetic pole for one voltage polarity, it would be just the opposite when the voltage source was reversed.

Figure 4-17B indicates what happens if the coil is wound on a form containing a soft magnetic core. Since the core is a magnetic material, it offers little opposition to the passage of magnetic flux. Thus, when current flows through the coil, the magnetic field within the core will be much more concentrated than it was with the nonmagnetic core. Consequently, a coil with a magnetic core exhibits considerably more inductance than it would if its core was a nonmagnetic material.

Note in Figure 4-17B that a point is reached where further increases in coil current **do not** produce any appreciable increase in flux density. This occurs because the magnetic field produced by the coil current, I_S, in Figure 4-17B, is sufficiently strong to align virtually all the molecular magnets in the core material. The core at this point is magnetized as much as it can get, and is said to be **saturated**. As you will see shortly, saturation is usually an undesirable condition for real inductors and, thus, should be avoided if at all possible.

If the coil current is reduced to zero, the core will return to its initial unmagnetized state, because the molecular magnets in a soft magnetic material can remain aligned only in the presence of an external magnetic field.

Figure 4-17C indicates what happens if the coil is wound on a form containing a hard magnetic core. Assuming the core is initially unmagnetized, the results of this experiment may be summarized as follows:

1. Starting from zero, increasing the coil current increases the flux density until saturation is reached at point 1.
2. As the coil current is decreased to zero, the flux density decreases from the saturation value to the value corresponding to point 2. If you removed the core from the coil at this time, you would have a permanent magnet, since the molecular magnets of a hard magnetic material have the ability to remain aligned when the external magnetic field is reduced to zero.
3. To demagnetize the core, you must reverse the direction of the coil current. Thus, if you reverse the polarity of the voltage source and gradually increase the voltage, sufficient current will flow to decrease the flux density from the value corresponding to point 2 to zero, which corresponds to point 3.
4. If the coil current is increased further, the flux density will increase until the core is saturated as indicated by point 4. Naturally, the polarity of the magnetic field at point 4 is just the opposite of what it is at point 1.
5. As the coil current is decreased to zero, the flux density decreases from the saturation value to the value corresponding to point 5.
6. Once again, to demagnetize the core the direction of the coil current must be reversed. Thus, reversing the polarity of the voltage source and gradually increasing the voltage produces sufficient current to decrease the flux density from the value corresponding to point 5 to zero, which is indicated by point 6. Further increases in current would bring you back to point 1, thus, completing the cycle.

The curve shown in Figure 4-17C is called a **hysteresis loop**. The area enclosed by a hysteresis loop is a measure of the energy required to reverse the directions of the molecular magnets in the core material, and represents a loss. Thus, hard-core coils with pulsating DC, or AC, always experience some hysteresis loss due to the fact that at least a portion of the hysteresis loop will be repeated each cycle. Also, since a constant amount of energy is expended each cycle due to hysteresis, these losses increase with frequency.

Recall that a voltage is induced in a conductor whenever there is relative motion between the conductor and a magnetic field. Thus, when the current through a coil is changing, the varying magnetic field that is generated induces voltages in the core. Since magnetic cores are good conductors of electricity, as well as flux, the induced voltages establish circulating core currents called **eddy currents**. Both hysteresis and eddy currents result in core heating, which, of course, represents a power loss. Eddy current losses also increase as the frequency of the pulsating DC or AC coil current increase.

Not all the flux generated by the changing coil current passes through the core. A small portion of the generated flux passes through the surrounding air rather than the core, and is referred to as **leakage flux**. Voltages can be induced in adjacent components by the leakage flux. This, of course, is undesirable. Moreover, as the core material approaches saturation, the so-called flux leakage loss increases substantially. For this reason, coils are normally operated well below the saturation point.

Permeability is a measure of the ability of a material to conduct magnetic lines of force. Thus, the permeability of a material in a magnetic circuit is analogous to the conductance of a material in an electrical circuit. By definition, the permeability of a vacuum is 1. By comparing the permeability of a material to that of a vacuum, you obtain the material's **relative permeability**, μ_r. Table 4-7 provides the relative permeabilities of selected materials.

Material	Relative Permeability, μ_r
Vacuum,	1
Air	1
Cobalt	170
Nickel	1,000
Iron	7,000
Permalloy*	100,000
Supermalloy**	1,000,000

*Permalloy — 17% iron, 4% molybdenum, 79% nickel.
**Supermalloy — 16% iron, 5% molybdenum, 79% nickel.

Table 4-7

Relative permeabilities for selected materials.

By knowing the relative permeability of a material, you can calculate the change in inductance resulting from different core materials. This is shown in the following example.

Example 4-5

A single-layer, 160-turn coil is wound on a hollow core that is 2 inches long, and has a diameter of 0.5 inches. Estimate the inductance of the coil. What is the new value of inductance if the core material is cobalt? Nickel?

Equation 4-7 can be used to compute the inductance for the air-core coil.

$$L_o = \frac{(rN)^2}{9r + 10L} = \frac{[(0.25)(160)]^2}{9(0.25) + 10(2)} = \frac{1600}{22.25}$$

$$L_o = 71.9\mu H$$

Table 4-7 indicates $\mu_r = 170$ for cobalt. Therefore,

$$L = \mu_r L_o = 170(71.9\mu H) = 12.2mH$$

Similarly, for nickel $\mu_r = 1000$ and

$$L = 1000\,(71.9\mu H) = 71.9mH$$

Mutual Inductance

Inductance, or more accurately self inductance, is the property by which a coil, whose magnetic field is varying, induces a voltage in itself. This process, referred to as self induction, is detailed in Unit 2 and Unit 3.

Since a voltage is induced in a conductor whenever there is relative motion between the conductor and a magnetic field, it is possible for the varying magnetic field of one coil to induce a voltage in an adjacent coil, and vice versa. This process is called **mutual induction**. Furthermore, if a current change of one ampere per second in one coil induces a voltage of one volt in an adjacent coil, the two coils are said to have a mutual inductance of one Henry.

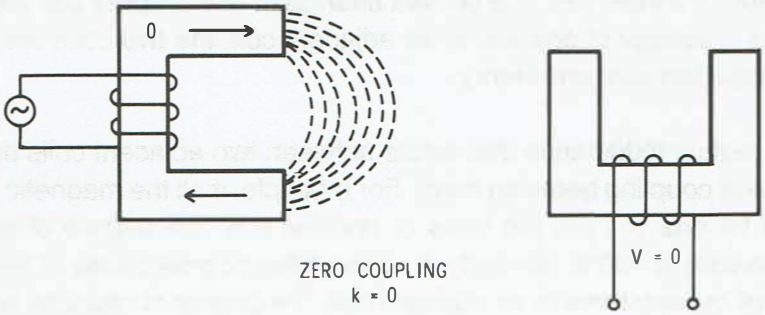
The amount of mutual inductance that exists between two adjacent coils depends upon the degree of coupling between them. For example, if all the magnetic lines of force produced by one coil cut the turns of another coil, the degree of coupling between the two coils is 100%. Similarly, if none of the magnetic lines of force produced by one coil cut any turns in an adjacent coil, the degree of coupling is 0%.

The degree of coupling between two coils is indicated by the **coefficient of coupling**, k. A coefficient of coupling of one, ($k = 1$) represents 100% coupling of the magnetic lines of force. Similarly, for degrees of coupling of 60%, 50%, and 0% k would have values of 0.6, 0.5, and 0, respectively. Clearly the possible range in values for k are:

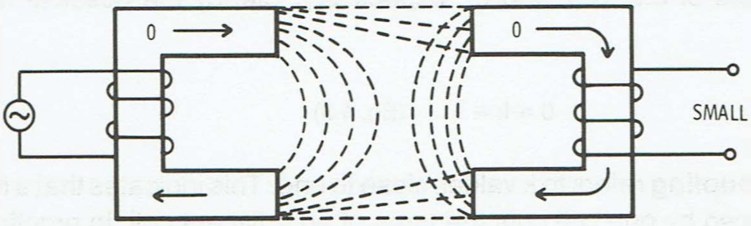
$$0 \leq k \leq 1 \qquad \text{(Eq. 4-9)}$$

The term **tight coupling** refers to k values close to one. This indicates that a majority of the flux produced by one coil cuts the turns of an adjacent coil. In practice, tight coupling is usually achieved by winding two coils on the same magnetic core, or by placing two air-core coils very close to each other. Similarly, the term **loose coupling** refers to k values close to zero and indicates that very little of the flux produced by one coil cuts the turns of an adjacent coil.

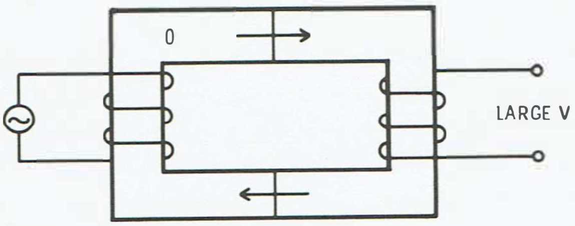
In practice, loose coupling results when two coils are very far apart, or at right angles to each other as shown in Figure 4-18A. In Figure 4-18B, the coils are closer together, and positioned so that some of the flux of the first coil cuts the turns of the second coil. Consequently, a voltage is induced in the second coil as shown. In Figure 4-18C, the two coils are effectively wound on the same magnetic core. Since the magnetic core offers little opposition to the passage of flux, nearly all the flux from the first coil cuts the turns of the second coil. In this case, the voltage induced in the second coil will be maximum.



ZERO COUPLING
k = 0



SOME COUPLING
0 < k < 1



MAXIMUM COUPLING
k = 1

Figure 4-18
The concept of mutual induction.
A. Zero coupling.
B. Some coupling.
C. Maximum coupling.

If you know the value of the coefficient of coupling for two coils, you can compute the mutual inductance from the following formula:

$$L_m = k\sqrt{L_1 L_2} \quad \text{(Eq. 4-10)}$$

where L_m = mutual inductance in Henries (H)
k = coefficient of coupling
$L_1 L_2$ = self-inductance of each coil in Henries (H)

The presence of mutual inductance complicates the calculation of the equivalent inductance of series-connected and parallel-connected coils. Specifically:

For two series connected coils:

$$L_T = L_1 + L_2 + 2L_m \quad \text{aiding fields} \quad \text{(Eq. 4-11)}$$

$$L_T = L_1 + L_2 - 2L_m \quad \text{opposing fields} \quad \text{(Eq. 4-12)}$$

Similarly, for two parallel connected coils:

$$\frac{1}{L_T} = \frac{1}{L_1 + L_m} + \frac{1}{L_2 + L_m} \quad \text{aiding fields} \quad \text{(Eq. 4-13)}$$

$$\frac{1}{L_T} = \frac{1}{L_1 - L_m} + \frac{1}{L_2 - L_m} \quad \text{opposing fields} \quad \text{(Eq. 4-14)}$$

If the coils are wound in the same direction, their magnetic fields aid each other. Consequently, you would use Equation 4-11 or Equation 4-13 to calculate the total inductance depending upon whether the coils were connected in series or parallel. Similarly, if the coils are wound in opposite directions, Equation 4-12 and Equation 4-14 are applicable, since the magnetic fields oppose each other.

Types of Coils

Coils are usually classified by the type of material used to make the core. The following is a brief description of the more common types of coils available.

Air-core. These coils are wound on either a hollow or solid nonmagnetic core. Here, the core simply provides support for the turns of wire. Where heavy rigid wire is used to wind the coil, an actual core may not be required. Air-core coils have low values of inductance, cannot be saturated, and are used primarily in high frequency applications.

Iron-core. Here, the core material is iron, or one of its alloys. Since iron has a large relative permeabilty, large values of inductance are possible. Typically, iron-core coils, or chokes, have inductances ranging from less than 1H to 40H or more. Hysteresis and eddy-current losses limit iron-core chokes to power line and audio frequency applications.

In an attempt to minimize hysteresis losses, the core is made from a soft magnetic material such as silicon steel. Figure 4-19 illustrates the steps taken to minimize eddy-current losses. As you can see, the core is constructed from numerous thin laminated sheets rather than one solid piece. A laminated core has the same magnetic properties as a solid core. However, since each lamination is coated with an insulating varnish, current cannot flow from one sheet to another. Consequently, the eddy currents are restricted to a single sheet of metal. Since the resistance of each sheet is relatively high, the magnitude of the eddy currents is greatly reduced as shown in Figure 4-19B and Figure 4-19C, respectively.

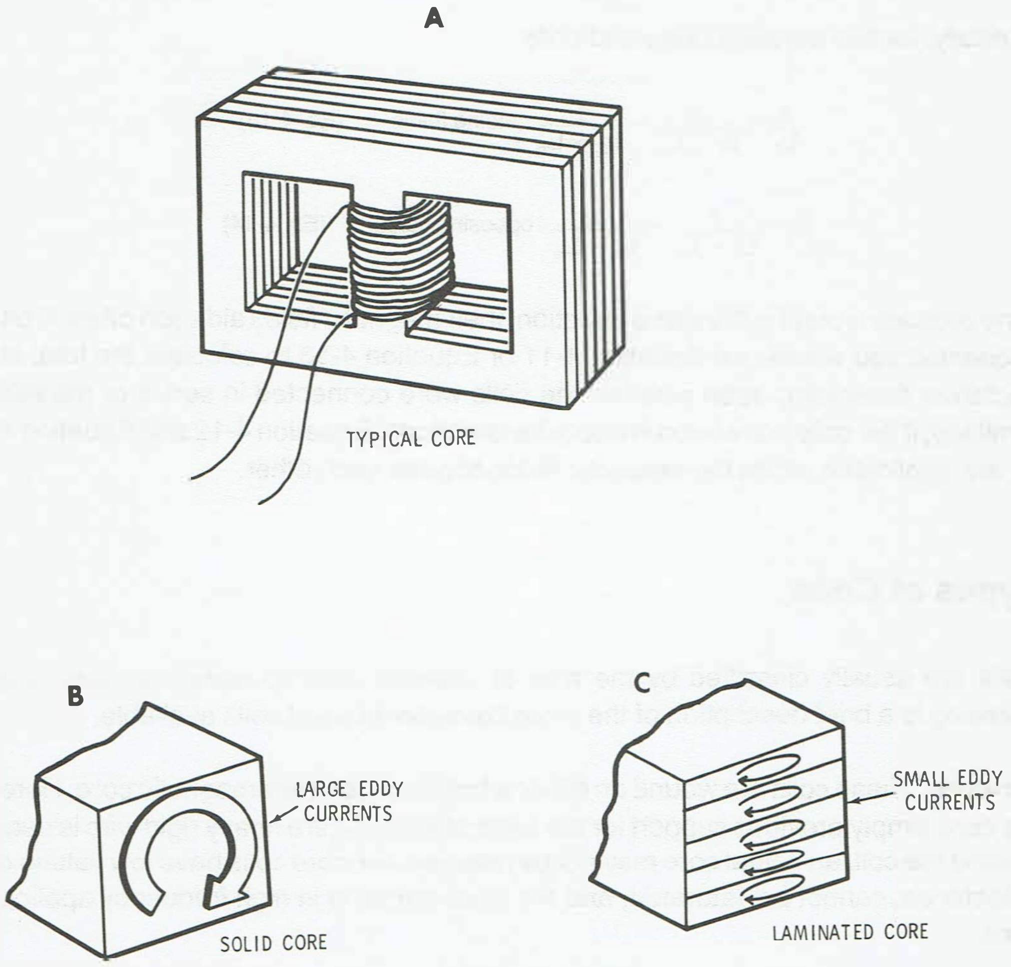


Figure 4-19

Laminated core construction of an iron-core choke.
A. Typical core.
B. Solid core.
C. Laminated core.

Powder-iron core. These cores consist of finely powered iron or iron alloys that have been mixed with a binder material. The function of the binder is not only to hold the powdered iron particles together, but also to electrically insulate one particle from another. This construction reduces eddy-current losses in the same way that laminated sheets do in iron-core chokes. Powder-iron core coils can be used at much higher frequencies than iron-core coils, although their inductance values are much lower.

Ferrite core. Ferrites are ceramic materials composed of iron and various other elements. These materials are excellent magnetic conductors and poor electrical conductors. Consequently, eddy-current losses are inherently small. The exact chemical composition of a ferrite determines its specific properties. Some ferrites are suitable for low frequency applications, while others are more suitable for high frequency applications.

One type of high frequency choke is made by sliding a small cylindrical-shaped ferrite bead on a wire. Due to its high permeability, the ferrite bead increases the inherent inductance of the wire. Naturally, only small inductance values may be obtained in this way, but such inductors are very useful in high frequency applications.

Toroidal core. This type of coil gets its name from the shape of the core, which resembles a donut. The core material is usually powdered-iron, or ferrite. Toroidal coils offer reasonably high values of inductance with comparatively little wire. In addition, virtually all the flux flows in the core, resulting in very little flux leakage loss.

Movable core. These coils have powdered-iron or ferrite cores that can be moved in and out of the coil via a screw-type adjustment. Consequently, the inductance value can be varied. These variable inductors are often used with capacitors to form tuned circuits in radio receivers.

Printed circuit board core. Here, the inductor is simply a spiral of copper foil on a printed circuit board. The principle advantage of this is that the coil is formed when the printed circuit board is being made. Obviously, only small inductance values are possible with this technique, which limits its use to high frequency applications.

Figure 4-20 illustrates the more common types of coils and their schematic symbols.

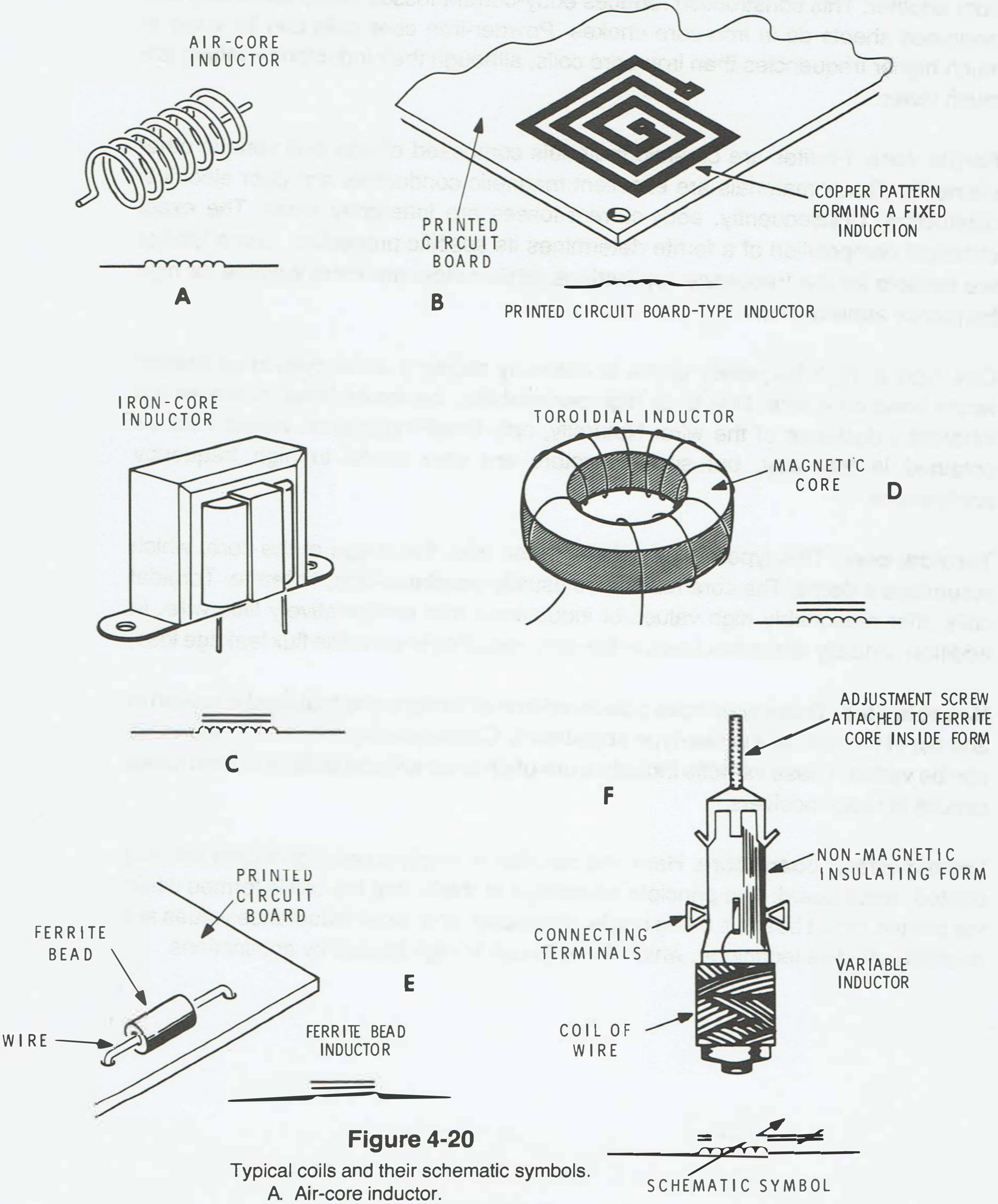


Figure 4-20

Typical coils and their schematic symbols.
A. Air-core inductor.
B. Printed circuit board inductor.
C. Iron-core inductor.
D. Toroidal inductor.
E. Ferrite bead inductor.
F. Variable inductor.

Coil Resistance

In addition to hysteresis, eddy-current, and flux-leakage losses, real coils dissipate power due to the resistance of their windings. These losses are referred to as **copper,** or **I^2R losses**, to distinguish them from the **core losses** discussed previously. In Unit 3 we reviewed the characteristics of series and parallel resonant circuits. At that time, however, the effects of coil resistance were not considered. As you will see, the resistance of real coils, especially in tuned circuits, directly effects the overall characteristics of the coil and the circuit in which it is used. To begin, consider the following example.

Example 4-6

> *Work out the input impedances for the circuits shown in Figure 4-21A and 4-21B. Also determine the Q for each circuit and discuss the results.*

By inspection, the impedance of the circuit in Figure 4-21A is:

$$Z_{in} = 100\Omega + j1k\Omega$$

Similarly, for the circuit in Figure 4-21B,

$$Z_{in} = 10.1k\Omega \| j1.01k\Omega$$

$$Z_{in} = \frac{10.1k\,(j1.01k)}{10.1k + j1.01k} = \frac{j10.201k^2}{10.1k + j1.01k}$$

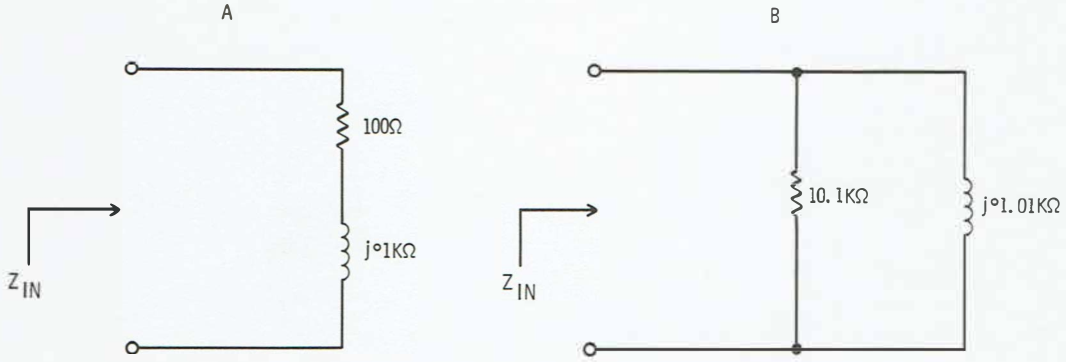


Figure 4-21

A series and parallel AC circuit.

Multiplying numerator and denominator by the conjugate of the denominator yields:

$$Z_{in} = \frac{j10.201k^2}{10.1k + j1.01k} \cdot \frac{10.1k - j1.01k}{10.1k - j1.01k}$$

$$Z_{in} = \frac{10.303k^3 + j103.03k^3}{(10.1k)^2 + (1.01k)^2}$$

$$Z_{in} = \frac{10.303k^3 + j103.03k^3}{103.03k^2} = 100\Omega + j1k\Omega$$

Recall that for a series circuit, the quality factor $Q = \frac{|X_L|}{R}$. Similarly, for a parallel circuit $Q = \frac{R}{|X_L|}$.

Thus, for the series circuit in figure 4-21A, $Q = 1k\Omega/100\Omega = 10$. The Q of the parallel circuit in Figure 4-21B is $Q = 1.01k\Omega/10.1\Omega = 10$.

Since the input impedance and quality factor of each circuit is the same, a signal source connected to either circuit would supply identical values of current, voltage, and power. Consequently, from the point of view of the signal source, the circuits are **identical**. In general then, we conclude that:

> *At a particular frequency, an AC circuit can be represented by either a series equivalent circuit, or a parallel equivalent circuit.*

For purposes of analysis and design, you can use either the series or parallel "model" of an inductor. Also, to convert from one model to the other, you can use the following formulas, which are derived in Appendix A.

Parallel-Series Conversion

Refer to Figure 4-22A.

$$R_s = \frac{R_p}{Q^2 + 1} \quad \text{(Eq. 4-15)}$$

$$L_s = \frac{L_p}{1 + \frac{1}{Q^2}} \quad \text{(Eq. 4-16)}$$

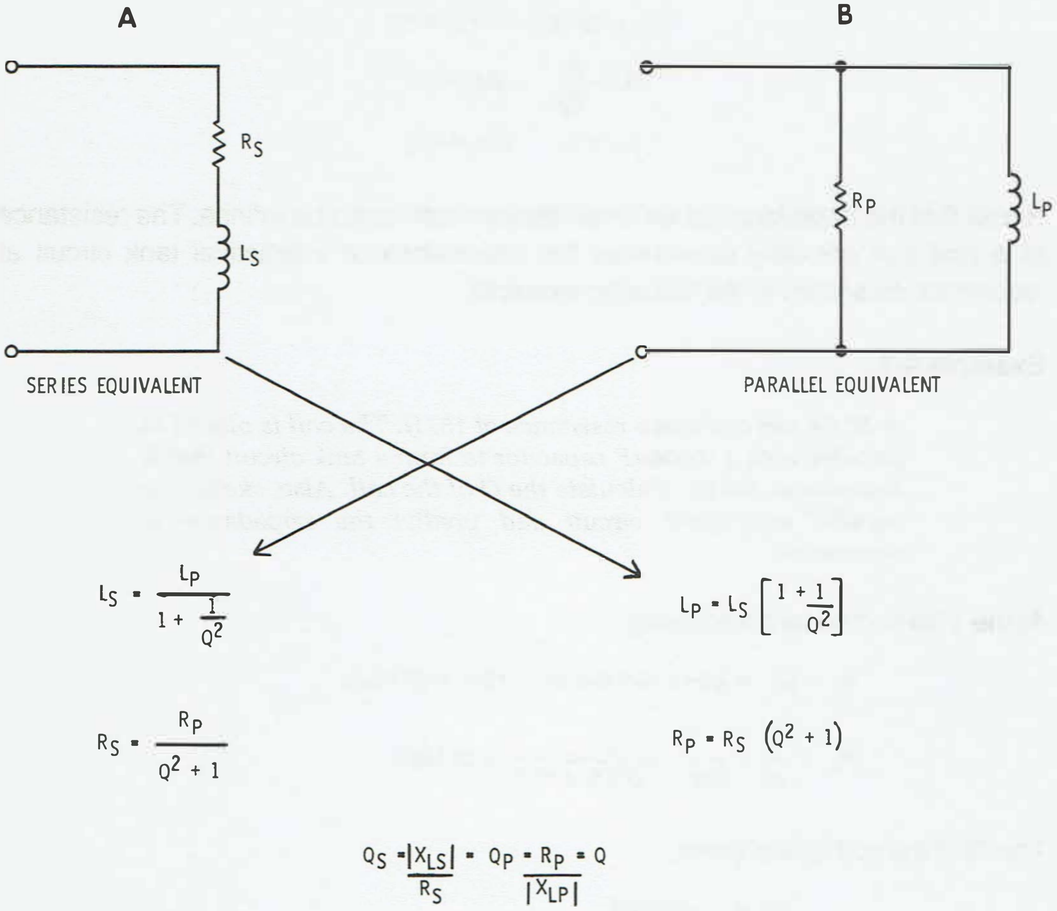

A
B
R_S
L_S
R_P
L_P
SERIES EQUIVALENT
PARALLEL EQUIVALENT
$L_S = \frac{L_P}{1 + \frac{1}{Q^2}}$
$L_P = L_S \left[1 + \frac{1}{Q^2}\right]$
$R_S = \frac{R_P}{Q^2 + 1}$
$R_P = R_S \left(Q^2 + 1\right)$
$Q_S = \frac{|X_{LS}|}{R_S} = Q_P = \frac{R_P}{|X_{LP}|} = Q$

Figure 4-22

At a particular frequency, a real inductor can be modeled by either a series or parallel equivalent circuit.
A. Series equivalent.
B. Parallel equivalent.

Series — Parallel Conversion

Refer to Figure 14-22B.

$$R_p = R_s(Q^2 + 1) \qquad \text{(Eq. 4-17)}$$

$$L_p = L_s\left[1 + \frac{1}{Q^2}\right] \qquad \text{(Eq. 4-18)}$$

For values of $Q \geqslant 10$, $(Q^2 + 1) \approx Q^2$ and $\left[1 + \frac{1}{Q^2}\right] \approx 1$. Consequently, for such "high Q" coils, you can use the following approximate formulas.

$$R_p = Q^2 RS \qquad \text{(Eq. 4-19)}$$

$$R_s = \frac{R_p}{Q^2} \qquad \text{(Eq. 4-20)}$$

$$L_s = L_p \qquad \text{(Eq. 4-21)}$$

Recall that the impedance of an ideal resonant tank circuit is infinite. The resistance of a real coil primarily determines the impedance of a practical tank circuit at resonance as shown in the following example.

Example 4-7

> *A 50.64 mH coil has a resistance of 157Ω. The coil is placed in parallel with a 0.005μF capacitor to form a tank circuit that is resonant at 10kHz. Calculate the Q of the coil. Also, sketch the parallel equivalent circuit and predict the impedance at resonance.*

At the 10kHz resonant frequency,

$$X_L = j\omega L = j(2\pi \times 10^4)(50.64 \times 10^{-3}) = j3.18k\Omega$$

$$X_c = \frac{-j}{\omega C} = \frac{-j}{(2\pi \times 10^4)(5 \times 10^{-9})} = j3.18k\Omega$$

The Q of the coil is, therefore,

$$Q = \frac{|X_{LS}|}{R_s} = \frac{3.18k\Omega}{157\Omega} = 20.3$$

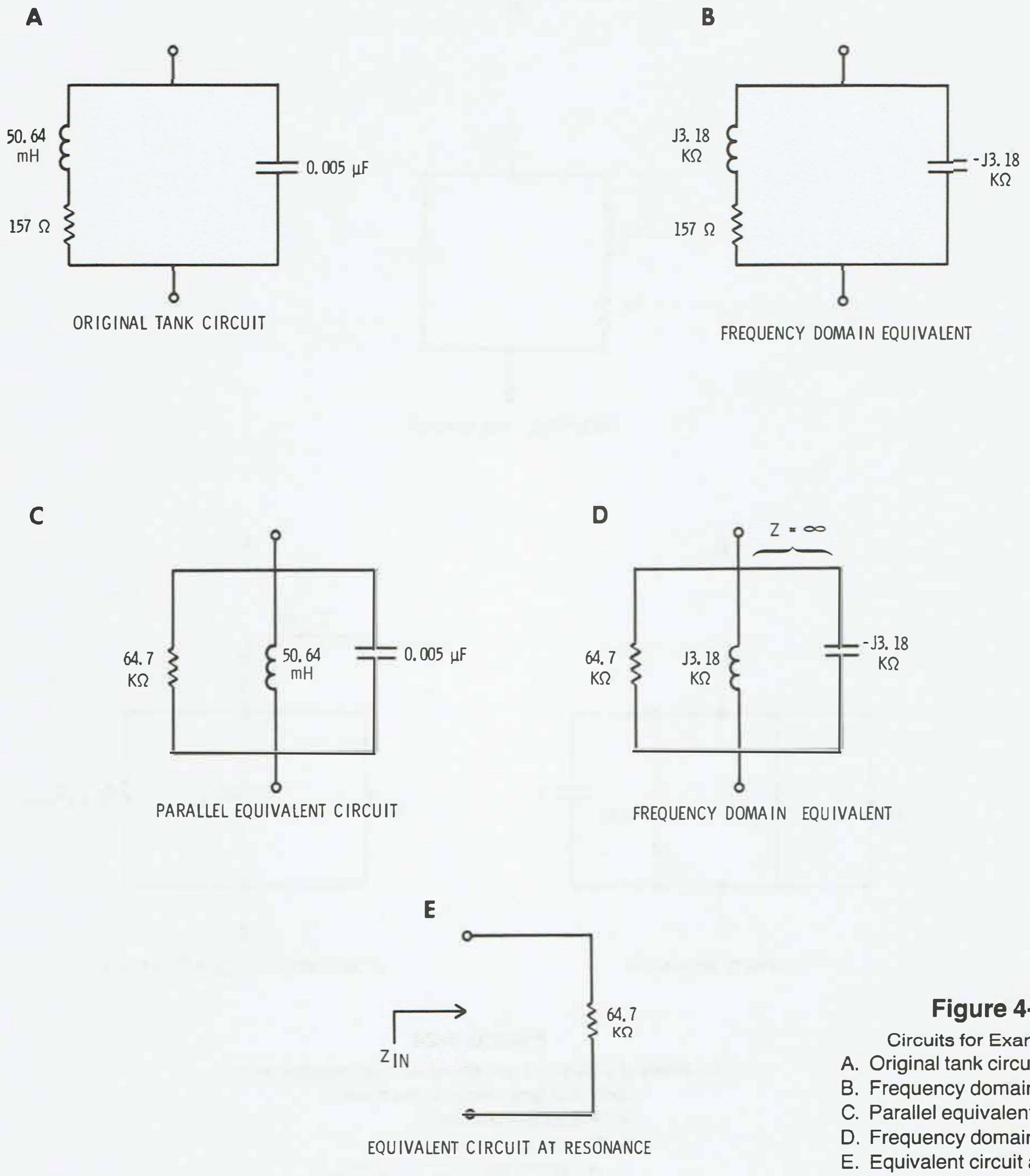
Since $Q > 10$, you can use the approximate formulas to determine the parallel equivalent circuit. Thus,

$$R_p = Q^2 R_s = (20.3)^2(157\Omega) = 64.7k\Omega$$

$$L_p = L_s = 50.64\text{mH}$$

Also, since $L_p = L_s$, $X_{LP} = X_{LS} = j3.18k\Omega$

Figure 4-23A and 4-23B illustrate the original tank circuit and its frequency domain equivalent circuit at 10kHz. Similarly, Figure 4-23C and Figure 4-23D illustrate the parallel equivalent circuit and its frequency domain equivalent circuit at 10kHz. Note in Figure 4-23D, that the impedance of $X_L \| X_c$ is infinite. Consequently, the total impedance of the circuit equals R_p, which is 64.7kΩ, as shown in Figure 4-23E.



ORIGINAL TANK CIRCUIT

FREQUENCY DOMAIN EQUIVALENT

PARALLEL EQUIVALENT CIRCUIT

FREQUENCY DOMAIN EQUIVALENT

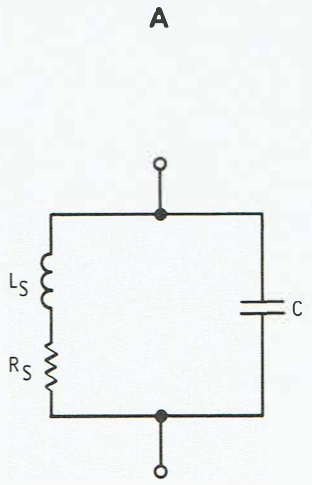
EQUIVALENT CIRCUIT AT RESONANCE

Figure 4-23

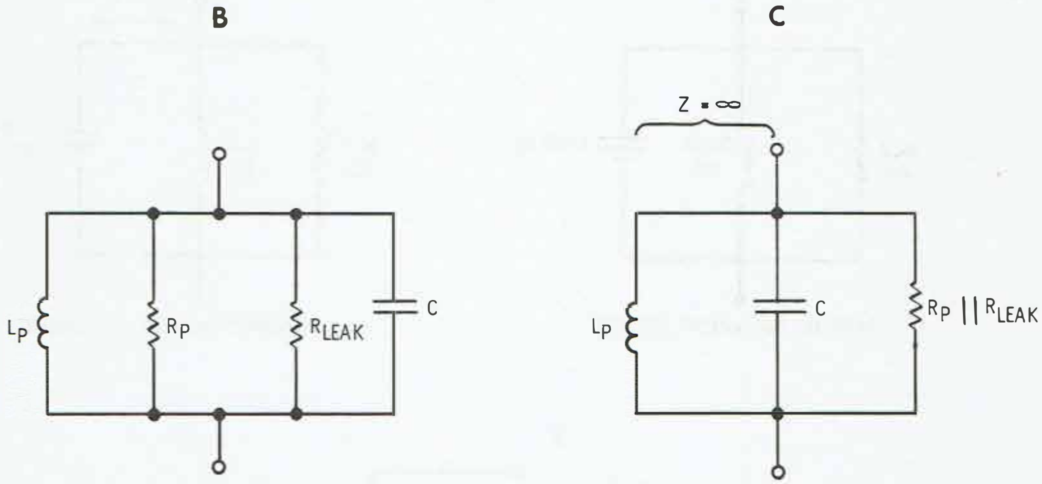
Circuits for Example 4-7.

A. Original tank circuit.
B. Frequency domain equivalent
C. Parallel equivalent circuit.
D. Frequency domain equivalent.
E. Equivalent circuit at resonance.

In example 4-7, the effect of the capacitor's leakage resistance was not considered. Figure 4-24 shows you how a capacitor's leakage resistances, R_{LEAK}, can be taken into account. As you can see, the tank impedance at resonance simply equals $R_p \| R_{LEAK}$. Since most capacitors have values of $R_{LEAK} >> R_p$, you can usually ignore the leakage resistance of a capacitor in a tank circuit.



PRACTICAL TANK CIRCUIT



PARALLEL EQUIVALENT

AT RESONANCE $Z_{TANK} = R_P \| R_{LEAK}$

Figure 4-24

The effect of series coil and shunt leakage resistance on a practical tank circuit at resonance.

A. Practical trank circuit.
B. Parallel equivalent.
C. At resonance $Z_{TANK} = R_p \| R_{LEAK}$.

Self-Test Review

30. The inductance of a single-turn air-core coil depends upon the number of turns and the coil's __________________ .
31. A __________________ magnetic material retains its magnetism when removed from a magnetic field.
32. Iron-core coils have laminated cores to reduce ________________ losses.
33. A saturated core exhibits significant __________________ .
34. The process by which one coil induces a voltage in an adjacent coil is called __________________ induction.
35. __________________ - core coils cannot be saturated.
36. __________________ - core coils have large inductance values, but can only be used at relatively low frequencies.
37. __________________ coils have very little flux leakage loss.
38. A coil with a nonmagnetic core has an inductance of 65 μH. If an iron core was used, the inductance would be _________ mH.
39. A 25μH and a 4μH inductor are connected in series so that their magnetic fields aid each other. If the coefficient of coupling is 0.8, the total inductance is _______________ .
40. A 253.3mH coil whose resistance is 106.05Ω in parallel with a 0.1μF capacitor. The impedance of the circuit is __________________ at the circuit's 1kHz resonant frequency.

Answers

30. geometry or shape
31. hard
32. eddy-current
33. flux-leakage
34. mutual
35. air
36. iron
37. toroidal
38. 455mH
39. 45μH
40. 23.86kΩ

The solutions to questions 38, 39, and 40 follow.

38. Table 4-7 indicates that the relative permeability for iron is 7000. Thus

$$L = \mu_r L_o = 7000\,(65\mu H) = 455 mH$$

39. The mutual inductance, L_m, is first determined.

$$L_m = k\sqrt{L_1 L_2} = 0.8\sqrt{25\,(4)\,10^{-12}}$$
$$L_m = 8\mu H$$

For two coils connected in series with aiding magnetic fields, the total inductance is:

$$L_T = L_1 + L_2 + 2\,L_m$$
$$L_T = 25 + 4 + 2(8)$$
$$L_T = 45\mu H$$

40. The impedance of the tank circuit at resonance essentially equals R_p. Thus:

$$X_L = j\omega L = j(2\pi \times 10^3)\,(253.3 \times 10^{-3})$$
$$X_L = j1590.7\Omega$$

$$Q = \frac{|X_{Ls}|}{R_s} \quad \frac{1590.7}{106.05} = 15$$

Since $Q > 10$, $R_p = Q^2 R_s = (15)^2\,(106.05)$

$$R_p = 23.86k\Omega$$

SUMMARY

The following points summarize the key ideas of this unit.

All real components exhibit the properties of resistance, capacitance, and inductance. Real resistors, capacitors, and inductors are designed to optimize one property while minimizing the other two. Typically, component values vary with changes in time, temperature, voltage, frequency, and many other factors. Generally speaking, the selection of a particular type of resistor, capacitor, or inductor should be based on the characteristics of the component and the circuit in which the component will be used.

Carbon composition resistors are inexpensive, and available in a wide range of standard resistance values. Their resistance changes with temperature, voltage, and frequency. Consequently, carbon composition resistors are not suitable for applications requiring precise resistance values. Wire-wound resistors are available in both precision and high power varieties. The disadvantages of wire-wound resistors include their relatively high cost and inherent inductive nature. This inductive nature makes wire-wound resistors unsuitable in high frequency applications. Numerous film resistors are available whose characteristics fall between the extremes of composition and wire-wound resistors. In many applications, film resistors are the preferred type.

The number of different types of capacitors available is quite large. This fact alone makes the task of selecting a particular capacitor for a given application more complex than the selection of a suitable resistor. Factors like leakage current, insulation resistance, and quality factor should be considered when selecting a capacitor for a particular application. The capacitor selection guide provided in this unit should help you become familiar with the major characteristics of the more common capacitor types.

Of the various components discussed in this unit, inductors are the least ideal. The inductance of a coil with a nonmagnetic core depends upon the number of turns and geometry of the coil. Using a magnetic core material, substantially increases the inductance of a coil.

Soft magnetic materials are used for inductor cores to minimize hysteresis losses. Eddy-current losses are reduced by using laminated cores rather than solid cores. Normally, saturation is avoided to minimize flux-leakage losses. The total inductance of series and parallel coil combinations is altered by the presence of mutual inductance. Formulas for computing the total inductance of two coils connected in series or parallel, with either aiding or opposing magnetic fields, were provided in the text.

The effects of coil resistance on the resonant impedance of a practical tank circuit can be determined by using the parallel equivalent circuit of the coil. The equations necessary to convert from the series equivalent to the parallel equivalent circuit and vice versa are provided in the text.

UNIT EXAMINATION

1. The two most widely used resistor types are the:
   - A. Cermet film and metal film.
   - B. Metal film and bulk film.
   - C. Carbon composition and bulk film.
   - D. Wire-wound and carbon composition.

2. Volume controls change resistance in a:
   - A. Linear manner.
   - B. Nonlinear manner.
   - C. Circular manner.
   - D. Constant manner.

3. The cross-sectional area of wire is usually given in:
   - A. Square inches.
   - B. Square centimeters.
   - C. Square mils.
   - D. Circular mils.

4. The resistor type that can dissipate the most power is the:
   - A. Wire-wound.
   - B. Carbon composition.
   - C. Carbon film.
   - D. Metal film.

5. Most materials have temperature coefficients that:
   - A. Are negative.
   - B. Are positive.
   - C. Are zero.
   - D. Vary greatly with time.

6. The type of capacitor least likely to change value with temperature is a:
   - A. Low-k ceramic capacitor.
   - B. Aluminum-oxide electrolytic capacitor.
   - C. Paper capacitor.
   - D. High-k ceramic capacitor.

7. Equalizing resistors are used with:
   - A. Series-connected electrolytic capacitors.
   - B. Shunt-connected tantalum oxide capacitors.
   - C. Shunt-connected electrolytic capacitors.
   - D. Series-connected mica capacitors.

8. A low-loss capacitor would have a:
    - A. Low Q value.
    - B. High D value.
    - C. Large ESR value.
    - D. Low D value.

9. A short shelf life is characteristic of a/an:
    - A. Aluminum-oxide electrolytic capacitor.
    - B. Polystyrene film capacitor.
    - C. Tantalum-oxide electrolytic capacitor.
    - D. Mica capacitor.

10. For effective coupling and bypassing:
    - A. $R_{TH} >> |X_C|$.
    - B. $|X_C| >> R_{TH}$
    - C. $R_{TH} = |X_C|$.
    - D. $|X_C| = R_{TH}^2$.

11. Hysteresis losses:
    - A. Decrease as frequency increases.
    - B. Are high in soft magnetic materials.
    - C. Are high in hard magnetic materials.
    - D. Are high in nonmagnetic materials.

12. A material whose relative permeability is 1:
    - A. Contains iron.
    - B. Is a ferrite.
    - C. Is a poor magnetic conductor.
    - D. Easily concentrates magnetic flux lines.

13. The process by which one coil induces a voltage in an adjacent coil is called:
    - A. Mutual induction.
    - B. Self induction.
    - C. Ionization.
    - D. Flux leakage.

14. Two coils that are tightly coupled have a coefficient of coupling that is:
    - A. Much less than 1.
    - B. Much greater than 1.
    - C. Slightly less than 1.
    - D. Zero.

15. A coil has a series resistance of 10Ω and a Q of 50. The equivalent parallel resistance is, therefore:
    - A. 10Ω
    - B. 25kΩ
    - C. 2500Ω
    - D. 1kΩ

16. A temperature coefficient of 200 ppm/°C is equivalent to:
    - A. 200% /°C.
    - B. 0.000200% /°C.
    - C. 0.002% /°C.
    - D. 0.02% /°C.

17. A capacitor marked NPO:
    - A. Is probably a high-k ceramic capacitor.
    - B. Has a positive temperature coefficient.
    - C. Has a negative temperature coefficient.
    - D. Has a zero temperature coefficient.

18. Saturation does not occur in:
    - A. Iron-core coils.
    - B. Ferrite-core coils.
    - C. Air-core coils.
    - D. Low-Q coils.

EXAMINATION ANSWERS

1. C — Carbon composition and bulk film resistors are the most widely used resistor types due to their low cost, and wide range of standard resistance values.

2. B — Volume controls are logarithmic or "audio taper" potentiometer, which change resistance in a nonlinear manner.

3. D — In the United States, the cross-sectional area of wires is expressed in circular mils.

4. A — Wire-wound resistors can dissipate large amounts of power. Some wire-wound resistors can dissipate 200W or more.

5. B — Most materials have positive temperature coefficients.

6. A — Low-k ceramic capacitors can be made that have a temperature coefficient that is essentially zero.

7. A — Due to their low values of leakage resistance, equalizing resistors are usually used for series-connected electrolytic capacitors.

8. D — Low-loss capacitors have high Q's. Since D = 1/Q, low-loss capacitors have low D's.

9. A — Aluminum-oxide electrolytic capacitors have short shelf lives.

10. A — For effective coupling and bypassing, $R_{TH} >> |X_C|$. As a rule of thumb, good coupling and bypassing occurs if $|X_C| = R_{TH}/20$.

11. C — Hard magnetic materials have large hysteresis losses.

12. C — A relative permeability of 1 indicates that a material has the same permeability as air, which is a poor magnetic conductor.

13. A — Mutual induction is the process by which one coil induces a voltage into an adjacent coil.

14. C — Tight coupling means the coefficient of coupling is slightly less than 1.

15. B — Since $Q > 10$, the equivalent parallel resistance is Q^2R_s or $(50)^2$ (10), which equals 25kΩ

16. D — 200 ppm/°C $= \frac{200}{1{,}000{,}000}$ /°C or $\frac{.02}{100}$ /°C $=$ .02% /°C. Remember, percent means parts per hundred.

17. D — The designation NPO means zero temperature coefficient.

18. C — Air-core coils cannot be saturated.

Appendix A

DERIVATION OF EQUATIONS 4-15 THROUGH 4-18

In Figure 4-22A, the impedance of the series circuit is, by inspection, $Z_s = R_s + j\omega L_s$.

Similarly in Figure 4-22B, the impedance of the parallel circuit is $Z_p = E_p \| j\omega L_p$. Thus:

$$Z_p = \frac{R_p(j\omega L_p)}{R_p + j\omega L_p} = \frac{j\omega L_p R_p}{R_p + j\omega L_p}$$

Multiplying the numerator and denominator by the conjugate of the denominator yields:

$$Z_p = \frac{j\omega L_p R_p}{R_p + j\omega L_p} \cdot \frac{R_p - j\omega L_p}{R_p - j\omega L_p}$$

$$Z_p = \frac{-j^2\omega^2 L_p{}^2 R_p + j\omega L_p R_p{}^2}{R_p{}^2 + \omega^2 L_p{}^2}$$

$$Z_p = \frac{\omega^2 L_p{}^2 R_p}{R_p{}^2 + \omega^2 L_p{}^2} + j\frac{\omega L_p R_p{}^2}{R_p{}^2 + \omega^2 L_p{}^2}$$

Since $Z_s = Z_p$, the real and imaginary parts of Z_s and Z_p are equal. Thus, equating the real parts:

$$R_s = \frac{\omega^2 L_p{}^2 R_p}{R_p{}^2 + \omega^2 L_p{}^2}$$

Dividing numerator and denominator by $\omega^2 L_p{}^2$

$$R_s = \frac{R_p}{\frac{R_p{}^2}{\omega^2 L_p{}^2} + 1} = \frac{R_p}{\left(\frac{R_p}{\omega L_p}\right)^2 + 1}$$

Since $Q_p = \frac{R_p}{\omega L_p} = Q$, we have

$$R_s = \frac{R_p}{Q^2 + 1} \qquad \text{(Eq. 4-15)}$$

Equating the imaginary parts

$$\omega L_s = \frac{\omega L_p R_p^2}{R_p^2 + \omega^2 L_p^2}$$

$$L_s = \frac{L_p R_p^2}{R_p^2 + \omega^2 L_p^2}$$

Dividing numerator and denominator by R_p^2

$$L_s = \frac{L_p}{1 + \frac{\omega^2 L_p^2}{R_p^2}} = \frac{L_p}{1 + {\frac{\omega L_p}{R_p}}^2}$$

Since $Q_p = \frac{R_p}{\omega L_p}$, the term $\frac{\omega L_p}{R_p} = \frac{1}{Q}$

Thus:

$$L_s = \frac{L_p}{1 + \frac{1}{Q^2}} \qquad \text{(Eq. 4-16)}$$

The formulas for R_p and L_p are obtained by solving Equation 4-15 for R_p, and Equation 4-16 for L_p.

$$R_p = R_s (Q^2 + 1) \qquad \text{(Eq. 4-17)}$$

$$L_p = L_s \; 1 + \frac{1}{Q^2} \qquad \text{(Eq. 4-18)}$$

Note that for $Q \geqslant 10$ the following approximations are useful:

$$R_s = \frac{R_p}{Q^2}$$

$$L_s = L_p$$

$$R_p = Q^2 R_s$$

Unit 5

UNREGULATED POWER SUPPLIES

CONTENTS

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INTRODUCTION

Virtually all electronic systems require a source of DC power, or power supply. In portable equipment, the power supply may simply consist of one or more batteries. Batteries, however, can be expensive and must be replaced periodically. Consequently, when a source of AC power is available, it is usually more economical to use a power supply that converts the AC power to DC power. In this unit, you will become familiar with the various sections of typical power supplies. In addition, you will learn how to design power supplies that can provide positive, negative, or both positive and negative voltages.

UNIT OBJECTIVES

When you have completed this unit, you should be able to:

1. State four applications for transformers.
2. Calculate the secondary voltage, primary and secondary currents, and effective primary resistance when the turns ratio and input voltage are known.
3. State typical turn-on voltages for Germanium and Silicon diodes.
4. Select component values so that a diode's forward resistance, reverse resistance, and turn-on voltage will be negligible.
5. Identify the following rectifier circuits:
    A. Half-wave.
    B. Center-tapped.
    C. Bridge.
    D. Dual-supply.
6. Calculate the following quantities for the four rectifier circuits: Average diode and load currents, peak diode and load currents, average diode and load voltages, and the peak inverse voltage.
7. Define percent ripple, percent voltage regulation, and smoothing factor.
8. Analyze and design a capacitor-input filter.
9. Analyze and design a choke-input filter.
10. Analyze and design cascaded filter sections.

UNIT ACTIVITY GUIDE

- ☐ Read section on “Transformers and Diodes.”
- ☐ Answer Self-Test Review questions 1-12.
- ☐ Perform Experiment 5 in Unit 8.
- ☐ Read section on “Rectifier Circuits.”
- ☐ Answer Self-Test Review questions 13-27.
- ☐ Perform Experiment 6 in Unit 8.
- ☐ Read section on “Smoothing Filters.”
- ☐ Answer Self-Test Review questions 28-41.
- ☐ Perform Experiment 7 in Unit 8.
- ☐ Study Summary.
- ☐ Complete Unit Examination.
- ☐ Check Examination Answers.

TRANSFORMERS AND DIODES

A transformer is a device that transfers energy from one circuit to another **without** a direct connection between the two circuits. In its simplest form, a transformer consists of two conductors that are magnetically coupled. Thus, the transfer of energy from one circuit to the other is accomplished by the process of mutual induction.

The construction details of transformers are similar to those discussed for inductors in Unit 4. Transformers designed for power line and audio frequencies, for example, use iron cores to achieve tight coupling between the primary and secondary circuits. Since core losses increase with frequency, transformers designed for high frequency operation normally have nonmagnetic cores. In this unit, the emphasis will be on iron-core transformers, since these are the type of transformers used in power supplies.

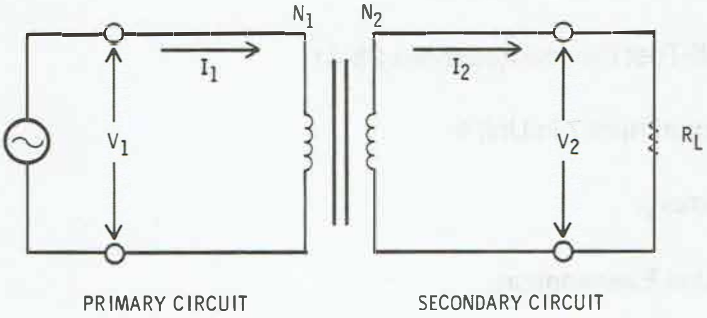


Figure 5-1

An ideal transformer.

The Ideal Transformer

Figure 5-1 illustrates the schematic symbol for an iron-core transformer. Note that the coil connected to the AC voltage source is called the primary winding, while the coil connected to the load, R_L, is called the secondary winding. The operation of the transformer may be summarized as follows:

1. A changing primary current, I_1, produces changing flux surrounding the primary winding.
2. The changing primary flux cuts the turns of the secondary winding — thereby inducing a voltage, V_2, in the secondary winding.

3. Since the voltage across R_L is V_2, the secondary current, I_2, equals V_2/R_L.
4. The effective resistance seen by the voltage source driving the primary winding is by ohms law: $R_1 = V_1/I_1$.

The efficiency of a transformer is defined as the ratio of the secondary, or output, power to the primary, or input, power. Since an ideal transformer is assumed to have no losses, all the power supplied to the primary circuit is delivered to the secondary circuit. Consequently, the efficiency of an ideal transformer is 100%. Based on this fact, we can derive the following relationships.

Since $P_{in} = P_{out}$, V_1I_1 must equal V_2I_2. Thus, the ratio of V_1 to V_2 is:

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} \qquad \text{(Eq. 5-1)}$$

The number of primary and secondary ampere turns are equal. Thus, I_1N_1 must equal I_2N_2. The ratio of I_1 to I_2 is therefore:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \qquad \text{(Eq. 5-2)}$$

From Equation 5-2, you can see that $\frac{I_2}{I_1} = \frac{N_1}{N_2}$.

Substituting $\frac{N_2}{N_1}$ for $\frac{I_2}{I_1}$ into Equation 5-1 yields:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad \text{(Eq. 5-3)}$$

Once again, assuming $P_{in} = P_{out}$, it follows that $I_1{}^2R_1$ must equal $I_2{}^2R_2$. Solving for R_1 you obtain:

$$R_1 = \frac{I_2{}^2R_2}{I_1{}^2} = \left(\frac{I_2}{I_1}\right)^2 R_2$$

Since $\frac{I_2}{I_1}$ equals $\frac{N_1}{N_2}$, you have:

$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2 \qquad \text{(Eq. 5-4)}$$

Equations 5-1 through 5-4 describe the characteristics of an ideal transformer. In these equations, each term is defined as follows:

V_1 = primary voltage in volts (V).
V_2 = secondary voltage in volts (V).
I_1 = primary current in amperes (A).
I_2 = secondary current in amperes (A).
R_2 = secondary load resistance in ohms (Ω).
R_1 = effective primary resistance in ohms (Ω).
N_1, N_2 are respectively the number of primary and secondary turns.

Collectively, Equations 5-1 through 5-4 suggest the following applications for a transformer.

1. **Step-up transformer** — If $N_2 > N_1$, the secondary voltage, V_2, will be larger than the primary voltage, V_1. Thus, if a load requires a larger AC voltage than the available line voltage, you could use a step-up transformer.

2. **Step-down transformer** — If $N_2 < N_1$, the secondary voltage, V_2, will be less than the primary voltage, V_1. Since the voltages required by most electronic devices are small compared to the 110V–120V rms line voltage, a step-down transformer is normally used in the "front-end" of most power supplies.

3. **Isolation transformer** — If $N_2 = N_1$, the secondary and primary voltages will be equal. Frequently, a repairman will use a 1:1 isolation transformer to isolate a transformerless chassis from the AC line voltage while he is repairing electronic equipment.

4. **Impedance matching transformer** — The ratio of N_1 to N_2 is called the **turns ratio.** Equation 5-1 indicates that the effective primary resistance, R_1 depends upon the value of the turns ratio, and the secondary load resistance, R_2. If a low resistance load is connected directly to the output of a high resistance source, very little power will be transferred from the source to the load. By placing a step-down transformer between the source and load, the low resistance load appears as a much larger resistance to the source. In this way, the low resistance load can be "matched" to the high resistance source in order to achieve the condition of maximum power transfer.

Example 5-1

Calculate the values of the primary current, secondary current, and secondary voltage for the ideal transformer in Figure 5-2. Also calculate the effective primary resistance, R_1.

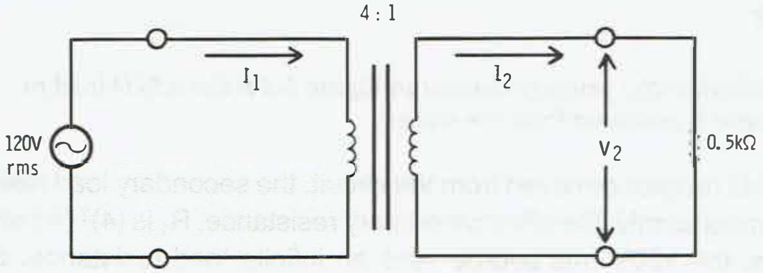


Figure 5-2

Circuit for Example 5-1.

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Thus:
$$V_2 = \frac{V_1 N_2}{N_1} = \frac{120V\,(1)}{4} = 30V\,\text{rms}$$

Since $V_2 = 30V$ and $R_2 = 0.5k\Omega$

$$I_2 = \frac{V_2}{R_2} = \frac{30V}{0.5k\Omega} = 60mA$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Thus:
$$I_1 = \frac{I_2(N_2)}{N_1} = \frac{60mA\,(1)}{4} = 15mA$$

Note that when the voltage is stepped down, the current is stepped up. Similarly, if voltage is stepped up, the current is stepped down.

$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2 = (4)^2 (0.5k\Omega) = 8k\Omega$$

R_1 represents the effective primary resistance as seen from the source. You can verify this fact by using ohms' law to calculate the primary current I_1. Specifically:

$$I_1 = \frac{V_1}{R_1} = \frac{120V}{8k\Omega} = 15mA$$

which agrees with the value of I_1 calculated previously.

Example 5-2

Calculate the primary current in Figure 5-2 if the 0.5kΩ load resistor is removed from the circuit.

With the 0.5kΩ resistor removed from the circuit, the secondary load resistance, R_2 is infinite. Consequently, the effective primary resistance, R_1 is $(4)^2(\infty)$ which is also infinite. Thus, the 120V rms source sees an infinite load resistance. Since $I_1 = 120V/\infty$, I_1 has a value of 0A. Ideally then, a transformer with no load resistance connected to its secondary winding draws no current from the source.

Example 5-3

The Thevenin equivalent circuit for an amplifier consists of a 5V rms AC source in series with a 2.5kΩ resistor as shown in Figure 5-3. What turns ratio is required for the impedance matching transformer, so that maximum power is delivered to the speaker whose effective resistance is 16Ω?

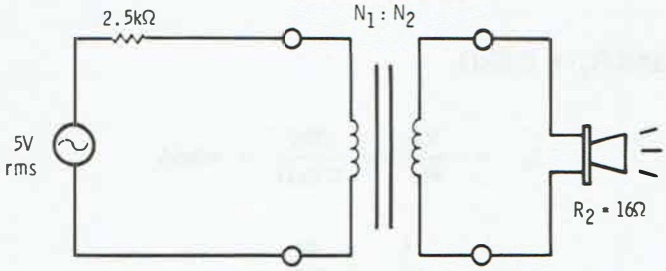


Figure 5-3

Circuit for Example 5-3.

For maximum power transfer, the effective primary resistance, R_1, should equal the Thevenin resistance of the amplifier. Solving Equation 5-4 for the turns ratio $\frac{N_1}{N_2}$ yields:

$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2$$

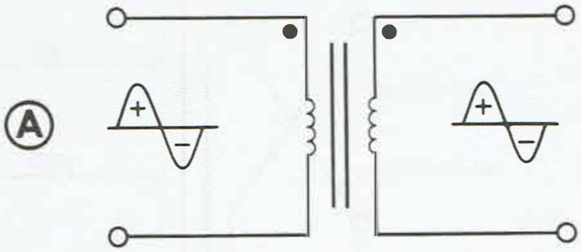
$$\left(\frac{N_1}{N_2}\right)^2 = \frac{R_2}{R_1}$$

$$\frac{N_1}{N_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{2.5k\Omega}{16\Omega}} = 12.5$$

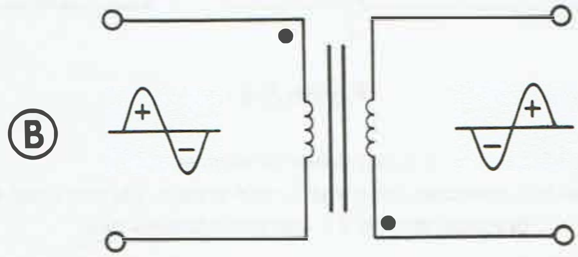
Thus, the required turns ratio is 12.5 to 1.

Real Iron-Core Transformers

Real transformers always have some hysteresis, eddy current, copper, and flux leakage loss. In designing transformers, manufacturers strive to keep losses to a minimum. They can minimize hysteresis and flux leakage losses by using core materials that have large relative permeabilities. As with coils, eddy currents are reduced by using a laminated core rather than a solid core. Finally, copper losses are minimized by using large diameter wires for the transformer windings. Consequently, real iron-core transformers are very efficient devices. In fact, efficiencies of 95% and more are common. Thus, the equations derived for the ideal transformer, which assumed 100% efficiency, are excellent approximations for real iron-core transformers.



IN PHASE PRIMARY AND SECONDARY VOLTAGES



PRIMARY AND SECONDARY VOLTAGES 180° OUT OF PHASE

Figure 5-4

The dot convention.

A. In phase primary and secondary voltages.
B. Primary and secondary voltages 180° out of phase.

In commercial transformers, depending upon whether or not the primary and secondary windings are wound in the same or opposite directions, the secondary voltage will be in phase or 180° out of phase with the primary voltage. This "winding sense" is often indicated by placing dots on the in-phase primary and secondary terminals as shown in Figure 5-4.

Figure 5-5 illustrates a transformer typical of the type found in power supplies. By providing two primmary windings, that can be connected in series or parallel, it is possible to operate the transformer from either 110V or 220V line voltages. Note that two center-tapped secondary windings are also provided. Center-tapped windings are especially useful for those applications requiring two equal voltages. As you can see in Figure 5-5, the various transformer leads have been color coded for easy identification.

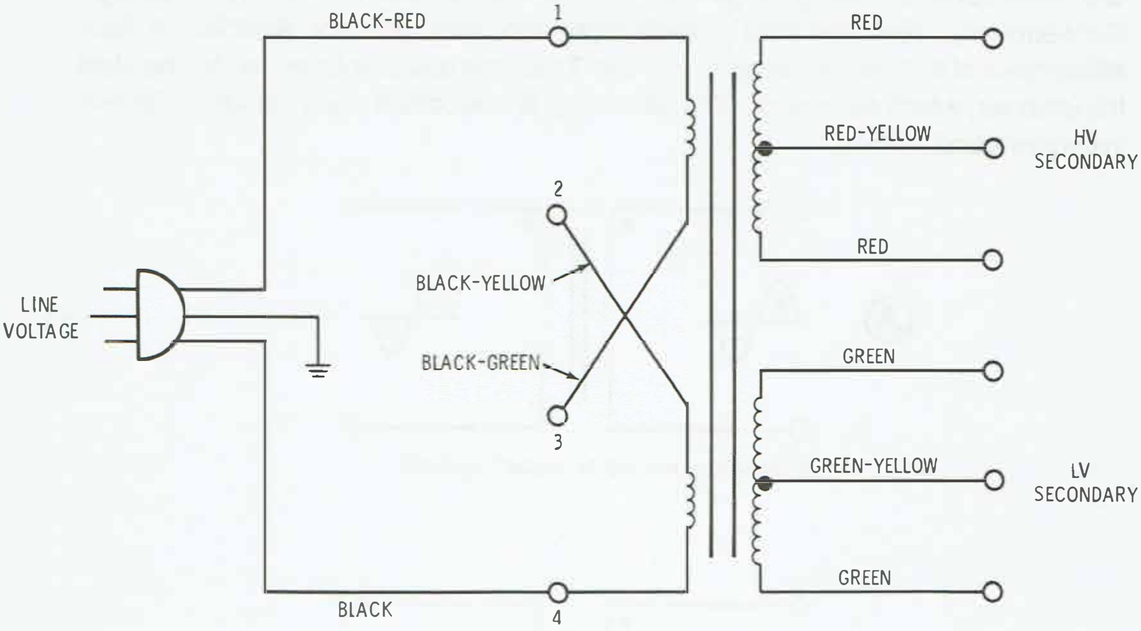


Figure 5-5

A typical power transformer.
For 110V operation terminal 2 is connected to terminal 1, and terminal 3 is connected to terminal 4. For 220V operation terminal 2 is connected to terminal 3.

The Ideal Diode

A diode is a two-terminal electronic device that conducts current in one direction only. The terminals of a diode are called the cathode and anode. Usually, the terminal connected to the cathode is indicated by a band or some other distinguishing mark on the body of the diode. The schematic symbol and current-versus-voltage curve for an ideal diode are illustrated in Figure 5-6. Here, you should note the following:

1. When the cathode is positive with respect to the anode, the diode acts like a perfect insulator. For this reason, the diode is modeled as an open switch whose resistance is infinite.

2. As soon as the cathode becomes negative with respect to the anode, the diode acts like a perfect conductor. Consequently, the diode is modeled by a closed switch whose resistance is zero ohms.

3. Current can only flow through the diode in the cathode-to-anode direction. Note that the direction of current flow is opposite the direction indicated by the arrow portion of the diode symbol.

A conducting diode is said to be forward biased, while a nonconducting diode is considered to be reverse biased.

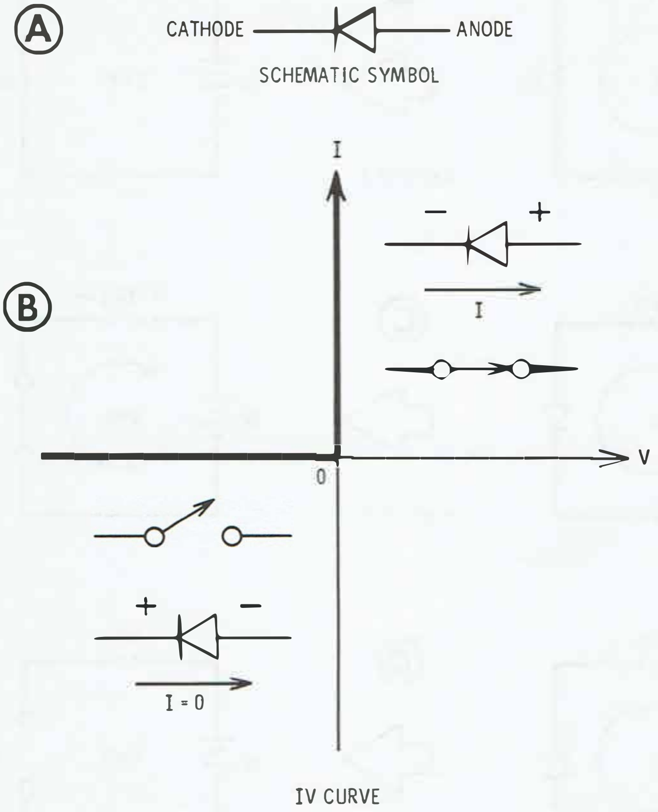


Figure 5-6

The ideal diode.
A. Schematic symbol.
B. IV curve.

Example 5-4

Determine the current through and voltage across each component in the circuits shown in Figure 5-7

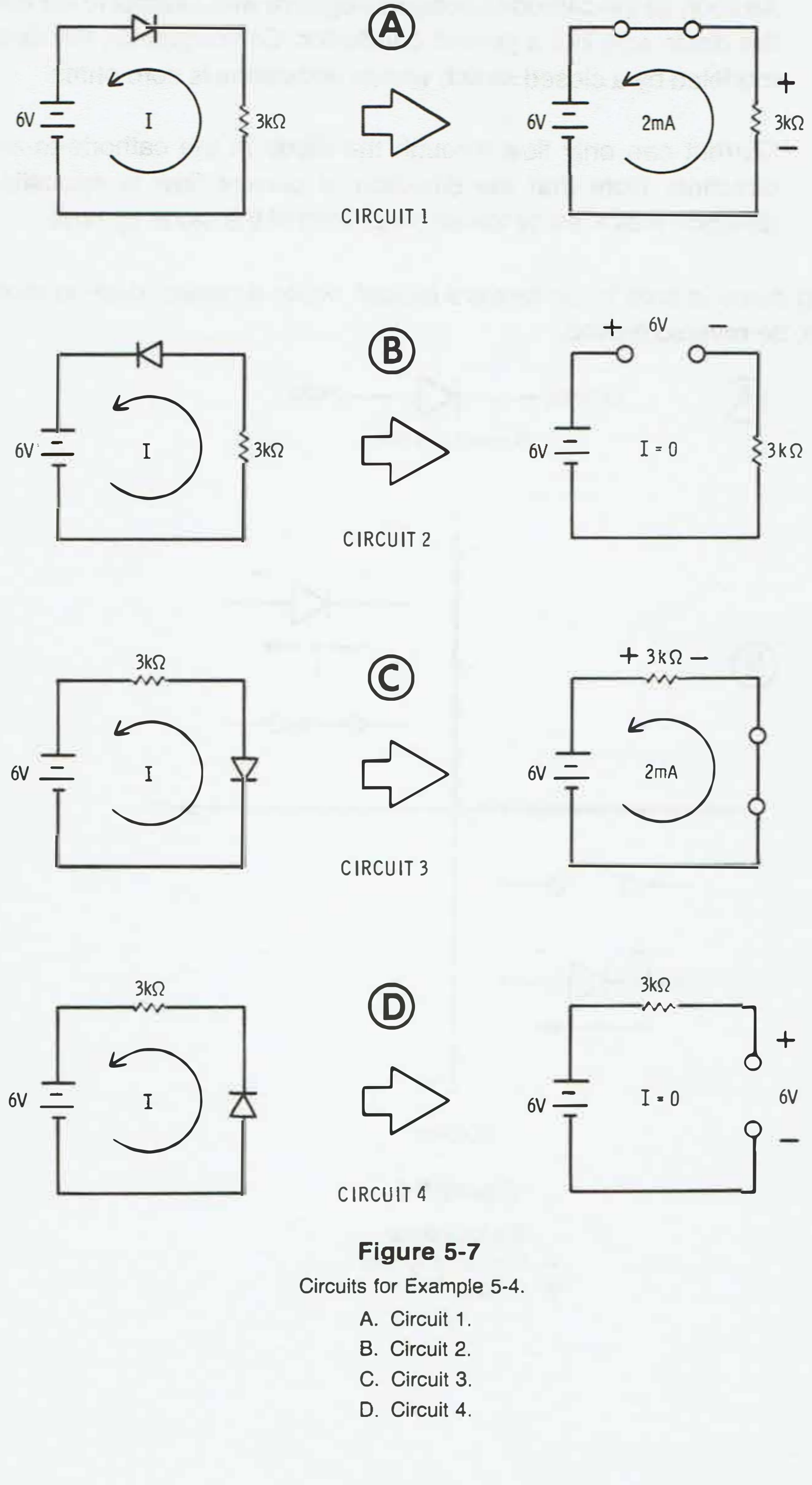


Figure 5-7

Circuits for Example 5-4.

A. Circuit 1.
B. Circuit 2.
C. Circuit 3.
D. Circuit 4.

Circuit 1.

The 6V source attempts to establish a counterclockwise loop current. Since the direction of the loop current is opposite the diode arrow, the diode is forward biased. Consequently, the diode appears as a closed switch, as shown. Thus:

$$I = \frac{6V}{3k\Omega} = 2mA$$

$$V_D = 0V$$

$$V_R = 2mA\,(3k\Omega) = 6V$$

Circuit 2.

Once again, the 6V source attempts to establish a counterclockwise loop current. However, since the direction of the loop current is the same as the diode arrow, the diode will be reverse biased. Thus, the diode appears as an open switch as shown. Therefore:

$$I = 0$$

$$V_R = 0V$$

$$V_D = 6V$$

Similarly, for the remaining circuits:

Circuit 3.

The diode is forward biased. Thus:

$I = 2mA$, $V_D = 0V$, and $V_R = 6V$

Circuit 4.

The diode is reverse biased. Thus:

$I = 0$, $V_D = 6V$, and $V_R = 0V$

Semiconductor Diodes

Figure 5-8 illustrates the current-versus-voltage curve for a typical semiconductor diode. By comparing the typical IV curve with the ideal IV curve in Figure 5-6, you should note the following:

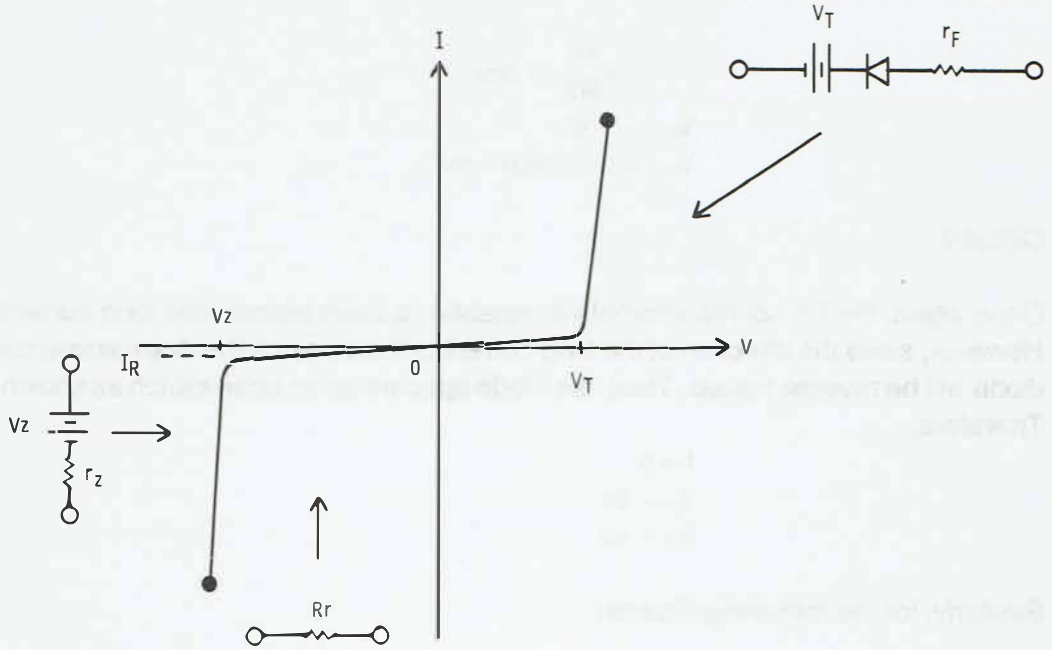


Figure 5-8

Typical IV curve for a semiconductor diode. Note the models used to represent the diode in the various regions of the curve.

FORWARD BIAS

1. As the voltage across the diode is gradually increased, very little current flows through the diode until the diode voltage exceeds the "turn-on" voltage, V_T.
2. Once the turn-on voltage is reached, very small increases in diode voltage produce very large increases in diode current. In this "linear region" of operation, the diode exhibits a forward resistance, r_F.

Thus, a forward biased semiconductor diode can be modeled as the series combination of a battery, V_T, an ideal diode, and a resistance, r_F. Most semiconductor diodes are made from Germanium, or Silicon. Typically, Germanium diodes have a turn-on voltage of 0.3V, while Silicon diodes have a turn-on voltage of

0.7V. Manufacturers usually list the current through a diode for a specified forward voltage. With this information, you can estimate the forward resistance of the diode from:

$$r_F = \frac{V_F - V_T}{I_F} \qquad \text{(Eq. 5-5)}$$

where:
r_F = forward resistance in ohms (Ω).
V_F = forward voltage in volts (V).
I_F = forward current in amperes (A).
V_T = diode turn-on voltage in volts (V).

REVERSE BIAS ($V_R < V_Z$)

As the reverse voltage across the diode is increased, a small "reverse current", I_R, flows through the diode. This reverse current is relatively constant. Thus, for reverse voltages less than V_Z, the diode acts like a large resistance, R_R. The value of the diode's reverse resistance is therefore:

$$R_R = \frac{V_R}{I_R} \qquad \text{(Eq. 5-6)}$$

where:
R_R = reverse resistance in ohms (Ω).
V_R = reverse voltage in volts (V).
I_R = reverse current in amperes (A).

REVERSE BIAS ($V_R = V_Z$)

As the reverse voltage is increased, a point is reached where the diode suddenly acts like a voltage source rather than a large resistance. The reverse voltage at which this sudden change occurs is called the breakdown, or zener, voltage, V_Z. Diodes specifically designed to operate in the zener region are called zener diodes. Zener diodes are used to provide stable reference voltages in regulator and control circuits. Since general purpose and rectifier-type diodes are not usually operated in the zener region, we will not pursue the matter further at this time.

The previous discussion reviewed the basic characteristics of ideal, and real semiconductor diodes. A discussion of why real diodes exhibit the characteristics previously described requires a discussion of the properties of semiconductor materials. If you are interested in these properties, you should consult our **Semiconductor Devices** course. Most of the semiconductor diodes you will encounter in practice will be Silicon diodes, since they have much smaller reverse currents than Germanium diodes. Rectifier circuits, for example, almost always use Silicon diodes for this reason.

Example 5-5

For a certain Silicon diode, a manufacturer lists the following typical values:

$I_F = 100mA$ *at* $V_F = 1V$

$I_R = 0.05\mu A$ *at* $25°C$

$I_R = 1\mu A$ *at* $75°C$

Estimate the following:

(a) The turn-on voltage, forward resistance, and reverse resistance of the diode.

(b) The diode current and load voltage, assuming the diode is used in the circuit of Figure 5-9A.

(c) The diode current and load voltage assuming the diode is used in the circuit of Figure 5-9B.

(d) The diode current and load voltage, assuming the diode is used in the circuit of Figure 5-9C.

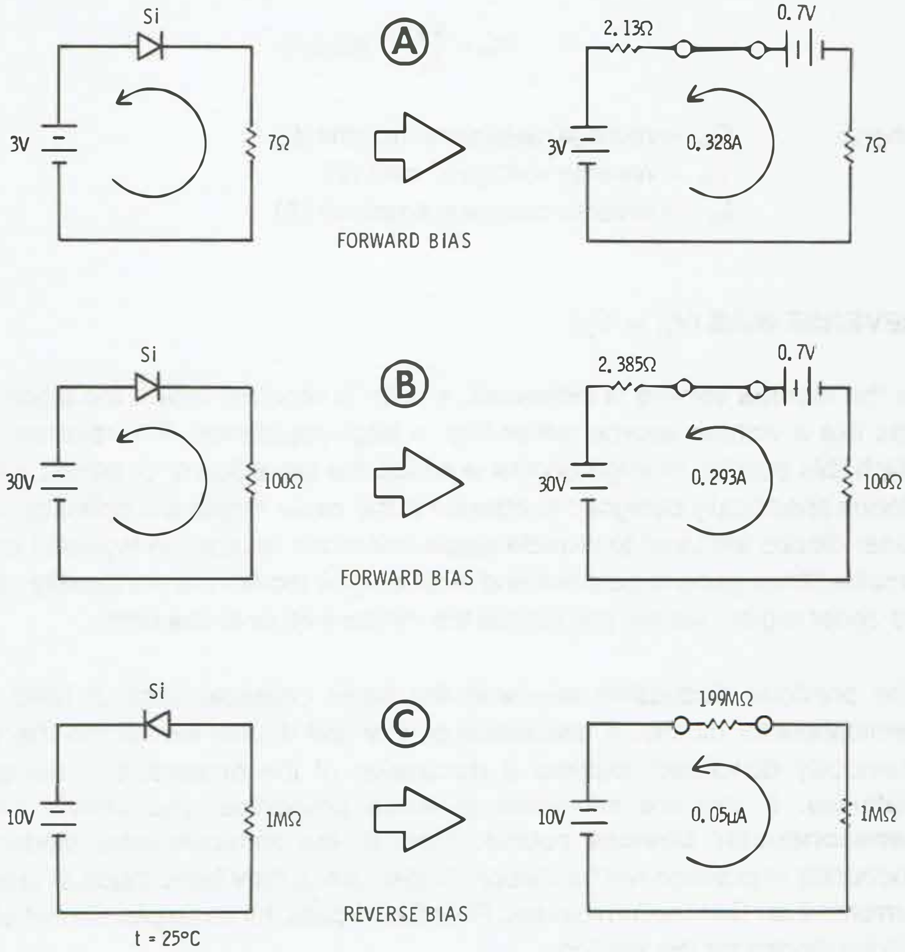


Figure 5-9

Circuits for Example 5-5.

A. Forward bias.
B. Forward bias.
C. Reverse bias.

(a) For a silicon diode $V_T = 0.7V$

$$r_F = \frac{V - V_T}{I_F} = \frac{1V - 0.7V}{100mA} = 3\Omega$$

The value of the diodes reverse resistance, R_R, depends upon the value of the reverse voltage, V_R, and the value of the reverse current, I_R. For example, at 25°C I_R, $= 0.05\mu A$. If $V_R = 10V$, the reverse resistance, R_R, is:

$$R_R = \frac{V_R}{I_R} = \frac{10V}{0.05\ \mu A} = 200M\Omega$$

Since reverse current increases with temperature, the value of the diode's reverse resistance, for a particular value of reverse voltage, will decrease as the temperature is increased. For example, at 75°C, $I_R = 1\mu A$. If $V_R = 10V$, the reverse resistance, R_R, is:

(b) If the diode in Figure 5-9A was ideal, the current, I, would equal $3V/7\Omega$ or 0.429A. Similarly, the load voltage would be 3V. Since the diode is not ideal, you must consider the effect of the diode's turn-on voltage and forward resistance as shown in the equivalent circuit. Thus:

$$I = \frac{V_L}{R_L} = \frac{3V - 0.7V}{7\Omega} = .328A$$

$$r_F = \frac{.7V}{.328A} = 2.13\Omega$$

Note that the actual value of diode forward resistance differs significantly from the ideal value.

(c) For an ideal diode in Figure 5-9B, the current, I, would equal $30V/100\Omega$ or 0.3A. Similarly, the load voltage would equal 30V.

For the real diode:

$$I = \frac{V - V_T}{R_L} = \frac{29.3V}{100} = 0.293A$$

$$r_F = \frac{.7V}{0.293A} = 2.385\Omega$$

Note that the actual and ideal values of diode forward resistance are approximately equal.

(d) In Figure 5-9C, the diode is reverse biased by the 10V source. Ideally, $R_R = \infty\,\Omega$ which means the ideal values of diode current and load voltage would be zero.

Since the real diode has a reverse current of 0.05μA, at 25°C, $I = 0.05\mu A$. Consequently:

$$V_L = I\ R_L = 0.05\mu A\ (1M\Omega) = 0.05V$$

Since $V_D = V - V_L$, the diode voltage is:

$$V_D = 10V - 0.05V = 9.95V$$

Thus, $R_R = 9.95V/0.05\ \mu A$ or 199 MΩ as shown in the equivalent circuit. Although the diode is not ideal, it closely approximates an ideal diode in this circuit.

The previous example illustrated the fact that, depending upon the specific component values, real diodes may or may not closely approximate the characteristics of ideal diodes. The nonideal properties of real diodes can be minimized by selecting component values that "mask out" the nonideal effects. For example, by driving a diode with a voltage that is large compared to the diode's turn-on voltage, the effects of the diode's turn-on voltage are minimized.

Similarly, the effect of a real diode's forward resistance is minimized if the load resistance is large compared to the diode's forward resistance.

Finally, a real diode that is reverse biased will closely approximate an ideal reverse biased diode if the load resistance is small compared to the reverse resistance of the diode. As a guide, you can use the 20:1 rule to define large and small. Thus, for near ideal operation of real diodes, you should select component values that are consistent with Table 5-1.

Parameter	Negligible When	Appropriate Component Value
V_T	$V >> V_T$	$V \geqslant 20\ V_T$
r_F	$R >> r_F$	$R \geqslant 20\ r_F$
R_R	$R_L << R_R$	$R_L \leqslant 20\ R_R$

Table 5-1

Guide for selecting component values for Near Ideal Operation of Real Diodes.

Self-Test Review

1. The ratio of secondary power to primary power indicates the ____________ of a transformer.
2. If the number of secondary turns is less than the number of primary turns, the transformer is called a ____________transformer.
3. The ___________ ___________ is the number of primary turns divided by the number of secondary turns.

Refer to Figure 5-10 for questions 4 through 7.

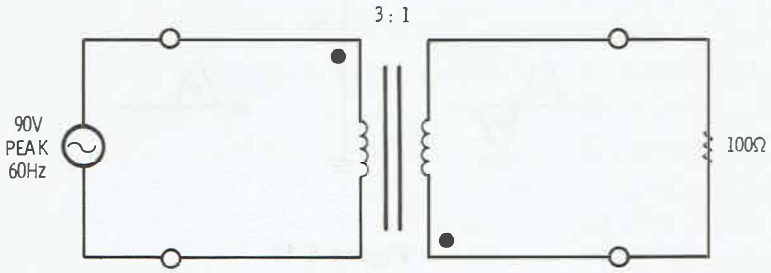


Figure 5-10

Circuit for questions 4 to 7.

4. The peak secondary voltage is ____________volts.
5. The peak primary current is ____________amperes.
6. The effective primary resistance is ____________ohms.
7. The primary and secondary voltages ____________in phase with each other.
   are/are not
8. In order for a diode to conduct, the cathode must be ____________with respect to the anode.
   positive/negative
9. A silicon diode has a turn-on voltage of approximately ____________volts.
10. A silicon diode has a forward current of 0.5A when the circuit voltage is 1V. Therefore, the forward resistance of the diode is approximately ______ ohms.
11. A silicon diode has a forward resistance of 10Ω, a turn-on voltage of 0.7V, and a reverse resistance of 100MΩ. For near ideal operation, the diode should be driven from a voltage source of at least _________ V.

12. The circuit shown in Figure 5-11 is called a half-wave rectifier. On the positive half cycle of the input voltage, the diode is forward biased and on the negative half cycle, the diode is reverse biased. Ideally, the output voltage consists of the positive portions of the input voltage as shown. To approximate this ideal case, R_L should be at least ____________Ω but not larger than ________ MΩ.

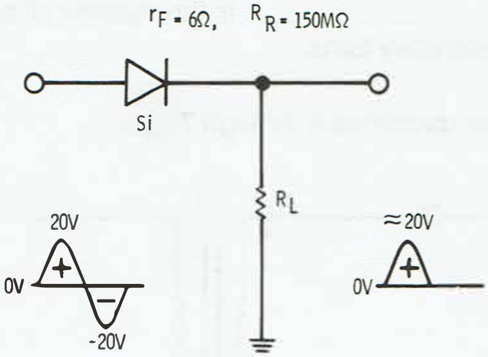


Figure 5-11

Circuit for question 12.

Answers

1. efficiency
2. step-down
3. turns ratio
4. 30
5. 0.1
6. 900
7. are not
8. negative
9. 0.7
10. 0.6
11. 14
12. 120, 7.5

The solutions to appropriate questions follow.

4. $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ Thus, $V_2 = \frac{V_1 N_2}{N_1} = \frac{90V\ (1)}{3} = 30V$ peak

5. Since $V_2 = 30V$ and $R_L = 100\Omega$.

$$I_2 = \frac{V_2}{R_L} = \frac{30V}{100\Omega} = 0.3A \text{ peak}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \text{ Thus, } I_1 = \frac{I_2 N_2}{N_1} = \frac{0.3A\ (1)}{3} = 0.1A \text{ peak}$$

6. $R_1 = \frac{V_1}{I_1} = \frac{90V}{0.1A} = 900\Omega$

10. $r_F = \frac{V - V_T}{I_F} = \frac{1V - 0.7V}{0.5A} = 0.6\Omega$

11. The source voltage, V, should be at least 20 times larger than the diode's turn-on voltage V_T. Thus, $V = 20\ (0.7) = 14V$.

12. In the forward biased state, the load resistance, R_L, should be at least 20 times larger than the diode's forward resistance, r_F. Thus, $R_L = 20(6\Omega) = 120\Omega$.

    In the reverse biased state, the load resistance, R_L, should be no larger than $R_R/20$. Thus:

$$R_L = \frac{150M\Omega}{20} = 7.5M\Omega$$

RECTIFIER CIRCUITS

Rectifier circuits convert an AC input voltage into a pulsating DC voltage. In this section, we will review the operation and characteristics of the four most commonly used rectifier circuits.

Half-wave Rectifier

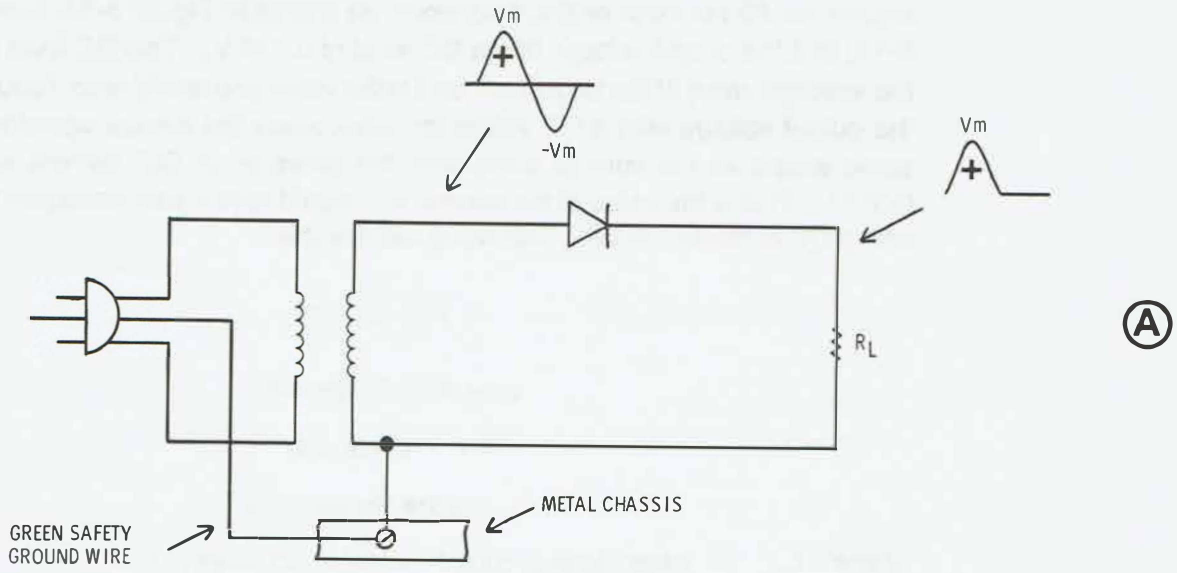
The circuit of a typical half-wave rectifier is illustrated in Figure 5-12A. The transformer is used to step up, or step down, the AC line voltage to the desired level. In addition, the transformer isolates the load from the AC line voltage. In Figure 5-12A, the middle prong of the three-wire plug is connected to earth ground through the wires between the electrical outlet and fuse panel. The green "safety ground" wire of the AC line cord is connected to the middle prong of the three-wire plug and should be connected to the metal chassis of the circuit. Thus, if a short develops between the circuit and chassis, there should be enough current to blow a fuse in the fuse panel. In this way, a person touching the metal chassis is protected from a potentially fatal electric shock.

Assuming an ideal diode, the operation of the half-wave rectifier may be summarized as follows:

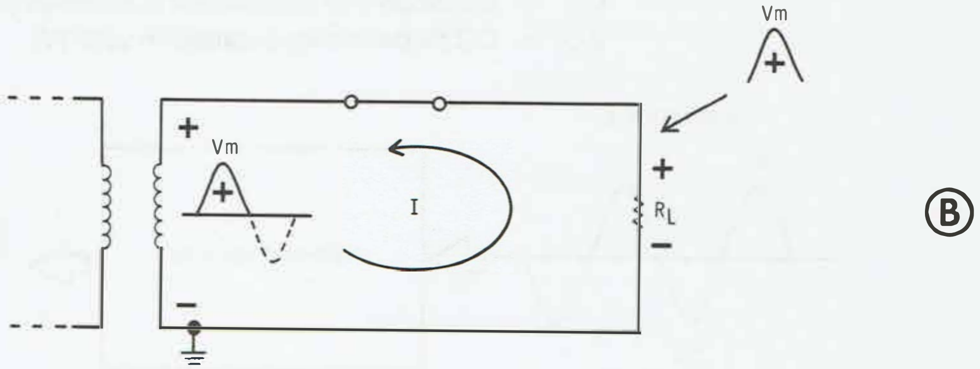
1. During the positive half cycle, the input voltage forward biases the diode. Consequently, the diode acts like a short circuit, as shown in Figure 5-12B. Clearly, the diode voltage will be 0V, and the load voltage will equal the input voltage.

2. During the negative half cycle, the input voltage reverse biases the diode. Thus, the diode acts like an open circuit, as shown in Figure 5-12C. Consequently, during the negative half cycle, the input voltage appears across the diode, and the load voltage equals 0V.

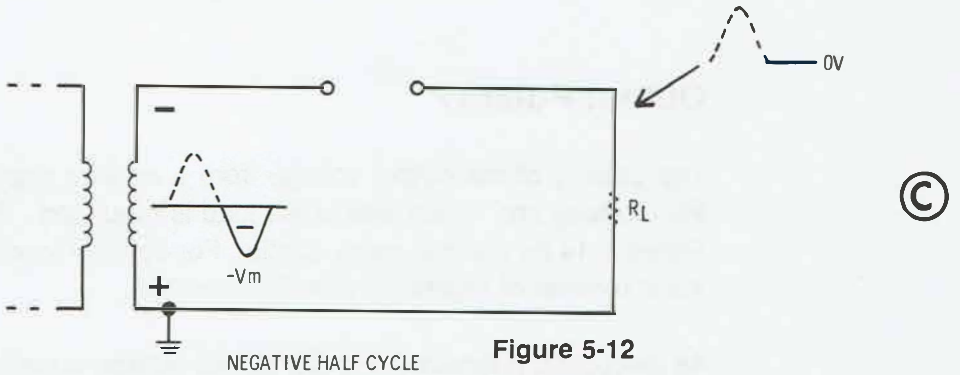
Note in Figure 5-12C that the maximum voltage developed across the diode equals the peak input voltage, and occurs when the diode is nonconducting. This maximum diode voltage is called the **peak inverse voltage**, or PIV for short. Since the output voltage consists of the positive alternations of the input voltage, the output frequency



CIRCUIT



POSITIVE HALF CYCLE



NEGATIVE HALF CYCLE

Figure 5-12

The half-wave rectifier.

A. Circuit.

B. Positive half cycle.

C. Negative half cycle.

equals the 60 Hz input or line frequency, as shown in Figure 5-13. Note in Figure 5-13, that the output voltage has a DC level of 0.318 V_m. This DC level represents the average value of the waveform, and is the value you would read if you measured the output voltage with a DC voltmeter. Also, since the current waveform has the same shape as the voltage waveform, the average, or DC, current would equal 0.318 I_m. This is the value of the current you would read if you measured the current with a DC ammeter. For the half-wave rectifier then:

$$I_m = \frac{V_m}{R_L} \text{ (Eq. 5-7)}$$

$$I_{DC} = 0.318\, I_m \text{ (Eq. 5-8)}$$

$$PIV = V_m \text{ (Eq. 5-9)}$$

$$V_{DC} = 0.318\, Vm \text{ (Eq. 5-10)}$$

where: I_m = peak diode and load current in amperes (A).
V_m = peak input voltage in volts (V).
I_{DC} = DC diode and load current in amperes (A).
V_{DC} = DC diode and load voltage in volts (V).

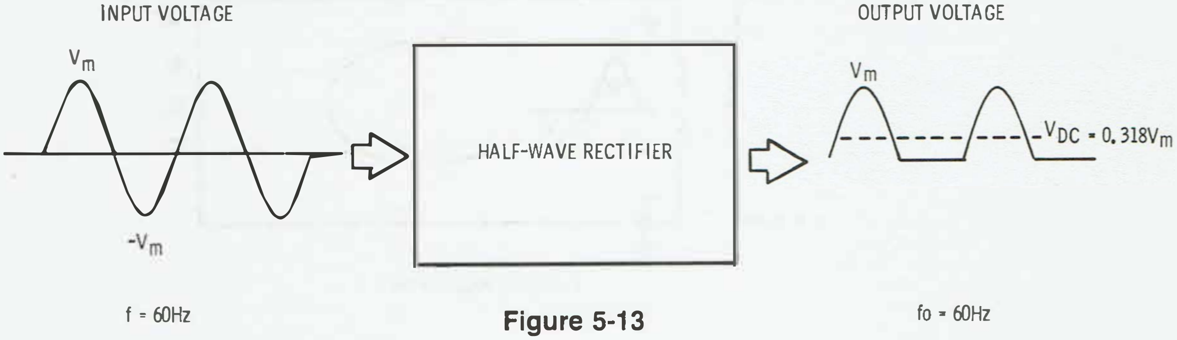


Figure 5-13

Input and output voltages for the half-wave rectifier.

Output Polarity

The polarity of the output voltage from a rectifier depends upon the direction of the diode(s) and which side of the load is grounded. This concept is illustrated in Figure 5-14 for the half-wave rectifier. For convenience, we have redrawn the half-wave rectifier of Figure 5-12 in Figure 5-14A.

As discussed previously, the DC output voltage equals 0.318 V_m. In Figure 5-14A note that terminal B is grounded. This makes the voltage from terminal A to ground postive as shown. If you want a negative output voltage, with respect to ground, you can either reverse the direction of the diode or reverse the ground connection as shown in Figure 5-14B and Figure 5-14C, respectively. If you reverse both the diode and ground connection, as shown in Figure 5-14D, the output voltage will once again be positive.

Although the output voltages in Figure 5-14 have a DC component equal to 0.318 V_m, they contain a great deal of ripple. Consequently, these voltages must be filtered before they are connected to the load. Nevertheless, by studying Figure 5-14, you should be able to see how a simple rectifier can provide either positive or negative voltages with respect to the common, or ground, connection.

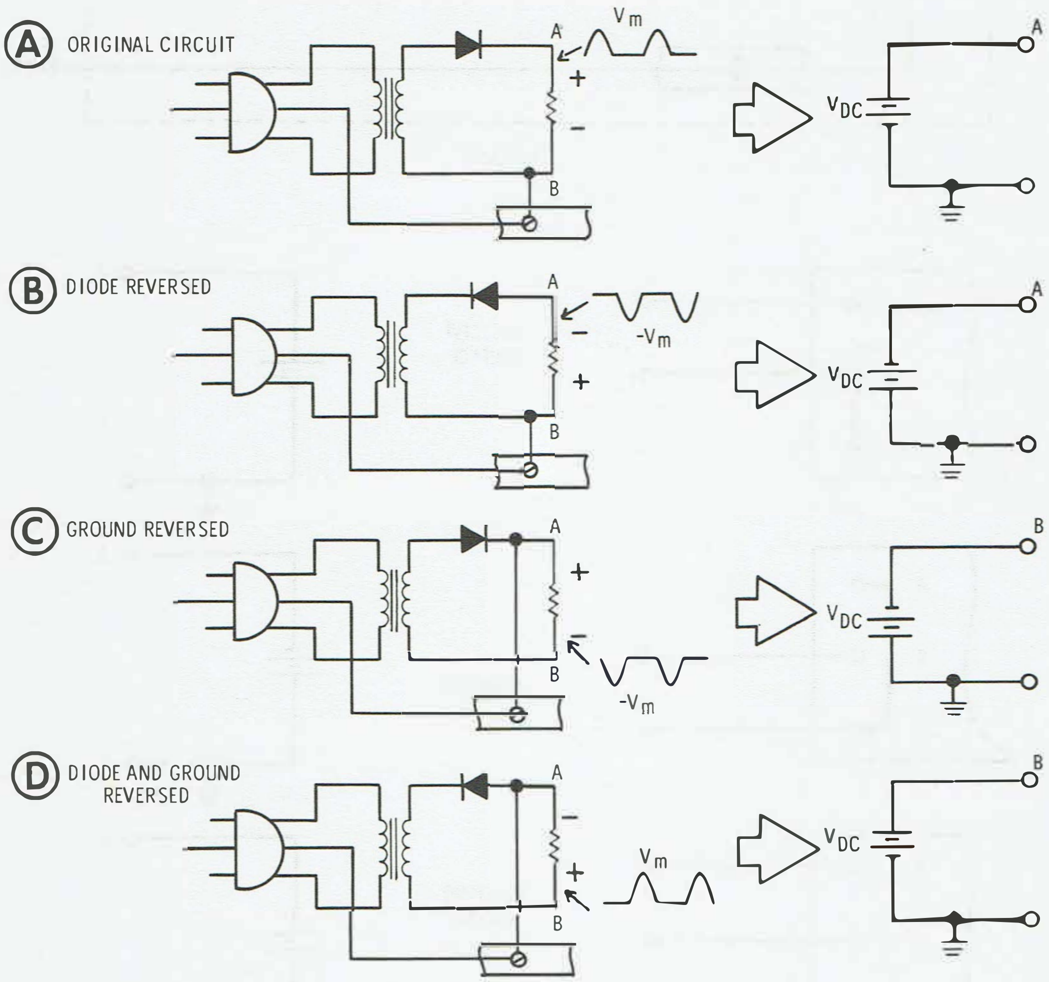


Figure 5-14

Polarity.
In each case the DC output voltage is 0.318 V_m. The polarity of the output voltage with respect to ground is determined by the direction of the diode, and which side, A or B, of the load is grounded.

A. Original circuit.
B. Diode reversed.
C. Ground reversed.
D. Diode and ground reversed.

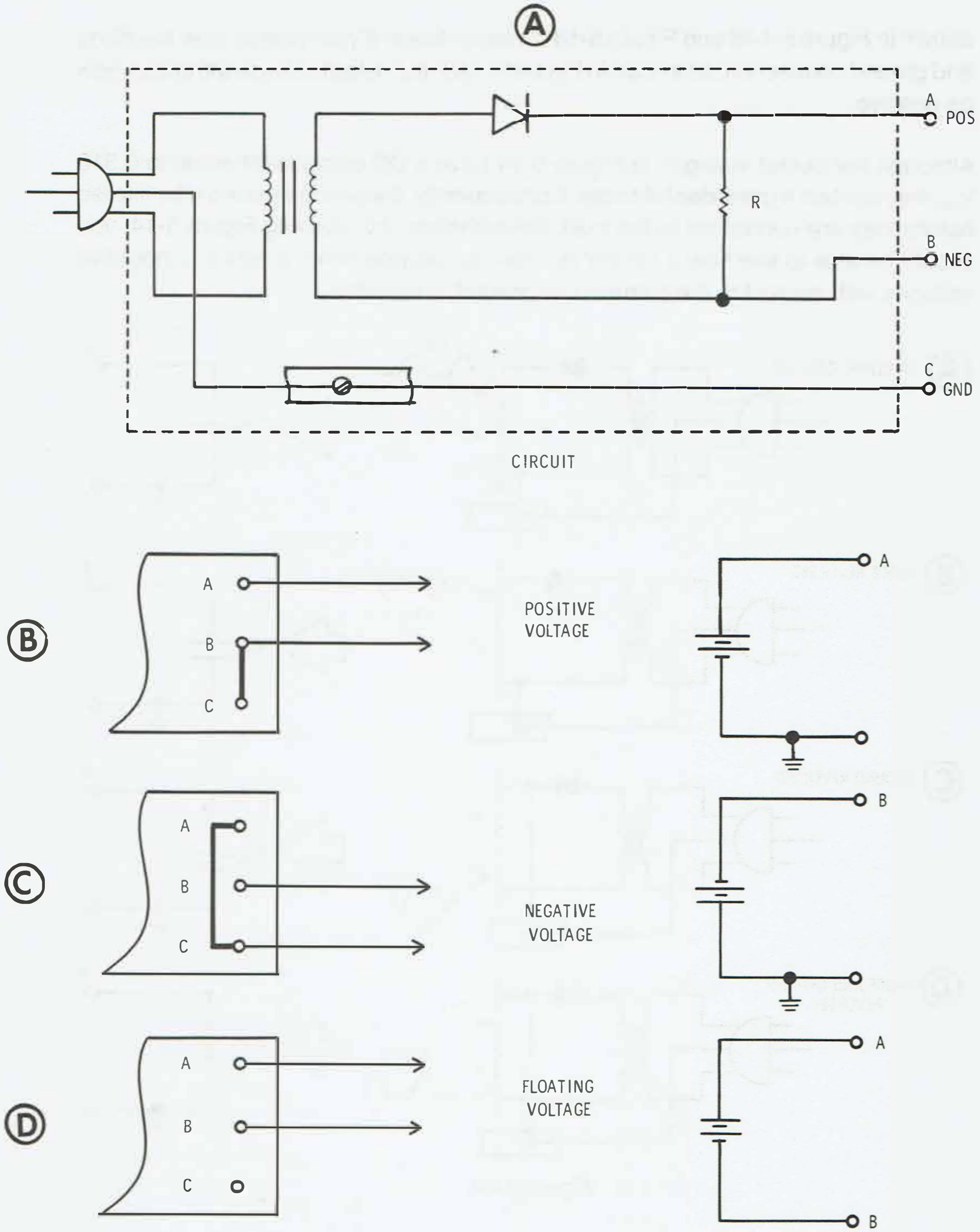


Figure 5-15

Commercial power supply concept.

By making the appropriate external connections the user can select a positive, negative, or floating output voltage.

Commercial power supplies are usually designed so that you can determine whether the output voltage is positive, negative, or floating with respect to ground. This concept is illustrated in Figure 5-15. Note that the ground wire, and the wires connected to each side of resistor R in Figure 5-15A are brought out to three terminals that are insulated from the chassis. Connecting terminal B and C together gives you a positive output voltage with respect to ground. Similarly, connecting terminals A and C together gives you an output voltage that is negative with respect to ground.

If neither end of resistor R is connected to ground, as shown in Figure 5-15D, a floating output voltage results. Some circuits require a non-ground referenced, or floating, voltage for proper operation. Consequently, commercial power supplies usually make this option available.

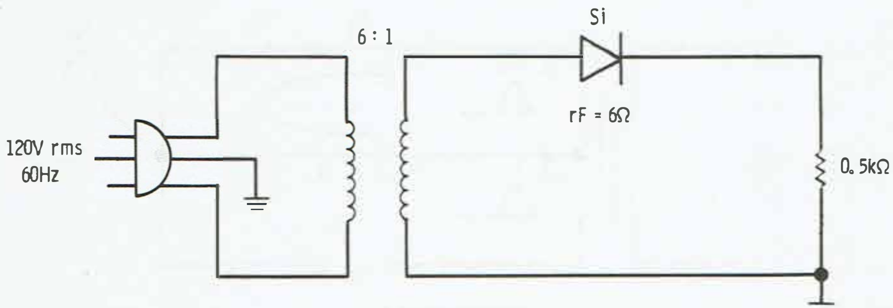


Figure 5-16

Circuit for Example 5-6.

Example 5-6

For simplicity, the chassis ground connection in Figure 5-16 is not shown. However, as you know, the green safety ground wire should always be connected to the metal chassis.

For the given circuit, calculate:
(a) The peak diode current.
(b) The PIV voltage.
(c) The DC load current.
(d) The DC load voltage.

(a) Since the step-down transformer has a 6:1 turns ratio, the secondary voltage is 120V/6 or 20V rms. Converting the rms value to a peak value yields:

$$V_2 = 1.414\ (20V) = 28.28V$$

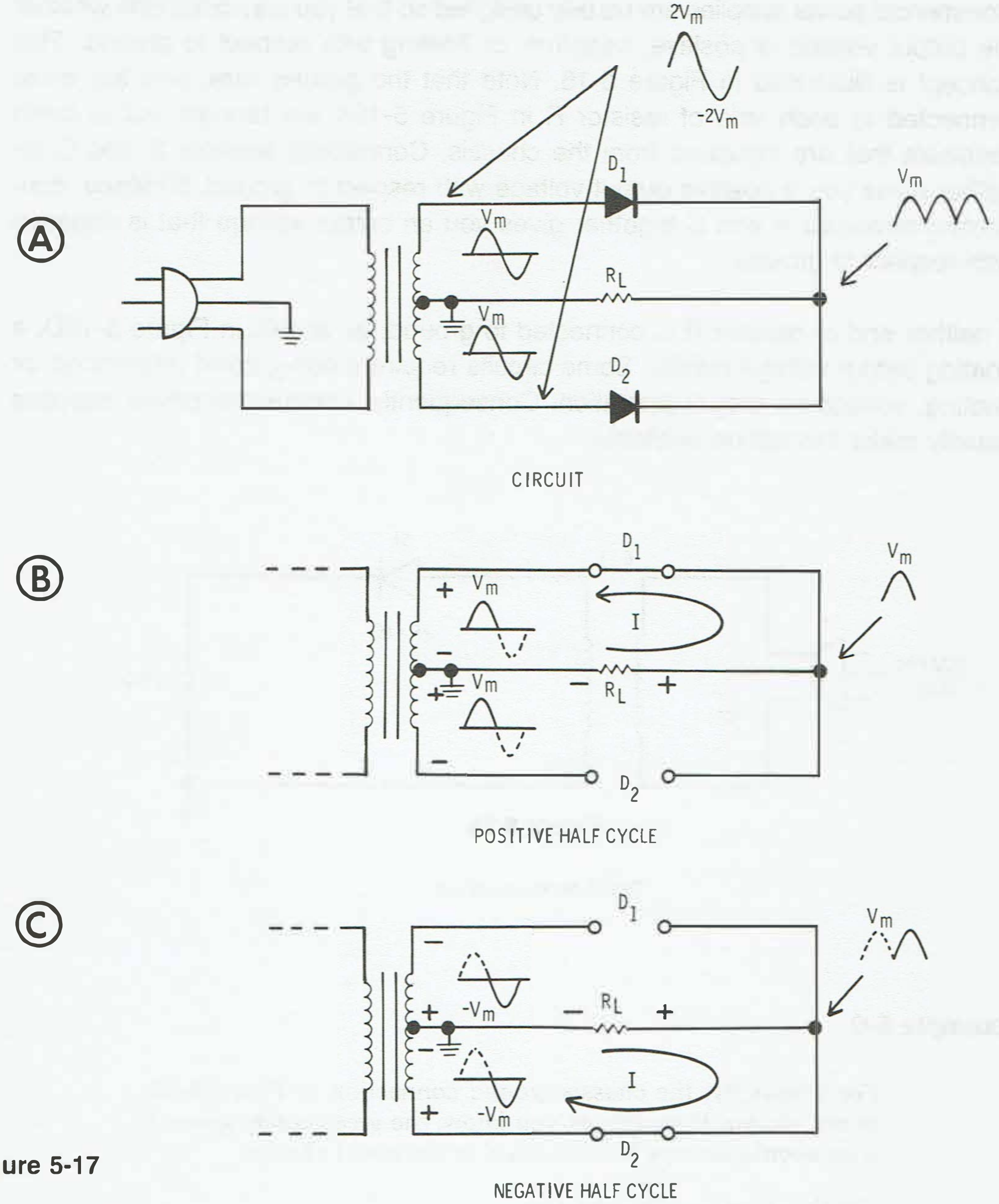


Figure 5-17

A full-wave rectifier.

Since the peak voltage driving the diode is large compared to 0.7V, which is the typical turn-on voltage for a silicon diode, you can neglect the diode's turn-on voltage. Also, since $R_L >> r_F$, you can neglect the diodes forward resistance. Thus:

$$I_m = \frac{V_m}{R_L} = \frac{28.28V}{0.5K\Omega} = 56.56mA$$

(b) For the half-wave rectifier, the peak inverse voltage equals V_m. Consequently:

$$PIV = 28.28V$$

(c)

$$I_{DC} = 0.318\ I_m = 0.318\ (56.56mA) = 17.98mA$$

(d)

$$V_{DC} = 0.318\ V_m = 0.318\ (28.28V) = 8.99V$$

Full-Wave Rectifiers

Figure 5-17A shows a full-wave rectifier with two diodes, and a center-tapped transformer. Note that the voltages between each end of the secondary winding and center tap equal one-half the full secondary voltage. During the positive half cycle, diode D_1 is forward biased and diode D_2 is reverse biased as shown in Figure 5-17B. Thus, current flows from the center tap through R_L and diode D_1.

During the negative half cycle, diode D_1 is reverse biased and diode D_2 is forward biased as shown in Figure 5-17C. In this case, current flows from the center tap through R_L and diode D_2. Note in Figure 5-17B and Figure 5-17C that the peak voltage across the nonconducting diode is the full secondary voltage or $2V_m$.

Since the direction of the current through R_L is the same for each half cycle, the load voltage consists of two positive alternations per cycle. Consequently, the frequency of the output voltage is twice the line frequency as shown in Figure 5-18. Also, since the output voltage consists of two alternations per cycle, its DC value is double the value obtained with a half-wave rectifier that uses the same V_m input. In addition, since each diode conducts for only half a cycle, the average, or DC, diode currents equal one-half the average load current.

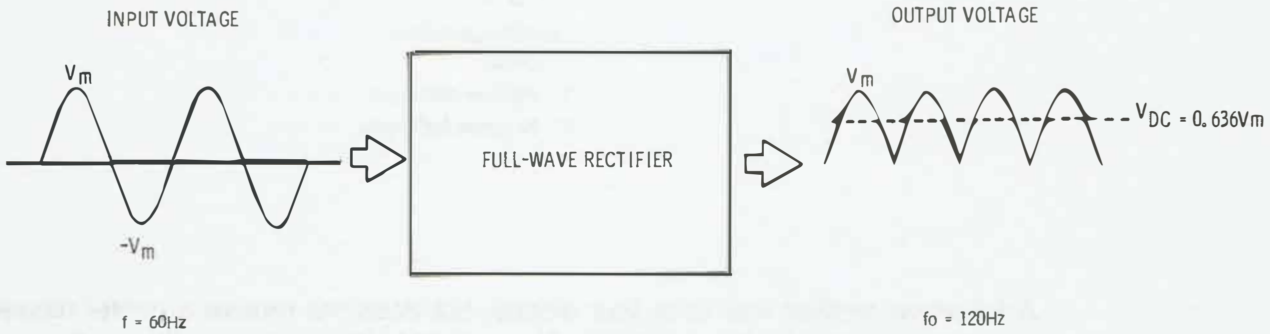
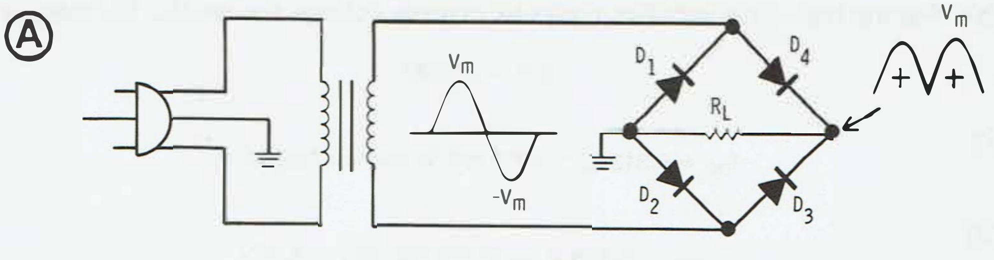
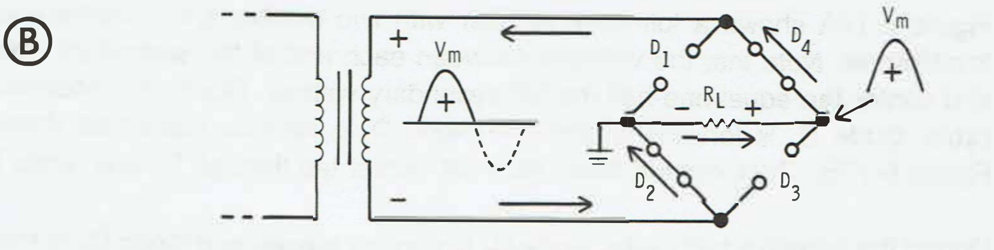


Figure 5-18

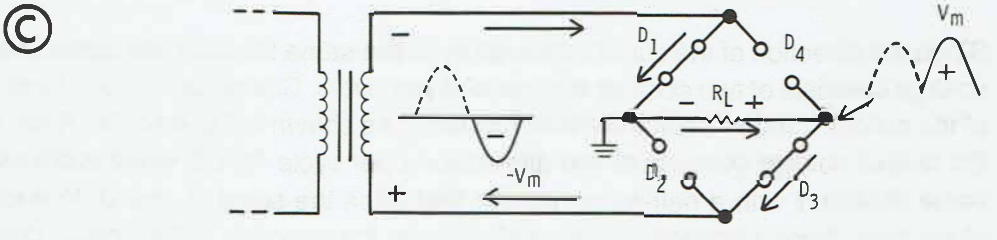
Input and output voltages for the full-wave rectifier.



CIRCUIT



POSITIVE HALF CYCLE



NEGATIVE HALF CYCLE

Figure 5-19

The bridge rectifer.

A. Circuit.

B. Positive half cycle.

C. Negative half cycle.

A full-wave rectifier that uses four diodes, but does not require a center-tapped transformer is the bridge rectifier in Figure 5-19A. During the positive half cycle, diodes D_2 and D_4 are forward biased and diodes D_1 and D_3 are reverse biased as shown in Figure 5-19B. Thus, current flows from the lower part of the secondary through diode D_2, R_L, and diode D_4.

On the negative half cycle, diodes D_1 and D_3 are forward biased and diodes D_2 and D_4 are reverse biased as shown in Figure 5-19C. Here, current flows from the upper part of the secondary through diode D_1, R_L, and diode D_3. Clearly, the direction of the load current is the same for each half cycle.

Thus, as is the case with the full-center-tapped rectifier, the load voltage consists of two alternations per cycle. The formulas to calculate the various currents and voltages for the half and full-wave rectifier circuits are provided in Table 5-2.

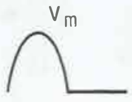
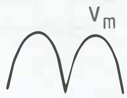
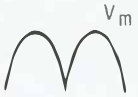
Parameter Load voltage/current waveform	Half wave 	Full-wave center-tapped 	Full-wave bridge 
Peak inverse voltage	V_m	$2\ V_m$	V_m
Peak diode/load current	V_m/R_L	V_m/R_L	V_m/R_L
DC load current	$0.318\ I_m$	$0.636\ I_m$	$0.636\ I_m$
DC diode current	$0.318\ I_m$	$0.318\ I_m$	$0.318\ I_m$
DC load voltage	$0.318\ V_m$	$0.636\ V_m$	$0.636\ V_m$
DC diode voltage	$0.318\ V_m$	$0.636\ V_m$	$0.318\ V_m$
Output frequency	f_{line}	$2f_{line}$	$2f_{line}$

Table 5-2

Characteristics of Half-Wave and Full-Wave Rectifiers with Resistive Loads.

Example 5-7

> *For the rectifiers in Figure 5-20, calculate the peak inverse voltage, the DC current through each diode, the DC load current, and the DC load voltage.*

For the center-tapped rectifier in Figure 5-20A, the turns ratio is 2:1. Thus, the full secondary voltage is:

$$V_s = \frac{120V}{2} = 60Vrms$$

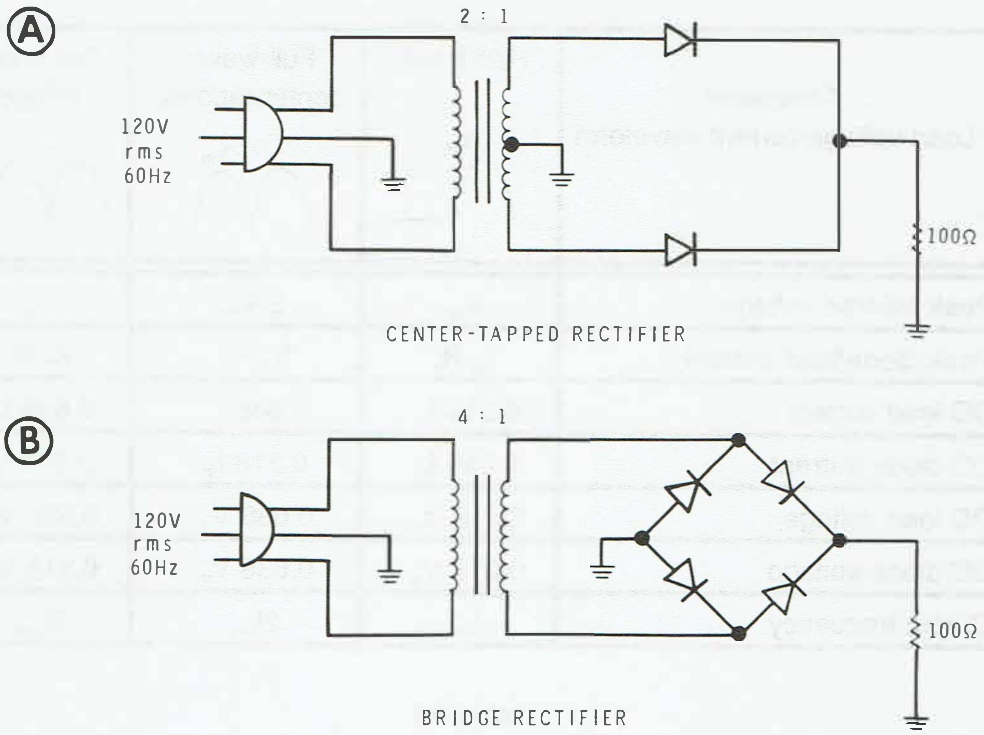


Figure 5-20

Circuits for Example 5-7.

A. Center-tapped rectifier.

B. Bridge rectifier.

The voltage from each end of the secondary to the center tap is one-half the full secondary value or 30V rms. Thus:

$$V_m = 1.414\ (30V) = 42.42V$$

Referring to Table 5-2

$$PIV = 2\ V_m = 2\ (42.42V) = 84.84V$$

$$I_m = \frac{V_m}{R_L} = \frac{42.42V}{100\Omega} = 424.2\ mA$$

$$I_{DC}\ (diode) = 0.318\ I_m = 0.318\ (424.2mA) = 134.9mA$$

$$I_{DC}\ (load) = 0.636\ I_m = 0.636\ (424.2mA) = 269.8mA$$

$$V_{DC}\ (load) = 0.636\ V_m = 0.636\ (42.42V) = 27V$$

For the bridge rectifier in Figure 5-20B, the turns ratio is 4:1. Thus, the secondary voltage is:

$$V_S = \frac{120V}{4} = 30Vrms$$

Converting from rms to peak:

$$V_m = 1.414\ (30V) = 42.42V$$

Refer to Table 5-2.

$$PIV = V_m = 42.42V$$

$$I_m = \frac{V_m}{R_L} = \frac{42.42V}{100\Omega} = 424.2\ mA$$

$$I_{DC}\ (diode) = 0.318\ I_m = 0.318\ (424.2mA) = 134.9mA$$

$$I_{DC}\ (load) = 0.636\ I_m = 0.636\ (424.2mA) = 269.8mA$$

$$V_{DC}\ (load) = 0.636\ V_m = 0.636\ (42.42V) = 27V$$

Although the output voltage, 27V, is the same for both examples, you should note that the turns ratio for the transformers in the examples are different. If the transformers in both examples were identical, the output voltage for the bridge rectifier would be twice that of the center-tapped rectifier.

Comparison of Rectifiers

Each of the rectifier circuits discussed previously has its own advantages and disadvantages. For this reason, we will compare the characteristics of each type.

HALF-WAVE RECTIFIER

Here, the advantages include simplicity and low cost. Since the output voltage consists of only one alternation per cycle, the average or DC output voltage is low and the 60 Hz ripple frequency is difficult to filter. In addition, since the load current always flows through the secondary winding in the same direction, its DC component tends to magnetize the transformer core. As you know, a magnetized core wastes power in the form of heat. Consequently, to minimize losses, half-wave rectifiers are usually restricted to low current applications.

An exception to the low current application rule just cited is shown in Figure 5-21. This is called a transformerless or universal AC-DC power supply, because it can be operated from either an AC or DC power line.

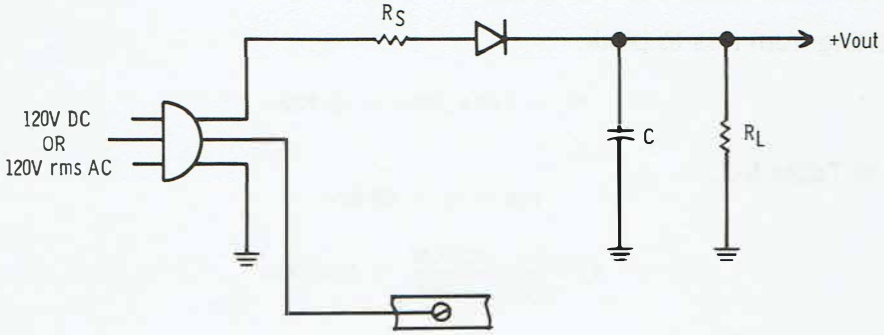


Figure 5-21

A universal AC-DC power supply.

The capacitor helps smooth out the pulsating DC voltage. Resistor R_s limits the peak diode current during the time the capacitor is charging.

Since it does not use a transformer, the principle advantage of the universal AC-DC power supply is its low cost. However, since a transformer is not used, the rectifier circuit is **not isolated** from the 120V rms AC line voltage. Consequently, a **potential shock hazard** exists — especially if a two-prong rather than a three-prong plug is used as shown in Figure 5-22. Since the two-prong plug can be connected in two ways, you never know which side of the AC line is connected to the chassis. Anyone touching the chassis and ground simultaneously could receive a dangerous shock.

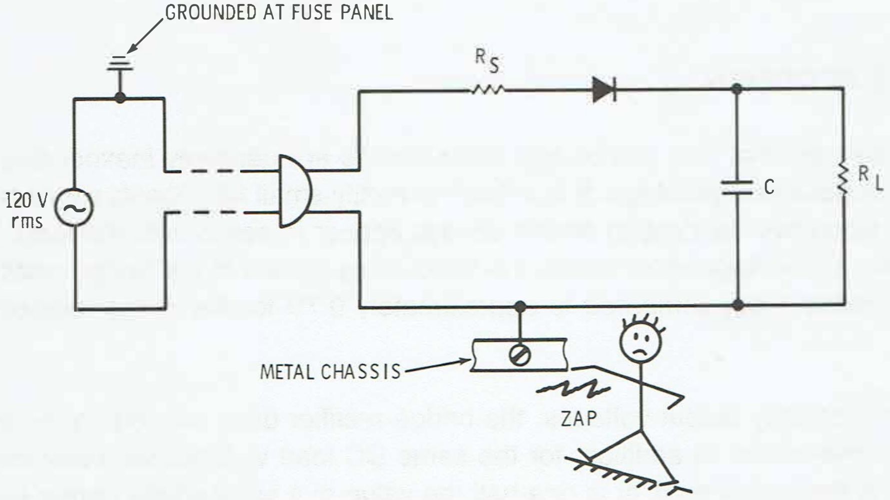


Figure 5-22

Shock hazard.

Without the isolation provided by the power transformer, this hazardous situation can result with the older two-prong line cords.

CENTER-TAPPED RECTIFIER

Since this circuit employs a center-tapped transformer, it is relatively expensive compared to the half-wave and bridge rectifiers. In addition, the peak inverse voltage is $2V_m$ as opposed to V_m for the half-wave and bridge rectifiers.

Compared to the half-wave rectifier the center-tapped rectifier is more efficient, since the load voltage consists of two alternations per cycle. Also, the load voltage is easier to filter because its frequency is double the 60Hz line frequency. Finally, since current flows through each half of the secondary winding in opposite directions, on alternate half cycles, the transformer core does not become magnetized.

BRIDGE RECTIFIER

This circuit requires four diodes but, since diodes are relatively inexpensive, this is not a serious disadvantage. It is difficult to rectify small AC signals with a bridge rectifier since two conducting diodes always appear in series with the load. Consequently, the voltage drop across the conducting diodes in the bridge rectifier is approximately 1.4V, compared to approximately 0.7V for the center-tapped rectifier.

For single-polarity output voltages, the bridge rectifier does not require a center-tapped transformer. In addition, for the same DC load voltage, the peak inverse voltage of the bridge rectifier is one-half the value of a comparable center-tapped rectifier. For these reasons, the bridge rectifier is the most commonly used rectifier in modern electronic equipment.

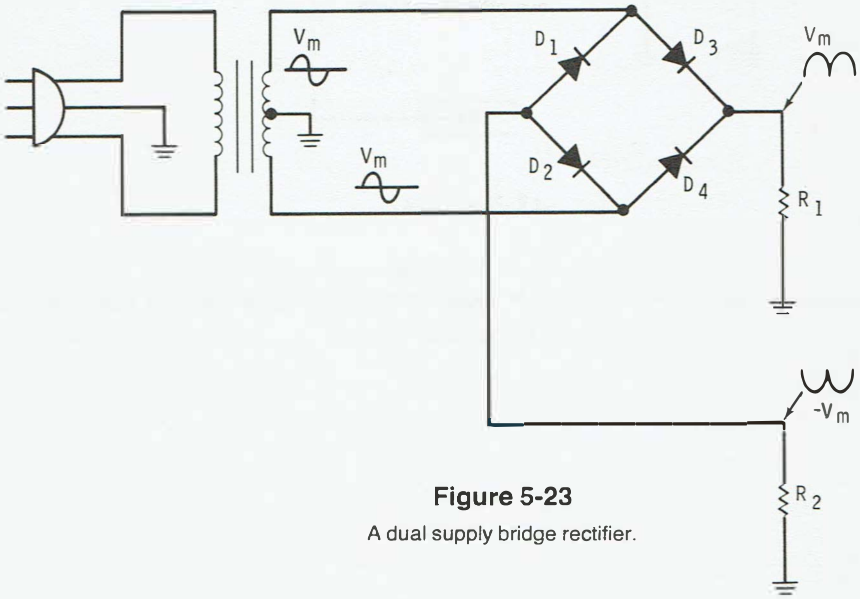
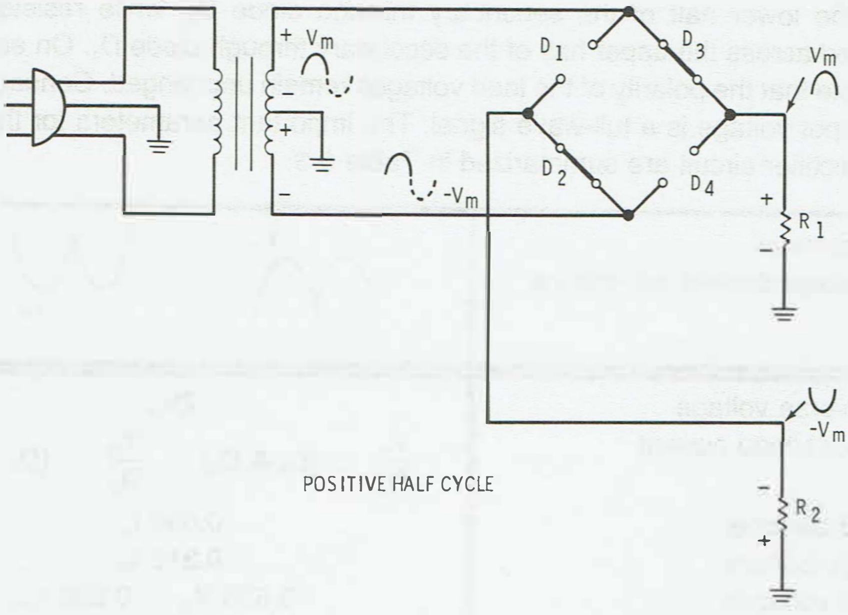


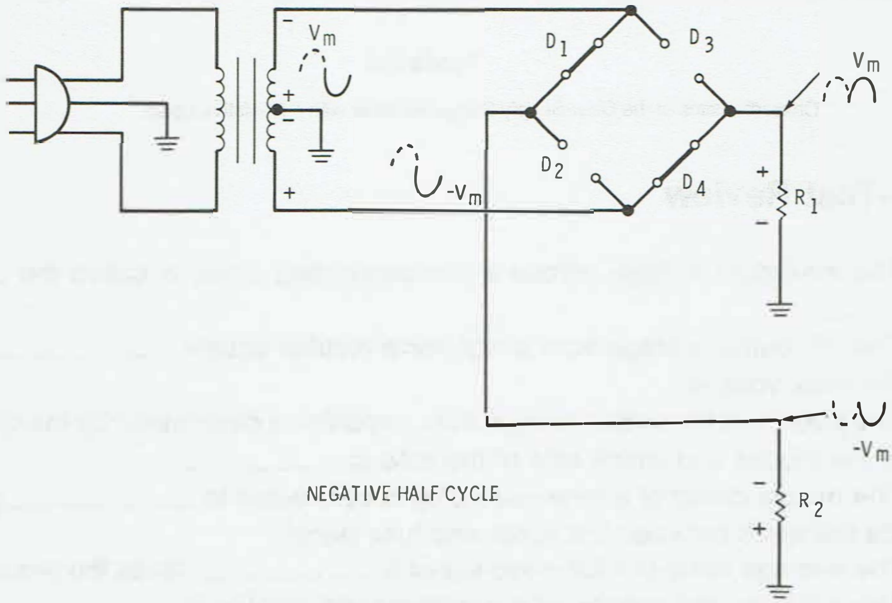
Figure 5-23

A dual supply bridge rectifier.

By combining a bridge rectifier and center-tapped transformer, you obtain the dual supply circuit shown in Figure 5-23. Here, two equal but opposite-polarity output voltages are supplied to the two loads, R_1 and R_2. The operation of the circuit is illustrated in Figure 5-24.



POSITIVE HALF CYCLE



NEGATIVE HALF CYCLE

Figure 5-24

Equivalent circuits for the dual supply bridge rectifier.

During the positive half cycle, diodes D_2 and D_3 are forward biased, and diodes D_1 and D_4 are reverse biased as shown in Figure 5-24A. Note that resistor R_1 is connected across the upper half of the secondary through D_3. Similarly, resistor R_2 is connected across the lower half of the secondary through D_2. During the negative half cycle, diodes D_1 and D_4 are forward biased, and diodes D_2 and D_3 are reverse biased as shown in Figure 5-24B. In this case, resistor R_1 is connected across the lower half of the secondary through diode D_4, while resistor R_2 is connected across the upper half of the secondary through diode D_1. On each half cycle, note that the polarity of the load voltages remain unchanged. Consequently, each output voltage is a full-wave signal. The important parameters for the dual-supply rectifier circuit are summarized in Table 5-3.

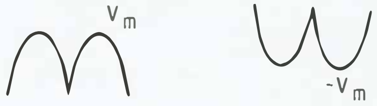
Parameter	
Load voltage/current waveforms.	
Peak inverse voltage	$2V_m$
Peak diode/load current	$\frac{V_m}{R_1}$ (D_3 & D_4) $\frac{V_m}{R_2}$ (D_1 & D_2)
DC load currents	$0.636\ I_m$
DC diode current	$0.318\ I_m$
DC load voltages	$+0.636\ V_m\ -\ 0.636\ V_m$
DC diode voltages	$0.636\ V_m$
Output frequency	$2\ f_{line}$

Table 5-3

Characteristics of the Dual-Supply Bridge Rectifier with a Resistive Load.

Self-Test Review

13. The maximum voltage across a non-conducting diode is called the ____________.
14. The DC output voltage from a half-wave rectifier equals ____________times the peak voltage.
15. The polarity of the output voltage from a rectifier is determined by the direction of the diodes and which side of the load is ____________.
16. The middle prong of a three-wire plug is connected to ____________ground via the wires between the outlet and fuse panel.
17. The average value of a full-wave signal is ____________times the peak value.
18. The peak inverse voltage of a center-tapped rectifier is ____________times the peak value.
19. The frequency of the output voltage from a full-wave rectifier equals ______ times the line frequency.
20. A ____________rectifier uses four diodes.
21. A dual supply rectifier uses a ____________transformer and bridge rectifier.

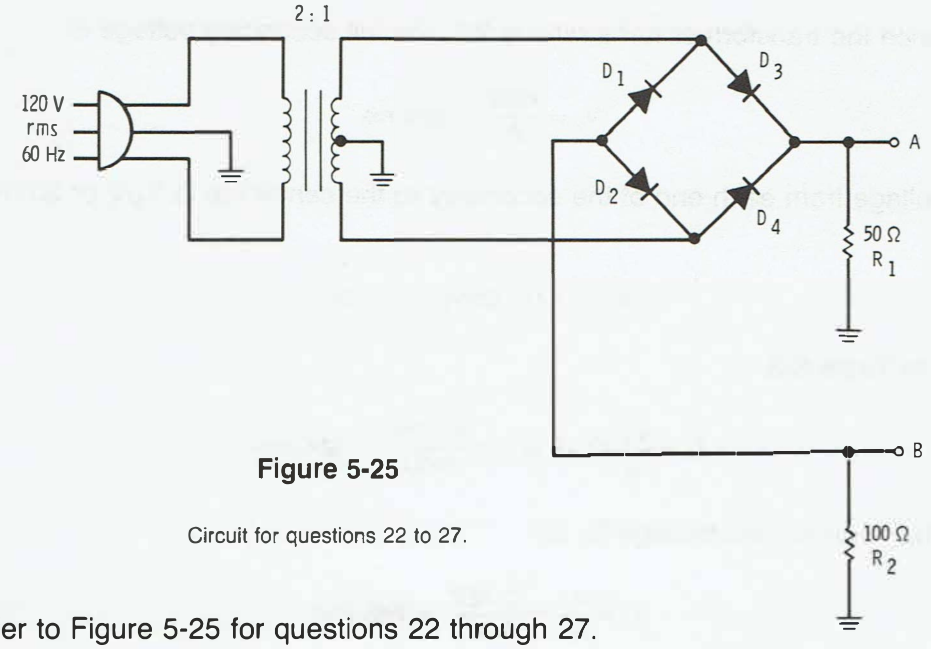


Figure 5-25

Circuit for questions 22 to 27.

Refer to Figure 5-25 for questions 22 through 27.

22. The peak current through D_1 is ____________mA.
23. A DC ammeter connected in series with R_1 would read ____________mA.
24. A DC voltmeter connected from point A to ground would read ____________ V.
25. A DC voltmeter connected from point B to ground would read ____________ V.
26. A DC ammeter connected in series with R_2 would read ____________A.
27. A DC voltmeter connected across D_4 would read ____________V.

Answers

13. peak inverse voltage
14. 0.318
15. grounded
16. earth
17. 0.636
18. 2
19. 2
20. bridge
21. center-tapped
22. 424.2mA
23. 539.6mA
24. 27V
25. −27V
26. 269.9mA
27. 27V

The solutions to questions 22 through 27 follow.

22. Since the transformer turns ratio is 2:1, the full secondary voltage is:

$$V_s = \frac{120V}{2} = 60Vrms$$

The voltage from each end of the secondary to the center tap is $V_S/2$ or 30Vrms. Thus:

$$V_m = 1.414\ (30V) = 42.42V$$

Refer to Table 5-3.

$$I_m = \frac{V_m}{R_2}\ (D_1\ \&\ D_2) = \frac{42.42V}{100\Omega} = 424.2mA$$

23. The peak current through R_1 is:

$$I_m = \frac{V_m}{R_1} = \frac{42.42V}{50\Omega} = 848.4mA$$

Consequently, the average, or DC current is:

$$I_{DC} = 0.636\ Im = 0.636\ (848.4mA) = 539.6mA$$

24. The voltage from point A to ground is the voltage across R_1. Thus:

$$V_A = V_{R1} = 0.636\ V_m = 0.636\ (42.42V) = 27V\ DC$$

25. The voltage from point B to ground is the voltage across R_2. Thus:

$$V_B = V_{R2} = -0.636\ (V_m) = -0.636\ (42.42V) = -27V\ DC$$

26. The peak current through R_2 is:

$$I_m = \frac{V_m}{R_2} = \frac{42.42V}{100\Omega} = 424.2mA$$

Thus, the average or DC current is:

$$I_{DC} = 0.636\ I_m = 0.636\ (424.2mA) = 269.9mA$$

27. Table 5-3 indicates that the DC diode voltages equal 0.636 Vm. Thus:

$$V_{D4} = 0.636\ (42.42V) = 27V$$

SMOOTHING FILTERS

The pulsating DC voltage from a rectifier contains too much AC ripple to be useful for most electronic circuits. For this reason, the rectifier section of a power supply is followed by a smoothing filter as shown in Figure 5-26. The smoothing filter converts the pulsating DC input voltage into a relatively pure DC output voltage. In this section, we will discuss the operation, analysis and design of the most popular types of smoothing filters.

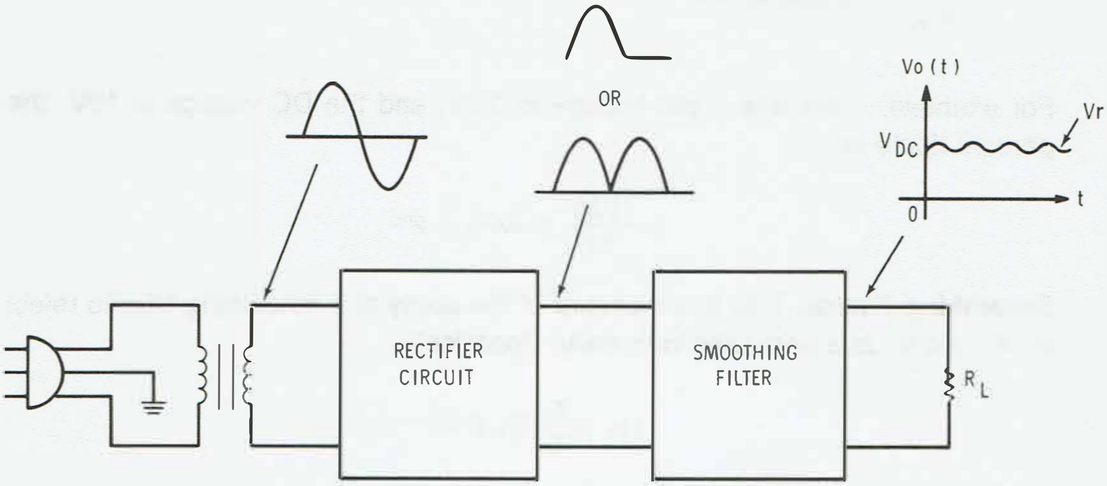


Figure 5-26

The unregulated power supply.

Definitions

The following terms are defined to help you understand the information in this section.

Percent Voltage Regulation. This is a measure of the amount of change in the DC output voltage of a power supply between no-load, $I_L = 0$, and full-load, $I_L = I_{Lmax}$. Specifically:

$$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% \text{ (Eq. 5-11)}$$

For example, if a power supply has a DC output voltage of 12V when the load current is zero, and a DC output voltage of 10.8V when maximum load current flows, the percent voltage regulation is:

$$VR = \frac{12V - 10.8V}{12V} \times 100\% = 10\%$$

Percent Ripple. This is a measure of the ability of a smoothing filter to pass DC and block AC. Stated mathematically:

$$r = \frac{V_r}{V_{DC}} \times 100\% \text{ (Eq. 5-12)}$$

where r = percent ripple
V_r = rms output ripple voltage
V_{DC} = DC output voltage
$\frac{V_r}{V_{DC}}$ = ripple factor

For example, if the rms ripple voltage is 0.2V, and the DC voltage is 10V, the percent ripple is:

$$r = \frac{0.2V}{10V} \times 100\% = 2\%$$

Smoothing Factor. This is a measure of the ability of a smoothing filter to reject an AC signal at a particular frequency. Specifically:

$$\alpha_n = \frac{V_1}{V_2} \text{ (Eq. 5-13)}$$

where α_n = smoothing factor at frequency f_n.
V_1 = AC input voltage at frequency f_n.
V_2 = AC output voltage at frequency f_n.

For example, assume the output voltage of a smoothing filter is 0.1V peak at 60Hz, and 0.01V peak at 120Hz. If in each case the input voltage was 10V peak, the 60Hz and 120Hz smoothing factors would be:

$$\alpha = \frac{10V}{0.1V} = 100 \quad \text{(60Hz)}$$

$$\alpha = \frac{10V}{0.01V} = 1000 \quad \text{(120Hz)}$$

Peak-Type Filter. Ideally, this type of filter provides a DC output voltage equal to the peak value of the input voltage.

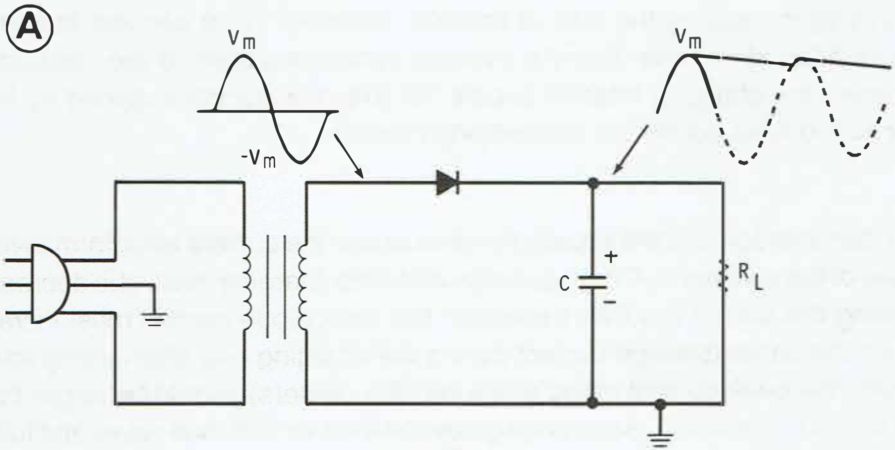
Average-Type Filter. Ideally, this type of filter provides a DC output voltage equal to the average, or DC value, of the input voltage.

In practice, real filters may approximate ideal peak-type or average-type filters. In addition, real filters often have characteristics that fall between those of ideal peak-type and average-type filters.

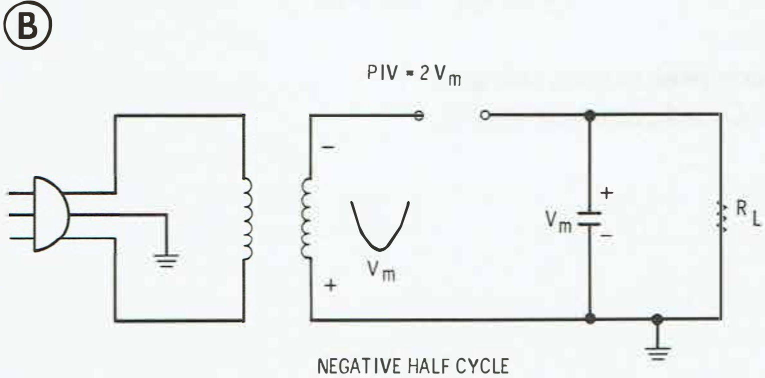
Capacitor Input Filter

The simple capacitor filter in Figure 5-27 approximates the characteristics of an ideal peak-type filter. Assuming an ideal diode, the operation of the filter may be described as follows:

1. At the instant the input voltage swings positive, the diode becomes forward biased. Consequently, current flows, which charges the capacitor to V_m during the first quarter cycle.
2. At the instant the input voltage falls below V_m, the diode becomes reverse biased. With the diode "open," the capacitor begins to discharge through the load, R_L.
3. The capacitor continues to discharge until the input voltage is once again sufficiently positive to forward bias the diode. At this time, the source begins to supply a charging current which causes the capacitor voltage to return to V_m rather quickly. From this point on, the process described in steps 2 and 3 is repeated each cycle.



CIRCUIT



NEGATIVE HALF CYCLE

Figure 5-27

Filtering the output from a half-wave rectifier with a capacitor.
A. Circuit.
B. Negative half-cycle.

From the discussion of RC discharging circuits in Unit 2, it is clear that the voltage across the discharging capacitor is described by:

$$\nu_c(t) = V_m e^{-t/R_L C} \qquad \text{(Eq. 5-14)}$$

Note in Figure 5-27A that the capacitor discharges for a period of time nearly equal to the period of one cycle. Component values are selected to make the $R_L C$ time constant large compared to the period of one cycle. For this reason, the capacitor voltage at the conclusion of the discharge portion of the cycle is only slightly less than V_m. Consequently, the average (DC) load voltage is approximately equal to V_m.

Note in Figure 5-27B that the peak inverse voltage equals $2V_m$, which is **double** the value of an unfiltered half-wave rectifier. Naturally, when selecting a diode for the circuit, you should choose one that has a PIV rating somewhat larger than $2V_m$. If you use a full-wave rectifier with a simple capacitor filter, the peak inverse voltages are the same as those encountered with purely resistive loads.

Equation 5-14 indicates that the ability of the filter to minimize AC ripple can be improved by increasing the size of the filter capacitor. This concept is illustrated in Figure 5-28. Here, note that the average current supplied to the capacitor and load during the charging interval equals the average current supplied by the capacitor to the load during the discharging interval.

Recall that average current equals the area under the current waveform divided by the base of the waveform. Consequently, when the charging interval is decreased by increasing the size of the filter capacitor, the peak diode current must increase to maintain the same average current during the charging and discharging intervals. Naturally, the peak current rating of the rectifier diode(s) should be larger than the peak current in the circuit. As a rough guideline then for both half-wave and full-wave rectifiers with capacitor filters:

$$I_p \geqslant 20\ I_{DC} \qquad \text{(Eq. 5-14)}$$

Where: I_p = diode peak current rating
I_{DC} = DC load current = V_{DC}/R_L

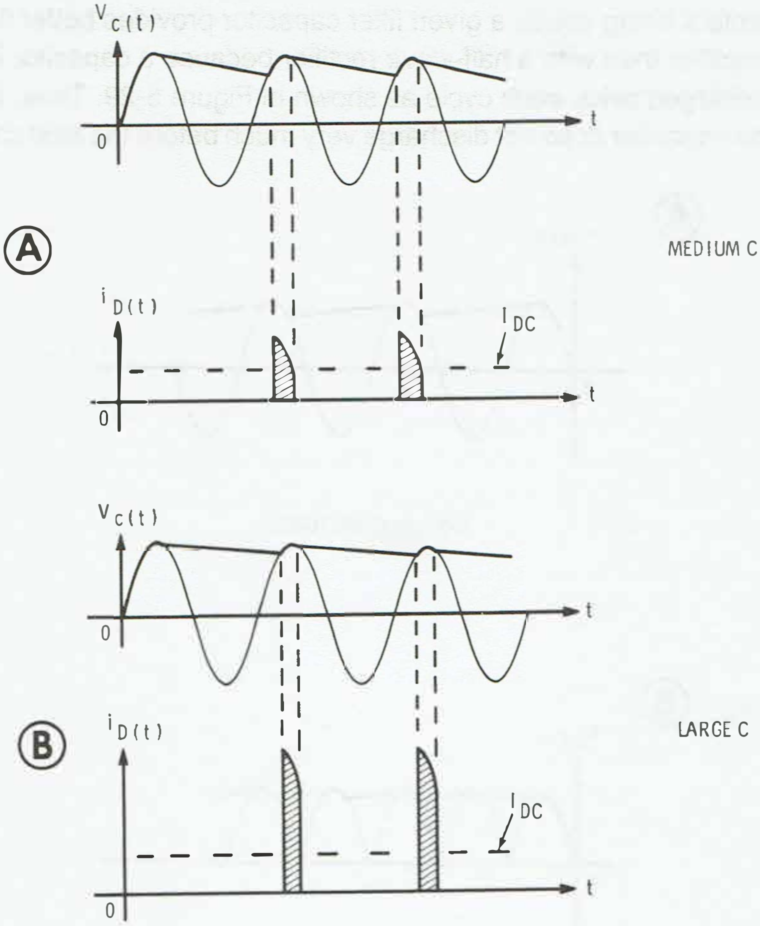
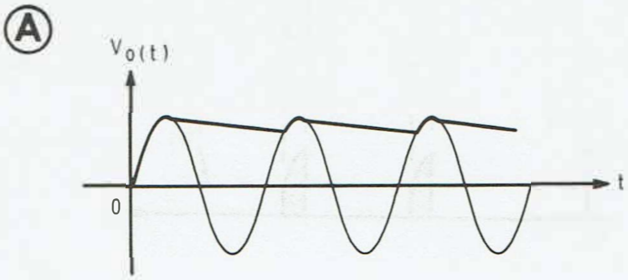


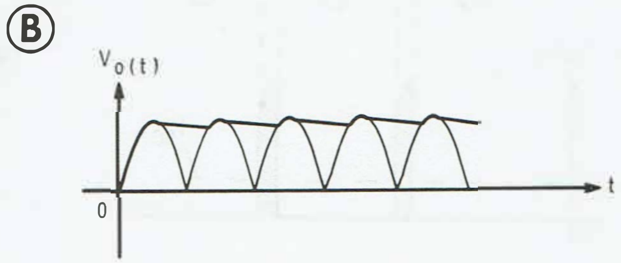
Figure 5-28

Capacitor voltage and diode current in a half-wave rectifier with a capacitor-input filter.
Note that the peak diode current increases with the size of the filter capacitor.

All other factors being equal, a given filter capacitor provides better filtering with a full-wave rectifier than with a half-wave rectifier because a capacitor in a full-wave rectifier is charged twice each cycle as shown in Figure 5-29. Thus, in a full-wave rectifier, the capacitor does not discharge very much before the next charging pulse occurs.



HALF-WAVE RECTIFIER



FULL-WAVE RECTIFIER

Figure 5-29

Comparison of half-wave, and full-wave output voltages with a capacitor-input filter. Note that the full-wave output voltage is easier to filter.

Capacitor Input Filter Analysis and Design Equations

Table 5-4 shows the formulas for analyzing and designing capacitor input filters. These formulas are based on numerous approximations. However, they yield results that are accurate enough for most design problems.

Example 5-8

A full-wave rectifier uses a capacitor-input filter to supply 9V DC at 100mA to a load. Calculate the capacitance required for the filter assuming 8 percent ripple, or less, is acceptable.

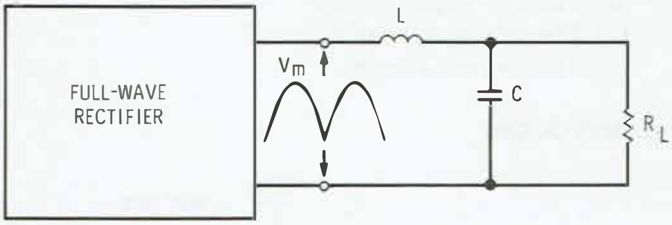
From Ohms law, the effective load resistance is:

$$R_L = \frac{9V}{100mA} = 90\Omega$$

From Table 5-4

$$C_{min} = \frac{2.4 \times 10^{-3}}{r\, R_L} = \frac{2.4 \times 10^{-3}}{0.08\ (90)} = 333.3\ \mu F$$

Naturally, the voltage rating of the capacitor should be larger than the DC load voltage. Also, since the calculated value of capacitance is a minimum value, you would select the next larger standard-size capacitance value. Since 470μF, 50V, electrolytic capacitors, are readily available, we select C = 470μF.



Parameter	Half-Wave Rectifier	Full-Wave Rectifier
DC output voltage	$V_{DC} = V_m \left[1 - \frac{8.34 \times 10^{-3}}{R_L C}\right]$	$V_{DC} = V_m \left[1 - \frac{4.17 \times 10^{-3}}{R_L C}\right]$
RMS ripple voltage	$V_r = \frac{4.8 \times 10^{-3} V_{DC}}{R_L C}$	$V_r = \frac{2.4 \times 10^{-3} V_{DC}}{R_L C}$
Ripple factor	$r = \frac{4.8 \times 10^{-3}}{R_L C}$	$r = \frac{2.4 \times 10^{-3}}{R_L C}$
Minimum capacitance	$C_{min} = \frac{4.8 \times 10^{-3}}{r R_L}$	$C_{min} = \frac{2.4 \times 10^{-3}}{r R_L}$

Table 5-4

Capacitor-input filter analysis and design equations for a 60Hz line frequency.

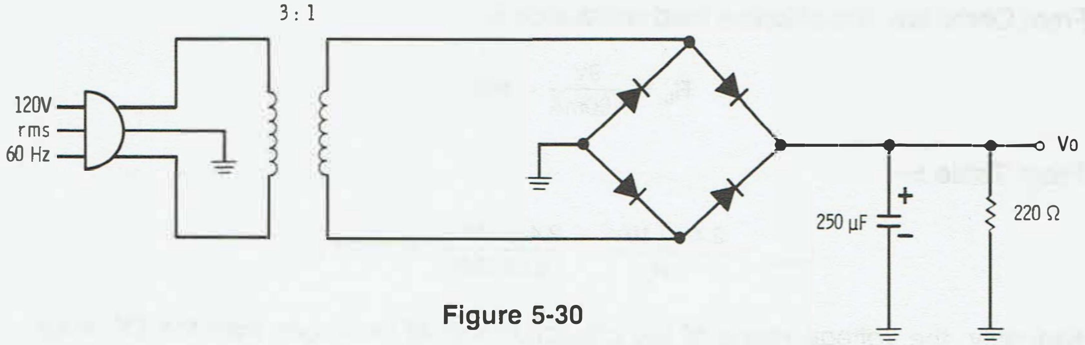


Figure 5-30

Circuit for Example 5-9.

Example 5-9

For the power supply in Figure 5-30 estimate:
(a) The DC output voltage.
(b) The rms ripple voltage.
(c) The percent ripple.

Assuming ideal diodes:

$$V_s = \frac{120V}{3} = 40V \text{ rms}$$

converting from rms to peak,

$$V_m = 1.414\ (40V) = 56.56V$$

from Table 5-4,

$$V_{DC} = V_m\left[1 - \frac{4.17 \times 10^{-3}}{R_LC}\right] = 56.56V\left[1 - \frac{4.17 \times 10^{-3}}{220\ (250)\ 10^{-6}}\right]$$

$$V_{DC} = 56.56V\ (0.9242) = 52.27V$$

$$V_r = \frac{2.4 \times 10^{-3}\ V_{DC}}{R_LC} = \frac{2.4 \times 10^{-3}\ (52.27V)}{220\ (250)\ 10^{-6}} = 2.28V \text{ rms}$$

Thus the percent ripple is:

$$\%\ r = \frac{V_r}{V_{DC}} \times 100\% = \frac{2.28V}{52.27V} \times 100\% = 4.36\%$$

Example 5-10

Assume the diodes in Figure 5-30 are silicon. Rework Example 5-9, taking the diode's turn-on voltage into account.

The diode's turn-on voltage reduces the effective peak voltage. In a full-wave bridge rectifier, two forward biased diodes are always in series with the load. Thus, the effective peak voltage, V_m', is:

$$V_m' = V_m - 2\,V_T = 56.56\text{V} - 2\,(0.7\text{V}) = 55.16\text{V}$$

Consequently:

$$V_{DC} = V_m' \left[1 - \frac{4.17 \times 10^{-3}}{R_L\,C}\right] = 55.16\text{V}\left[1 - \frac{4.17 \times 10^{-3}}{220\,(250)\,10^{-6}}\right]$$

$$V_{DC} = 55.16\text{V}\,(0.9242) = 50.98\text{V}$$

$$V_r = \frac{2.4 \times 10^{-3}\,V_{DC}}{R_L\,C} = \frac{2.4 \times 10^{-3}\,(50.98\text{V})}{220\,(250)\,10^{-6}} = 2.22\text{V rms}$$

$$\%\,r = \frac{V_r}{V_{DC}} \times 100\% = \frac{2.22\text{V}}{50.98\text{V}} \times 100\% = 4.36\%$$

Initial Surge Current

In Unit 2, you learned that capacitor voltage cannot change instantaneously. For this reason, when power is first applied to a circuit containing an uncharged capacitor, the capacitor acts like a short circuit. Consequently, the initial charging, or surge, current may be quite large. In rectifier circuits that use capacitor-input filters, the initial surge current is limited by the value of the input voltage and the equivalent resistance in series with the capacitor. This equivalent resistance consists of the forward resistance of the diodes and the effective transformer resistance. Since the equivalent series resistance is small, the initial surge current may be large enough to destroy the rectifier diodes.

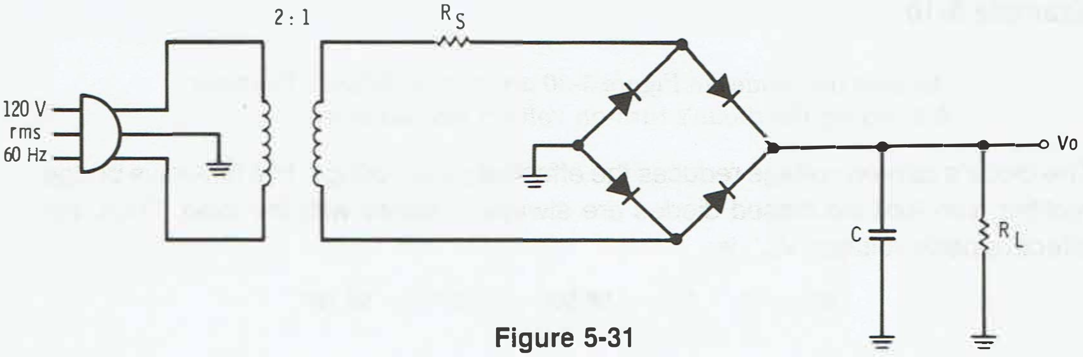


Figure 5-31

Limiting surge current with a surge limiting resistor.

In order to limit the initial surge current to a safe value, a "surge limiting resistor" can be placed in series with the capacitor and load as shown in Figure 5-31. In the worst-case the circuit is plugged in when the secondary transformer voltage is maximum. Thus, the worst-case surge current is approximately:

$$I_s = \frac{V_m}{R_S} \quad \text{(Eq. 5-15)}$$

Example 5-11

Calculate the value of the surge limiting resistor in Figure 5-31 in order to limit the initial surge current to 3A or less.

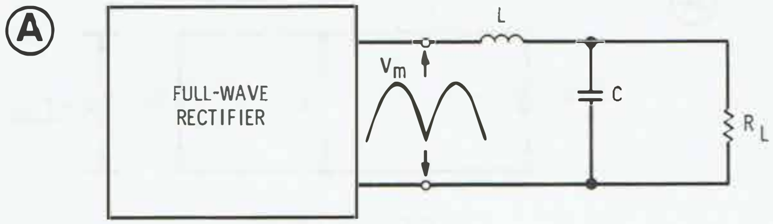
$$V_s = \frac{120V}{2} = 60Vrms$$

$$V_m = 1.414\ (60V) = 84.84V$$

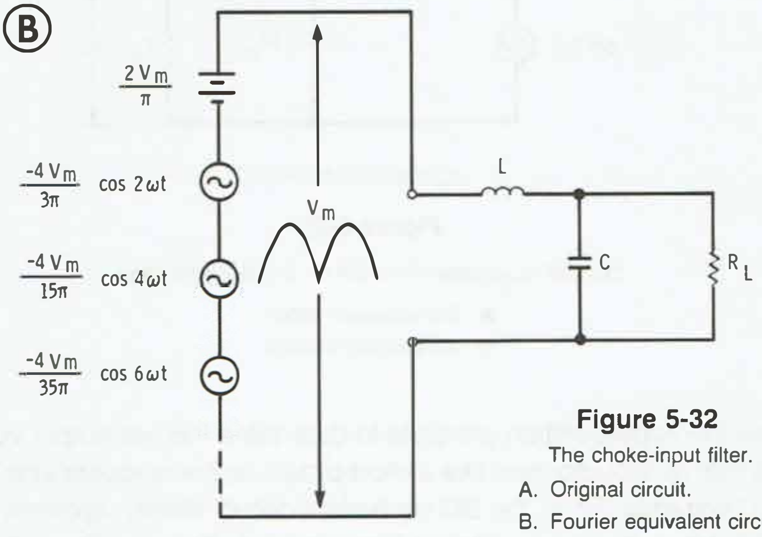
$$R_s = \frac{Vm}{I_s} = \frac{84.84V}{3A} = 28.28\Omega$$

Choke-Input Filter

Capacitor-input filters are characterized by large surge and peak diode currents. However, an inductor connected in series with the filter capacitor and load greatly minimizes these problems, as shown in Figure 5-32A. For obvious reasons, the resulting LC filter is called a choke-input filter. This type of smoothing filter is a popular choice for circuits that require relatively large DC load currents. Consequently, choke-input filters are normally used with full-wave rectifiers.



ORIGINAL CIRCUIT


Ⓑ
$\frac{2V_m}{\pi}$
$\frac{-4V_m}{3\pi}$ cos 2ωt
$\frac{-4V_m}{15\pi}$ cos 4ωt
$\frac{-4V_m}{35\pi}$ cos 6ωt
V_m
L
C
R_L

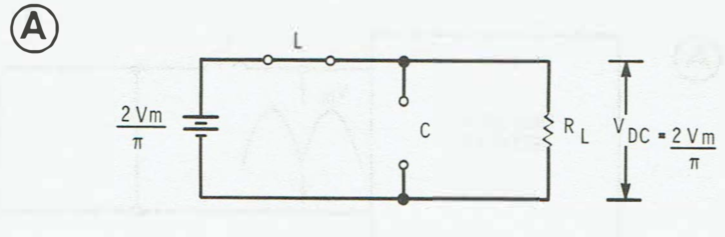
FOURIER EQUIVALENT CIRCUIT

Figure 5-32
The choke-input filter.
A. Original circuit.
B. Fourier equivalent circuit.

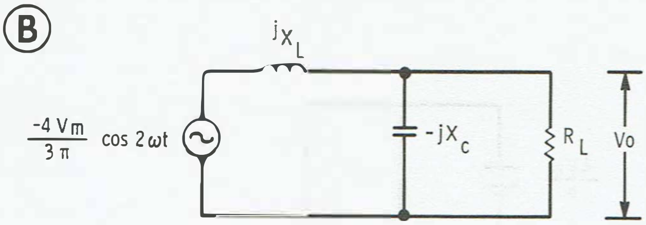
Since the output voltage of a full-wave rectifier is a complex waveform, it may be described by a Fourier series. Specifically:

$$v(t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos 2\omega t - \frac{4V_m}{15\pi}\cos 4\omega t - \frac{4V_m}{35\pi}\cos 6\omega t \ldots \text{(Eq. 5-16)}$$

Equation 5-16 suggests the equivalent circuit shown in Figure 5-32B. Here, the full-wave input voltge is represented as the series combination of a DC voltage source, equal to the average value of the input voltage, and an infinite series of harmonic, AC voltage sources.



DC EQUIVALENT CIRCUIT



AC EQUIVALENT CIRCUIT

Figure 5-33

DC and AC equivalent circuits for a choke-input filter.

A. DC equivalent circuit.
B. AC equivalent circuit.

You can use the superposition principle to determine the net output voltage of the filter. Recall that an inductor acts like a short circuit, and a capacitor acts like an open circuit for DC voltages. Thus, the DC equivalent circuit, ideally, appears as shown in Figure 5-33A. Here, the coil resistance is assumed to be zero. Thus, the DC output voltage, ideally, equals the average (DC) value of the input voltage. Consequently, the choke-input filter is an example of an **average-type** smoothing filter. If the coil resistance is too large to be neglected, the DC output voltage will equal the DC input voltage minus the drop across the coil resistance.

The AC equivalent circuit, for the first AC voltage source, is illustrated in Figure 5-33B. Here, component values are selected so that:

$$|X_C| << R_L$$

$$|X_L| >> |X_C|$$

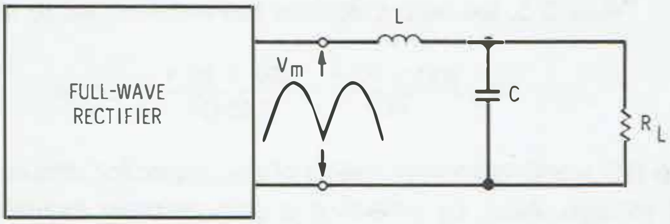
Since $|X_C| << R_L$ the parallel combination of $|X_C|$ and R_L approximately equals $|X_C|$. Also, since $|X_L| >> |X_C|$ most of the AC input voltage will be dropped across the coil. Thus, very little AC voltage appears across the load.

The filter is even more effective in blocking the remaining AC terms for two reasons. First, the amplitudes of the higher order harmonics are small compared to the amplitude of the first AC term. Consequently, the filter output voltages due to these harmonics are small. Secondly, $|X_L|$ increases and $|X_C|$ decreases with increasing frequency. Therefore, the proportion of input voltage dropped across the coil increases with frequency.

For these reasons, most of the net AC ripple voltage appearing at the output of the filter is due to the first AC term. For purposes of analysis and design then, the effects of the higher order harmonics can usually be neglected.

Choke-Input Filter Analysis and Design Equations

Approximate formulas for the analysis and design of choke-input filters are summarized in Table 5-5. Note that for proper operation of the filter a minimum value of inductance is required. This minimum inductance value is referred to as the **critical inductance**.



Parameter	Full-wave Rectifier
DC output voltage (ideal)	$V_{DC} = 0.636\ V_m$
Actual DC output voltage	$V_{DC} = \dfrac{0.636\ V_m\ (R_L)}{R_C + R_L}$ or $V_{DC} = 0.636\ V_m - I_{DC}\ R_C$
Ripple factor	$r = \dfrac{830 \times 10^{-9}}{LC}$
Minimum inductance	$L_{min} = \dfrac{R_L}{1000}$
Required capacitance	$C = \dfrac{830 \times 10^{-9}}{rL}$
Smoothing factor (ideal)	$\alpha = 5.68 \times 10^5\ LC - 1$

Table 5-5
Choke-input filter analysis and design equations for a 60Hz line frequency, full-wave rectifier.

Example 5-12

> *Design a choke-input filter for the output of a full-wave rectifier. The DC load voltage is 250V at 0.5A. Also, the ripple factor should be 3% or less.*

Since $V_{DC} = 250V$ and $I_{DC} = 0.5A$, the effective load resistance is:

$$R_L = \frac{V_{DC}}{I_{DC}} = \frac{250V}{0.5A} = 500\Omega$$

The minimum inductance required is therefore:

$$L_{min} = \frac{R_L}{1000} = \frac{500}{1000} = 0.5H$$

Good design practice dictates that we select a commercially available choke with an inductance greater than 0.5H. Consulting a manufacturer's catalog we choose a 1H inductor whose DC resistance is 40Ω.

Referring to Table 5-5, we next calculate the required value of capacitance.

$$C = \frac{830 \times 10^{-9}}{r\,L} = \frac{830 \times 10^{-9}}{0.03\,(1)} = 27.7\mu F$$

Clearly, the DC working voltage rating of the capacitor should be larger than the 250V load voltage. Also, by selecting a commercially available capacitor larger than the calculated value, we can be confident that the ripple factor will be less than 3%. Referring to a manufacturer's catalog then, we choose a 47μF, 450V electrolytic capacitor for the design.

Example 5-13

> *Show how to determine the specifications for the rectifier and transformer sections of the power supply in Example 5-12.*

The rectifier diodes should be able to conduct an average, DC, current at least equal to the DC load current. Similarly, the PIV rating of the diodes should exceed V_m for a bridge rectifier, and $2V_m$ for a center-tapped rectifier. The choice of a bridge rectifier reduces the cost of the transformer and is, therefore, recommended for the design. Also, bridge rectifiers are available as "single unit" packages, which simplifies power supply construction. Consequently, we recommend a single-package bridge unit for the rectifier portion of the power supply.

To determine the required turns ratio for the transformer, we begin by considering the DC voltage requirements. Specifically:

1. The load requires 250V DC.
2. Since the choke has a resistance of 40Ω, the DC voltage drop across the choke is 0.5A (40Ω) or 20V DC.
3. the DC voltage dropped across the diodes in the bridge rectifier is approximately 2(0.7V) or 1.4V DC.
4. The transformer has an effective DC resistance that depends upon the resistance of the windings, turns ratio, source impedance, and several other factors. An "exact" calculation for this resistance is rather complex. For this example, we will assume the effective transformer resistance is 100Ω. Consequently, the DC voltage drop is approximately 0.5A (100Ω) or 50V DC.

Adding up the various DC voltages, we have:

$$V_{DC} = 250V + 20V + 1.4V + 50V = 321.4V$$

Since $V_{DC} = 0.636$ Vm, for a full-wave rectifier, the required peak voltage for the transformer secondary is:

$$V_P = \frac{321.4V}{0.636} = 505.3V$$

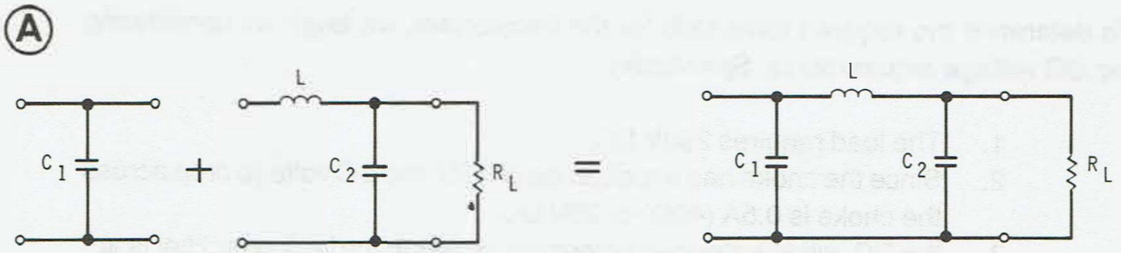
converting to rms:

$$V_{rms} = 0.707\,(505.3V) = 357.3V \text{ rms.}$$

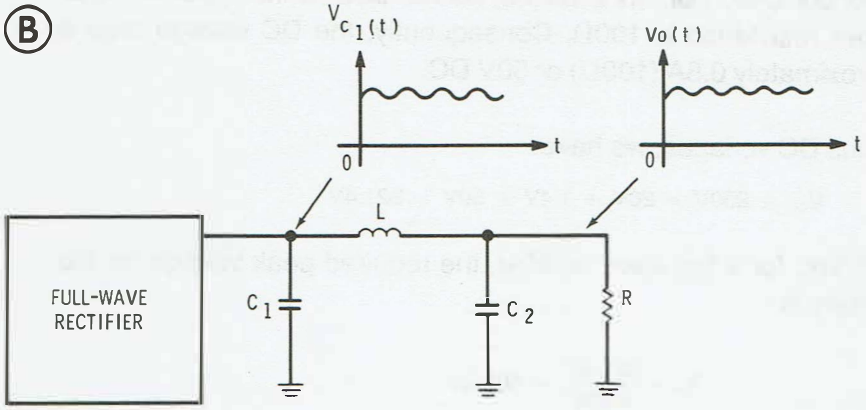
Thus, the required turns ratio is 120/357.3 or approximately 1:3. Naturally, the transformer selected for the design must be capable of supplying the required load current.

Power supplies similar to the one in Example 5-13 were popular during the era of vacuum tubes. Due to the uncertainty in the value of the effective transformer resistance, and other factors, the power supply would often be built and then modified to meet the specific requirements of the load.

The principle advantages of a choke-input filter compared to a capacitor input filter are improved regulation and ripple reduction. Disadvantages include relatively highcost and weight. Today, numerous three-terminal IC voltage regulators are readily available that have excellent ripple reduction characteristics. Consequently, by using a simple capacitor filter in conjunction with a three-terminal IC regulator, there is usually no need for a choke-input filter.



FORMING AN INDUCTOR π - TYPE FILTER



RIPPLE REDUCTION

Figure 5-34

A π-type filter that uses an inductor.

A. Forming an inductor π-type filter.
B. Ripple reduction.

Cascaded Filter Sections

The ripple voltage across a capacitor-input filter can be further reduced by following the capacitor with a choke-input filter as shown in Figure 5-34A. Component values are chosen so that the following inequalities are satisfied.

$$|X_{C_2}| << R_L$$

$$|X_L| >> |X_{C_2}|$$

$$|X_{C_1}| << |X_L|$$

Assuming the inequalities are satisfied, the analysis and design of the filter is rather straightforward.

Example 5-14

The output of a full-wave rectifier that has a capacitor input filter, C_1, consists of 12V DC with 1V rms ripple. To further reduce the ripple voltage, an LC section is added as shown in Figure 5-35. Calculate values of L and C that will reduce the ripple voltage by a factor of 100 or more.

From Table 5-5.

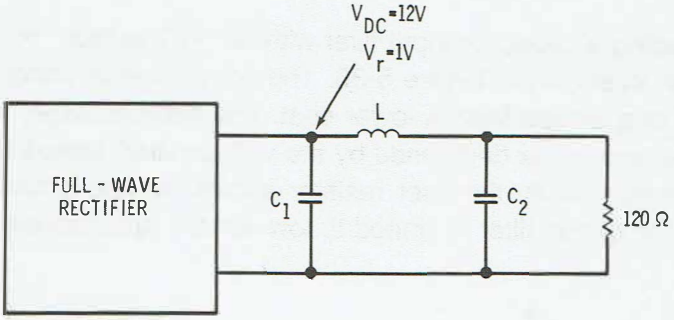


Figure 5-35

Circuit for Example 5-14.

$$L_{min} = \frac{R_L}{1000} = \frac{120}{1000} = 0.12\text{H}$$

Consulting a manufacturer's catalog, we select a 0.5H choke for the filter. The manufacturer indicates that this particular choke has a DC resistance of 15Ω.

Table 5-5 indicates that the smoothing factor of a choke-input filter is:

$$\alpha = 5.68 \times 10^5\ LC - 1$$

Solving for C yields

$$C = \frac{\alpha + 1}{5.68 \times 10^5\,(L)} = \frac{101}{5.68 \times 10^5\,(0.5)} = 355.6\ \mu\text{F}$$

To ensure that the smoothing factor is at least 100, we select a standard value capacitor larger than the calculated value. So, we chose a 470μF, 25V electrolytic capacitor for the filter.

Example 5-15

Predict the DC load voltage and rms ripple voltge for the filter designed in Example 5-14.

The DC input voltge divides between the choke and load resistance. Applying the voltage division principle:

$$V_{OUTDC} = \frac{12V\ (120\Omega)}{15\Omega + 120\Omega} = 10.67V$$

Assuming the smoothing factor is 100:

$$V_{OUTAC} = \frac{V_{INAC}}{100} = \frac{1V}{100} = 10mV \text{ rms}$$

By cascading a capacitor-input filter with an "RC section" you obtain a resistor π-type filter as shown in Figure 5-36. The advantage of using a resistor instead of a choke in a π-type filter is lower cost. The disadvantage, of course, is that the DC output voltage is decreased by the voltage drop across the resistor. Also, the power dissipated by the filter resistor serves no useful function. Consequently, the resistor π-type filter is limited to low current applications.

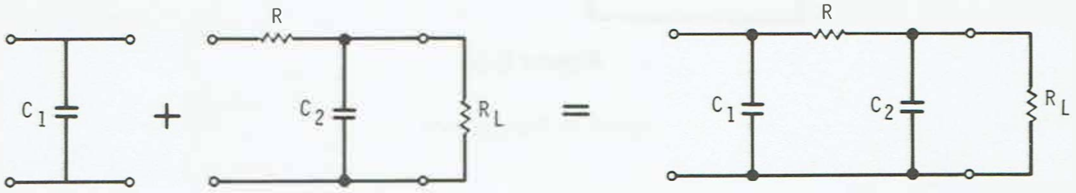


Figure 5-36

Forming a resistor π-type filter.

In Figure 5-36 component values are chosen so that:

$$|X_{C2}| << R_L$$

$$R >> |X_{C2}|$$

$$|X_{C1}| << R$$

The filtering should be effective, assuming that inequalities are satisfied. Incidentally, the DC output voltage and smoothing factor for the "RC section", assuming a full-wave rectifier with a 60Hz line frequency, are:

$$V_{DC} = \frac{0.636\ V_m R_L}{R + R_L} \qquad \text{(Eq. 5-17)}$$

$$\alpha = 754RC \qquad \text{(Eq. 5-18)}$$

As a final example, we will consider the design of a filter composed of a choke-input and RC section as shown in Figure 5-37.

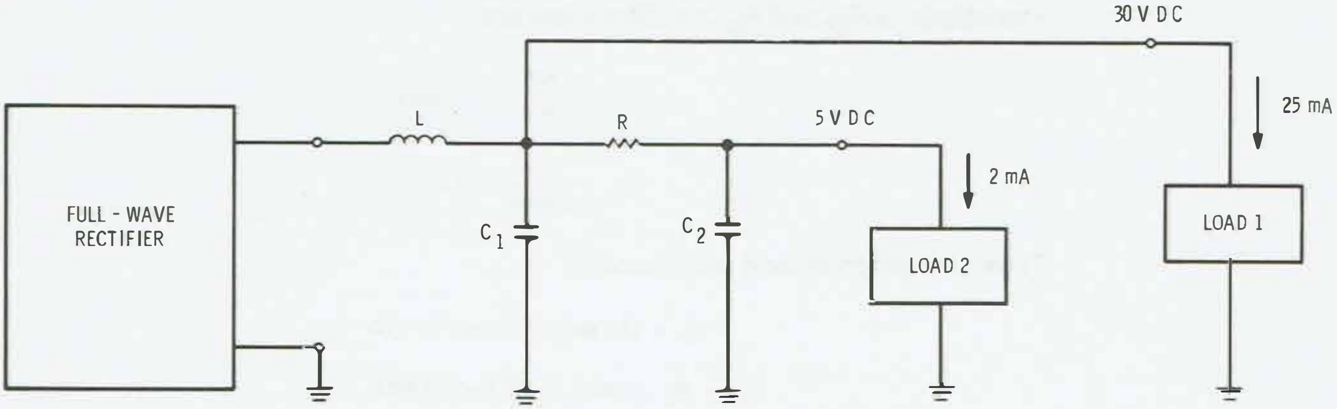


Figure 5-37

Circuit for Example 5-16.

Example 5-16

> *The two-stage filter in Figure 5-37 supplies two different loads. The first load requires 30V DC at 25mA, with an rms ripple voltage no greater than 1.6V. The second load requires 5V DC at 2mA with a maximum rms ripple voltage of 35mV. Calculate the values of the required components, assuming the output from the rectifier consists of 34V DC with 16V rms ripple.*

Starting with the RC section, we note that the DC current through R is 2mA. Similarly, the DC voltage across R is 30V – 5V or 25V. Thus:

$$R = \frac{25\text{V}}{2\text{mA}} = 12.5\text{k}\Omega$$

The smoothing factor for the RC section is the ratio of the AC input voltge to the AC output voltge. Therefore:

$$\alpha = \frac{1.6\text{V}}{35\text{mV}} = 45.7$$

Since $\alpha = 754RC$ for the RC section, the required value of C is:

$$C_2 = \frac{\alpha}{754R} = \frac{45.7}{754\,(12.5\text{k}\Omega)} = 4.8\mu\text{F}$$

Consequently, a standard 10µF, 16V electrolytic capacitor is selected for the filter.

In the LC section, to calculate the minimum value of inductance, you need to estimate the effective load resistance of the LC filter. If you assume the capacitors in Figure 5-37 to be open, you can see:

$$R_L = (R + R_{L2}) \parallel (R_{L1})$$

The values of R_{L2} and R_{L1} via Ohm's law are:

$$R_{L2} = \frac{5V}{2mA} = 2.5k\Omega$$

$$R_{L1} = \frac{30V}{25mA} = 1.2k\Omega$$

Thus, the effective load resistance is:

$$R_L = (12.5k\Omega + 2.5k\Omega) \parallel 1.2k\Omega$$

$$R_L = 15k\Omega \parallel 1.2k\Omega = 1.11k\Omega$$

The minimum inductance is therefore:

$$L_{min} = \frac{R_L}{1000} = \frac{1.11k}{1000} = 1.11\ H$$

Consequently, a commercially available 2.5H inductor is selected for the filter.

Since the DC input voltage to the LC filter is 34V and the DC output voltage is 30V, the choke must drop 4V. Also, the DC current through the choke is 27mA. Thus, the required choke resistance is:

$$R_C = \frac{4V}{27mA} = 148.1\Omega$$

Since it is unlikely you would find a commercially available, 2.5H, choke whose resistance was exactly 148.1Ω, you would choose one whose resistance was **less than** 148.1Ω. The difference could easily be made up with a series resistor. The LC section has an AC input of 16V rms and a maximum AC output of 1.6V rms. The desired smoothing factor is therefore:

$$\alpha = \frac{16V}{1.6V} = 10$$

From Table 5-5,

$$\alpha = 5.68 \times 10^5\ LC - 1$$

Solving for C yields

$$C = \frac{(\alpha + 1)}{5.68 \times 10^5\ L}$$

Thus, the minimum value of C_1 is:

$$C_1 = \frac{10 + 1}{5.68 \times 10^5\ (2.5)} = 7.7\mu F$$

You could use standard 10µF, 50V electrolytic capacitor.

Finally, you should calculate the reactances of the inductor and capacitors, at 120Hz, to make sure that the inequalities given previously are satisfied. In most smoothing filter design problems, a 5:1 rule is sufficient for defining large and small. Thus:

$$|X_L| = \omega L = 754(2.5) = 1.89\text{k}\Omega$$

$$|X_{C1}| = |X_{C2}| = \frac{1}{\omega C} = \frac{1}{754\,(10)\,(10^{-6})} = 132.6\Omega$$

Note that:

$$|X_L| > 5\,|X_{C1}|$$

$$|X_{C2}| < 5\,R_{L2}$$

$$|X_{C1}| < 5\,R_L\ \text{(effective)}$$

Self-Test Review

28. The capacitor input smoothing filter is an example of a ____________-type filter.
29. The choke-input smoothing filter illustrates an ____________-type filter.
30. The ripple factor is the ratio of AC output voltage to the ____________ (DC or AC) output voltage.
31. The smoothing factor is the ratio of AC input voltage to the ____________ (DC or AC) output voltage.
32. A full-wave rectifier drives a 10kΩ resistor in parallel with a filter capacitor. The minimum value of capacitance required so that the percent ripple is 3% or less is ______________.
33. A choke-input filter is connected between a 10kΩ load and the output of a full-wave rectifier. The critical inductance is therefore __________ H.
34. Surge limiting resistors are often used with ____________filters.
35. Most of the ripple appearing at the output of a smoothing filter is due to the ____________AC term in the Fourier series describing the input voltage.

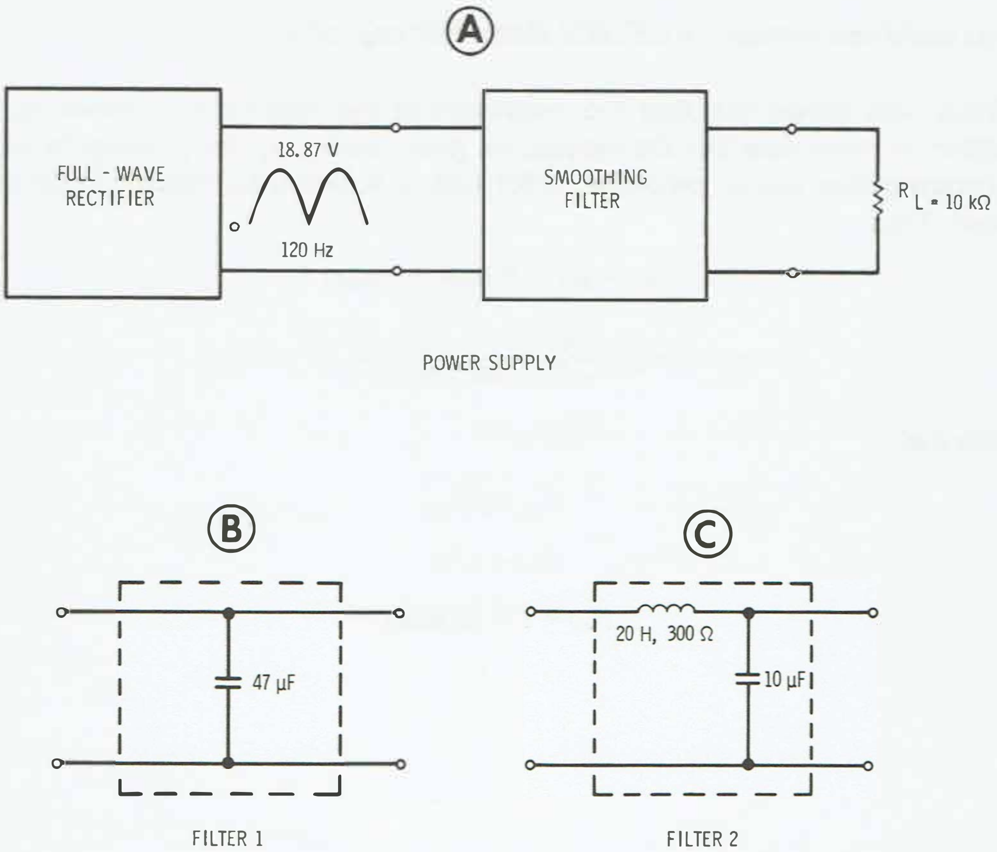


Figure 5-38

Circuit for questions 36 to 41.

Refer to Figure 5-38 for questions 36 through 41.

36. The DC input voltage to the filter is ____________V.
37. The rms value of the AC input voltage to the filter is ____________V.
38. Assuming filter number 1 is used in Figure 5-37A the DC output voltage is ____________V.
39. In question 38, the rms ripple voltage across the load is ____________V. Similarly, the percent ripple is ____________%.
40. Assuming filter number 2 is used in Figure 5-37A, the DC output voltage is ___ V.
41. In question 40, the rms ripple voltage across the load is ____________mV. Similarly, the percent ripple is ____________%.

Answers

28. peak
29. average
30. DC
31. AC
32. 8uF
33. 10H
34. capacitor-input
35. first
36. 12V
37. 5.66V
38. 18.7V
39. 0.0955V, 0.51%
40. 11.65V
41. 48.35mV, 0.415%

The solutions to appropriate questions follow.

32. From Table 5-4, $C_{min} = \frac{2.4 \times 10^{-3}}{r R_L}$ Thus:

$$C_{min} = \frac{2.4 \times 10^{-3}}{0.03\,(10)\,(10^3)} = 8\mu F$$

36. From Equation 5-16, the DC voltage is $\frac{2V_m}{\pi}$. Thus:

$$V_{DC} = \frac{2V_m}{\pi} = 0.636\,V_m = 0.636\,(18.87V) = 12V$$

37. From Equation 5-16, the peak value of the first AC term is $\frac{4\,V_m}{3\pi}$. Thus:

$$V_P = \frac{4V_m}{3\pi} = \frac{4\,(18.87V)}{3\pi} = 8V \text{ peak}$$

Converting to rms yields:

$$V_{rms} = 0.707\,(8V) = 5.66V$$

38. From Table 5-4, $V_{DC} = V_m \left[1 - \frac{4.17 \times 10^{-3}}{R_L C}\right]$. Thus:

$$V_{DC} = 18.87V \left[1 - \frac{4.17 \times 10^{-3}}{10(10^3)(47)(10^{-6})}\right] = 18.87V\,(0.991)$$

$$V_{DC} = 18.7V$$

39. From Table 5-4, $V_r = \dfrac{2.4 \times 10^{-3}\, V_{DC}}{R_L\, C}$ Thus:

$$V_r = \frac{2.4 \times 10^{-3}\,(18.7V)}{10(10^3)(47)(10^{-6})} = 0.0955V \text{ rms}$$

$$\%r = \frac{V_r}{V_{DC}} \times 100\% = \frac{0.0955}{18.7} \times 100\% = 0.51\%$$

40. From Table 5-5, $V_{DC} = \dfrac{0.636\, V_m\,(R_L)}{R_C + R_L}$. Thus:

$$V_{DC} = \frac{0.636\,(18.87V)\,(10K\Omega)}{10.3K\Omega} = 11.65V$$

41. From Table 5-5, $r = \dfrac{830 \times 10^{-9}}{LC}$ Thus:

$$r = \frac{830 \times 10^{-9}}{20(10)(10^{-6})} = 4.15 \times 10^{-3}$$

Since $r = \dfrac{V_r}{V_{DC}}$, $V_r = r\ V_{DC}$. Therefore:

$$V_r = 4.15 \times 10^{-3}\,(11.65V) = 48.35 \text{ mV rms}$$

$$\%\, r = r \times 100\% = 4.15 \times 10^{-3} \times 100\% = 0.415\%$$

SUMMARY

The following points summarize the key ideas of this unit.

Transformers are used to step up or step down currents and voltages, provide isolation between two circuits, and match impedances. Since the efficiency of modern transformers is very high, little error is introduced by assuming ideal operation.

Commercial transformers, suitable for power supplies, often have two primary windings that may be connected in series or parallel, to operate the transformer from either a 110V or 220V supply. In addition, many commercial transformers have one or more tapped secondary windings. With tapped windings, a given transformer can be used in a large number of circuits.

Ideally, diodes permit current to flow through them in one direction only. In a given circuit, the component values and source voltage determine how closely a real diode approximates an ideal diode. For near ideal operation, the following conditions must be met.

1. The peak source voltage should be large compared to the diode's turn-on voltage.
2. The load resistance should be large compared to the diode's forward resistance.
3. The load resistance should be small compared to the diode's reverse resistance.

Consequently, component values should be selected so that the conditions just stated are satisfied.

Rectifier circuits convert AC into pulsating DC. The most frequently encountered rectifier configurations include:

1. Half-wave rectifiers.
2. Full-wave center-tapped rectifiers.
3. Full-wave bridge rectifiers.
4. Dual supply rectifiers.

The various tables provided in this unit summarize the characteristics of the four configurations. Generally speaking, the half-wave rectifier is suitable only for low current applications.

Smoothing filters convert the pulsating DC voltage from the rectifier into a relatively pure DC voltage, suitable for the load. A peak-type filter provides a DC output voltage approximately equal to the peak value of the input voltage. Similarly, an average-type filter provides a DC output voltage approximately equal to the DC average value of the input voltage.

The analysis and design equations for capacitor-input and choke-input filters are summarized in Table 5-4 and Table 5-5 respectively. Examples illustrating the analysis and design of a variety of smoothing filters were provided in this unit.

UNIT EXAMINATION

The following multiple choice examination is designed to test your understanding of the material presented in this unit. Place a check beside the multiple choice answer (A, B, C, or D) that you feel is most correct. When you have completed the examination, compare your answers with the correct ones that appear after the exam.

Refer to Figure 5-39 for questions 1 through 4.

1. V_{CD} equals:
   - A. 10V rms.
   - B. 30V rms.
   - C. 3.3V rms.
   - D. 90V rms.
2. The effective resistance between terminals A and B equals:
   - A. 0Ω.
   - B. 4.5KΩ.
   - C. 500Ω.
   - D. 40.5KΩ.
3. The primary current equals:
   - A. 6.67mA rms.
   - B. 2.22mA rms.
   - C. 10mA rms.
   - D. 20mA rms.
4. The power supplied to the primary circuit is approximately:
   - A. 0.2W
   - B. 22.2mW.
   - C. 0W.
   - D. 62.5mW.

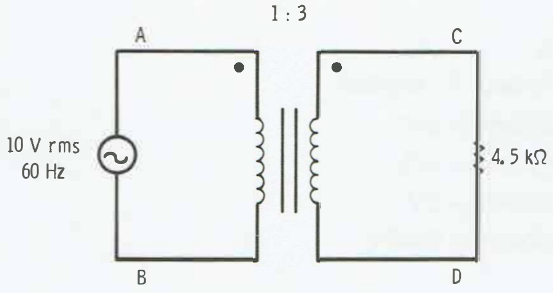


Figure 5-39

Circuit for questions 1 to 4.

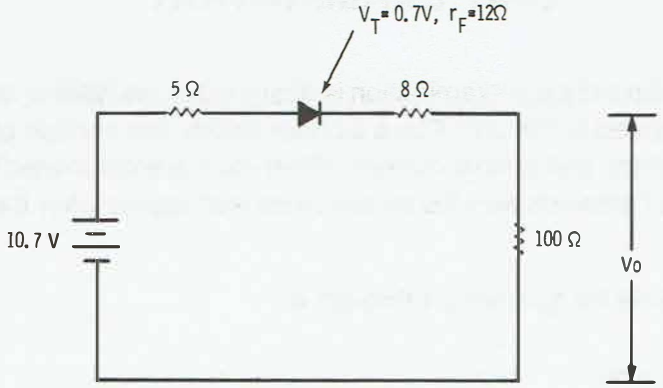


Figure 5-40

Circuit for questions 5 to 8.

Refer to Figure 5-40 for questions 5 through 8.

5. Assuming an ideal diode, V_o equals:
   - A. 0V.
   - B. 9.47V.
   - C. 8.56V.
   - D. 10V.
6. Taking the real diode characteristics into account, V_o equals:
   - A. 0V.
   - B. 9.47V.
   - C. 8.56V.
   - D. 8V.
7. The DC diode current equals:
   - A. 0A.
   - B. 0.8A.
   - C. 80mA.
   - D. 94.7mA.
8. If the diode is reversed, V_o equals:
   - A. Approximately 0V.
   - B. Approximately 10V.
   - C. Approximately 8V.
   - D. Approximately 9.47V.

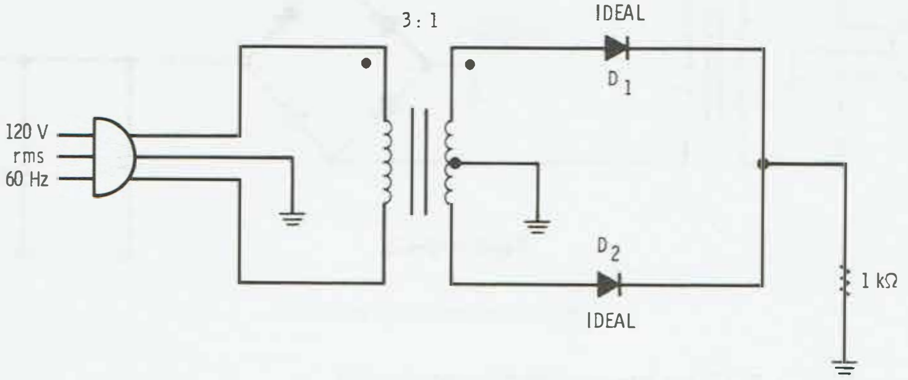


Figure 5-41

Circuit for questions 9 to 12.

Refer to Figure 5-41 for questions 9 through 12.

9. The peak current through D_1 equals:
   - A. 20mA.
   - B. 10mA.
   - C. 28.28mA.
   - D. 14.14mA.
10. The DC load current equals:
    - A. 18mA.
    - B. 20mA.
    - C. 9mA.
    - D. 10mA.
11. The DC current through D_2 equals:
    - A. 10mA.
    - B. 20mA.
    - C. 18mA.
    - D. 9mA.
12. The peak inverse voltage across D_1 equals:
    - A. 20V.
    - B. 28.28V.
    - C. 14.14V.
    - D. 56.56V.

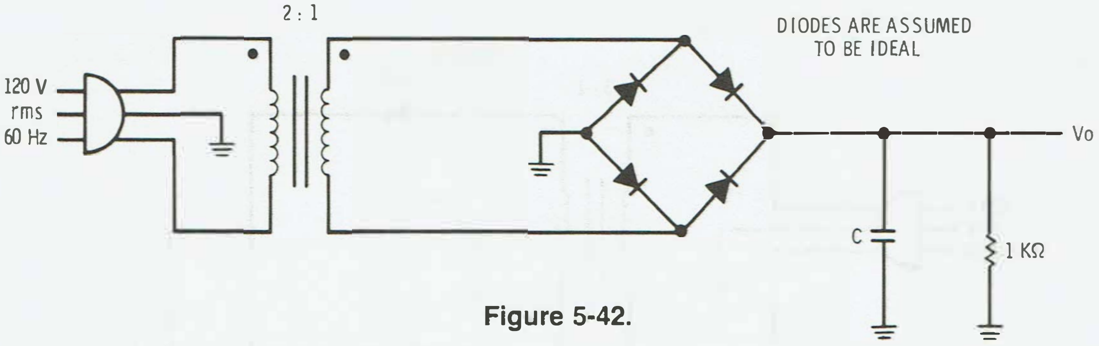


Figure 5-42.

Circuit for questions 13 to 16.

Refer to Figure 5-42 for questions 13 through 16.

13. Assuming a 2% ripple factor, the minimum value for C is:
    - A. 240μF.
    - B. 120μF.
    - C. 2400μF.
    - D. 12μF.

14. The DC output voltge equals:
    - A. 81.9V.
    - B. 60V.
    - C. 120V.
    - D. 30V.
15. The rms ripple voltage equals:
    - A. 3.28V.
    - B. $0.636V_m$ V.
    - C. $0.318V_m$ V.
    - D. 1.64V.
16. The working voltage rating for C must be at least:
    - A. 81.9V.
    - B. 60V.
    - C. 30V.
    - D. 70V.

The output of a full-wave rectifier consists of 80V DC with 24V rms ripple. A choke-input filter is used to obtain a ripple factor of 4%. The choke and load resistances are 60Ω and 800Ω respectively.

17. The critical inductance is:
    A. 800H.
    B. 80H.
    C. 8H.
    D. 800mH.
18. The DC output voltage is approximately:
    A. 125.8V.
    B. 74.4V.
    C. 80V.
    D. 24V.
19. Assuming L = 10H, the required capacitance is:
    A. 2.08μF.
    B. 1μF.
    C. 21μF.
    D. 6.32μF.

EXAMINATION ANSWERS

1. B — Since the turns ratio is 1:3, the secondary voltage is 3 times the primary voltage. Thus, $V_{CD} = 3(10V) = 30V$ rms.

2. C — $R_1 = \left(\frac{N_1}{N_2}\right)^2 \qquad R_2 = \left(\frac{1}{3}\right)^2 4.5K\Omega = 0.5K\Omega = 500\Omega.$

3. D — $I_1 = \frac{V_1}{R_1} = \frac{10V}{500\Omega} = 20mA$

4. A — Since the efficiency of a real transformer is close to 100%, the primary power is approximately equal to the secondary power.

   Thus:

$$P_2 = \frac{V_2^2}{R_2} = \frac{(30V)^2}{4.5K\Omega} = 0.2w \approx P_1$$

5. B — Replacing the "ideal" diode by a short, and applying the voltage division principle yields:

$$V_o = \frac{10.7V\,(100\Omega)}{113\Omega} = 9.47V$$

6. D — In this case, the diode's turn-on voltage and forward resistance must be considered. Consequently, the diode current is:

$$I = \frac{V - V_T}{r_F + R} = \frac{10.7V - 0.7V}{12\Omega + 113\Omega} = 0.08A$$

   Since the diode current flows through the 100Ω resistor, the output voltage is 0.08A (100Ω) or 8V.

7. C — 0.08A = 80mA.

8. A — If the diode is reversed, it will be reverse biased by the source. Since a reverse biased diode approximates an open circuit, the loop current, and hence output voltage, are approximately zero.

9. C — Since the turns ratio is 3:1, the full secondary voltage is 120V/3 or 40V rms. Converting to a peak value yields: $V_s = 40V(1.414) = 56.56V$ peak. In the center-tapped rectifier, V_m is one-half the full peak secondary voltage or 28.28V peak. From Table 5-2:

$$I_m = \frac{V_m}{R_L} = \frac{28.28V}{1k\Omega} = 28.28mA$$

10. A — From Table 5-2, $I_{DC} = 0.636\ I_m$. Thus, $I_{DC} = 0.636\,(28.28mA) = 17.99mA \approx 18mA$.

11. D — From Table 5-2, you can see that the DC diode current is one-half the DC load current. Thus, $I_{D2} = 18mA/2 = 9mA$.

12. D — Table 5-2 indicates that the peak inverse voltage of a center-tapped rectifier equals $2V_m$. Thus, PIV = 2(28.28V) or 56.56V.

13. B — From Table 5-4,

$$C_{min} = \frac{2.4 \times 10^{-3}}{r\ R_L} = \frac{2.4 \times 10^{-3}}{0.02\ (10^3)} = 120\mu F$$

14. A — From Table 5-4,

$$V_{DC} = V_m \left[1 - \frac{4.17 \times 10^{-3}}{R_L\ C} \right]$$

$$V_{DC} = 84.84V \left[1 - \frac{4.17 \times 10^{-3}}{10^3(120)(10^{-6})} \right]$$

$$V_{DC} = 84.84V\ (0.965) = 81.9V$$

15. D — From Table 5-4,

$$V_r = \frac{2.4 \times 10^{-3}\ V_{DC}}{R_L C}$$

$$V_R = \frac{2.4 \times 10^{-3}\ (81.9V)}{10^3(120)\ (10^{-6})} = 1.64V$$

16. A — Since the DC capacitor voltage is approximately 81.9V, the WVDC rating must at least equal this value.A higher value would, of course, be desirable.

17. D — From Table 5-5, $L_{min} = R_L/1000$. Thus:

$$L_{min} = \frac{800}{1000} = 800mH$$

18. B — From Table 5-5, $V_{DC} = \frac{0.636\ V_m\ (R_L)}{R_C + R_L}$

Since 0.636 V_m is the DC input voltage, we have:

$$V_{DC} \quad \frac{(80V\ (800\Omega)}{860\Omega} = 74.4V$$

19. A — From Table 5-5 $C = \frac{830 \times 10^{-9}}{rL}$. Thus:

$$C = \frac{830 \times 10^{-9}}{0.04\ (10)} = 2.08\mu F$$

Unit 6

ADDITIONAL PASSIVE CIRCUITS

CONTENTS

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INTRODUCTION

Generally speaking, waveshaping circuits change the shape, DC level or both the shape and DC level of the input signal waveforms.

In this unit, you will learn how to design a number of frequently encountered waveshaping circuits. As you will see, circuit design often involves compromise when selecting appropriate component values. Applications for the circuits discussed in this unit will be covered in Unit 7.

UNIT OBJECTIVES

When you have completed this unit, you should be able to:

1. Design clippers.
2. Design clampers.
3. Design peak and peak-to-peak detectors.
4. Design low-pass and high-pass RC filters.
5. Sketch Bode plots for series RC circuits.
6. State the conditions necessary for a series RC circuit to function as:
    - (a) An integrator.
    - (b) A differentiator.
    - (c) A 0 to 90° phase shifter.
    - (d) A 0 to –90° phase shifter.
    - (e) A coupling circuit.
    - (f) A bypass circuit.

7. Select component values so that the circuits discussed in this unit approximate the characteristics of ideal circuits.

UNIT ACTIVITY GUIDE

☐ Read section on “Diode Clippers.”

☐ Answer Self-Test Review questions 1-10.

☐ Perform Experiment 8 in Unit 8.

☐ Read section on “Diode Clampers and Voltage Multipliers.”

☐ Answer Self-Test Review questions 11-20.

☐ Perform Experiment 9 in Unit 8.

☐ Read section on “Amazing RC Circuit.”

☐ Answer Self-Test Review questions 21-30.

☐ Study Summary.

☐ Complete Unit Examination.

☐ Check Examination Answers.

DIODE CLIPPERS

Clipping circuits remove or “clip off” voltages above or below a particular reference level. As you will see, the reference level may be positive, negative, or zero.

Positive clippers remove voltages more positive than the reference level. Similarly, negative clippers remove voltages more negative than the reference level. For this reason, clipping circuits are also referred to as limiting circuits, or limiters.

Negative Clippers

In its simplest form, a clipper is nothing more than an unfiltered, half-wave rectifier as shown in Figure 6-1A. Here, the output voltage, ideally, equals the input voltage when the diode is forward biased. Similarly, the output voltage equals zero when the diode is reverse biased. Clearly, the diode in Figure 6-1A will be forward biased when the input voltage is positive, and reverse biased when the input voltage is negative.

Consequently, voltages below the 0V reference level are removed by the circuit.

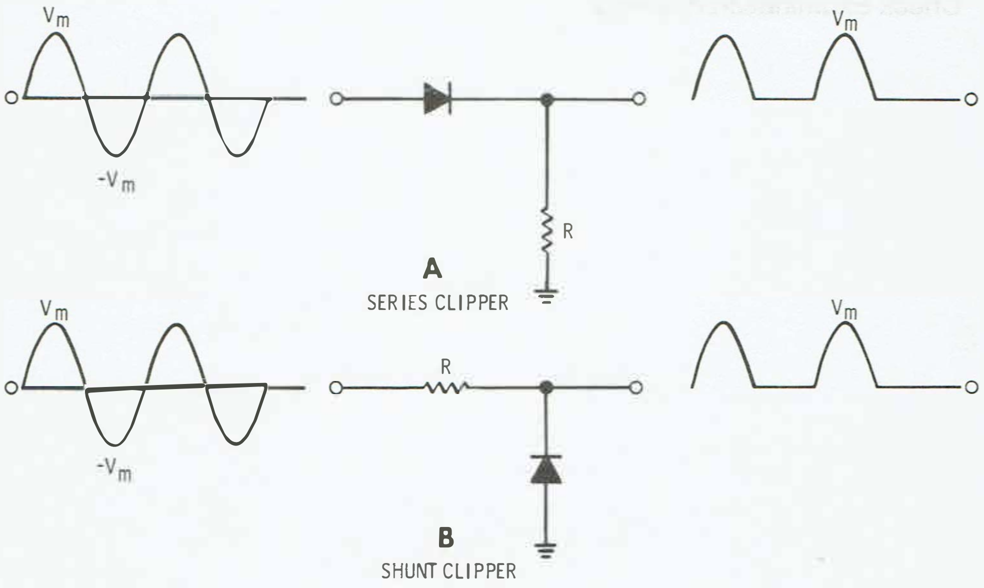


Figure 6-1

Ideal negative diode clippers.
A. Series clipper.
B. Shunt clipper.

The same result may be obtained by the shunt-type clipper illustrated in Figure 6-1B. Here, the output voltage equals the input voltage when the diode is reverse biased. Similarly, the output voltage equals zero when the diode is forward biased. Since the diode in Figure 6-1B is reverse biased when the input voltage is positive, and forward biased when the input voltage is negative, the output voltage consists of the positive alternations of the input voltage.

Positive Clippers

If the diodes in the negative clippers of Figure 6-1 are reversed, you obtain the positive clippers illustrated in Figure 6-2. Here, the diodes again function as a switch to pass or block the positive or negative portion of the input voltage. Since the diodes are reversed, however, voltages above the 0V reference level are removed by the circuits.

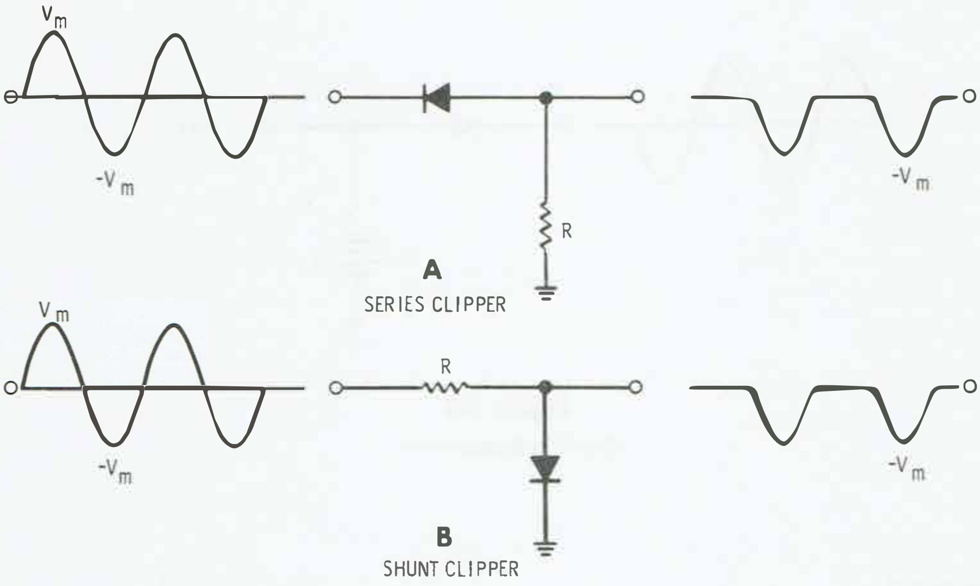


Figure 6-2

Ideal positive diode clippers.

A. Series clipper.

B. Shunt clipper.

Biased Clippers

Biased clippers have reference levels that are either positive or negative. To obtain these nonzero reference levels, a DC voltage source is required. Since the DC voltage source can be placed in the circuit with either one of two possible polarities, quite a few clipper configurations are possible.

When analyzing a particular clipper configuration, the following procedure is useful:

1. Describe the output voltage assuming the diode is forward biased (closed switch).
2. Describe the output voltage assuming the diode is reverse biased (open switch).
3. Mentally reduce the input voltage to zero. Next, determine if the DC voltage source forward biases or reverse biases the diode.
4. Assume the input voltage is positive. Determine the output voltage if $|\nu_{in}| < V$ and if $|\nu_{in}| > V$.
5. Repeat step 4 assuming the input voltage is negative.

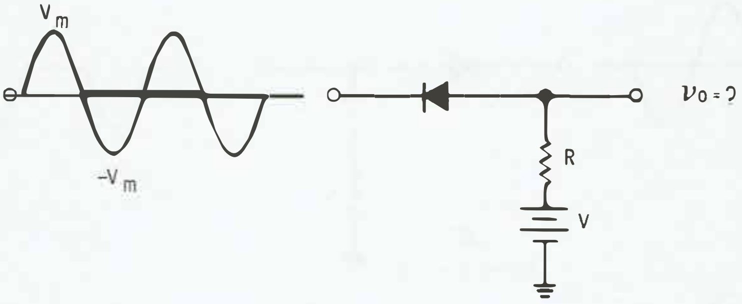


Figure 6-3

Circuit for Example 6-1.

Example 6-1

Sketch the output voltage for the circuit shown in Figure 6-3. Assume the diode is ideal, and that $V_m > V$.

Applying the five-step procedure given earlier:

1. When the diode is forward biased, it acts like a closed switch as shown in Figure 6-4A. In this case, ν_o equals ν_{in}.

2. When the diode is reverse biased, it acts like an open switch as shown in Figure 6-4B. Since the current through R is zero, the voltage across R is also zero. Consequently, the output voltage equals the battery voltage, V.

3. Reducing the input voltage to zero produces the circuit shown in Figure 6-4C. Here, the battery establishes a clockwise loop current which forward biases the diode.

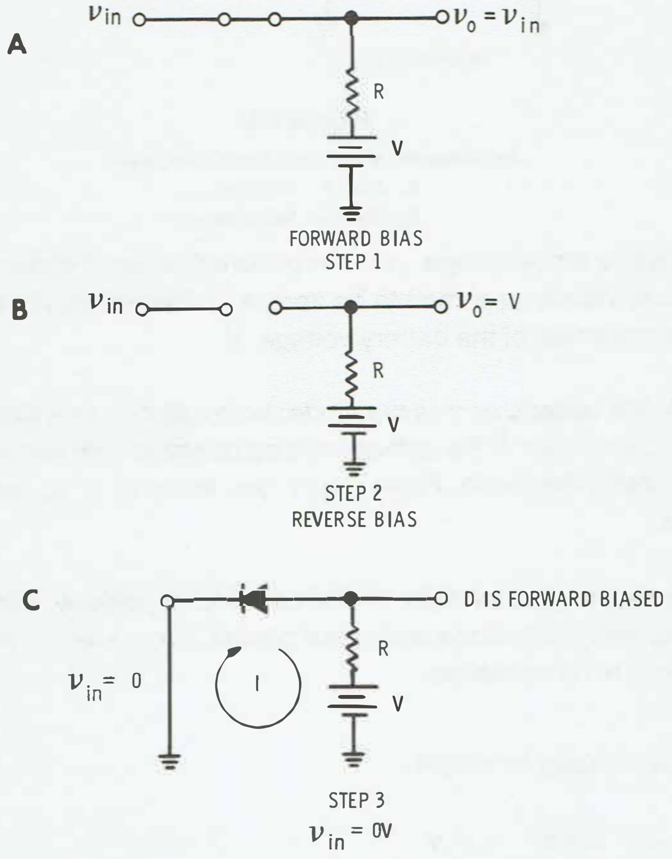


Figure 6-4

Implementing steps 1-3 for Example 6-1.

A. Step 1 Forward bias.
B. Step 2 Reverse bias.
C. Step 3 $\nu_{in} = 0V$.

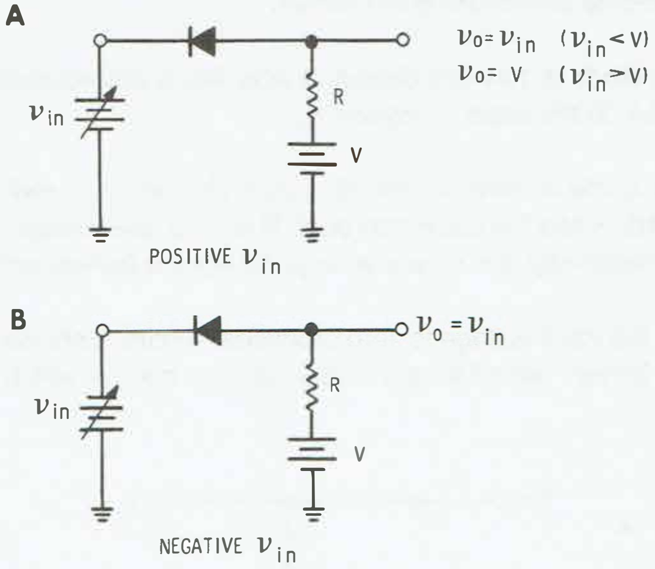


Figure 6-5

Implementing steps 4 and 5 for Example 6-1.

A. Step 4 Positive ν_{in}.

B. Step 5 Negative ν_{in}.

4. For a positive input voltage, you can picture the circuit shown in Figure 6-5A. Here, ν_{in} is initially assumed to be zero, and then gradually increased until it exceeds the value of the battery voltage, V.

In Figure 6-5A, the voltage across the diode, from cathode to anode, equals $\nu_{in} - V$. For values of ν_{in} less than V, the cathode will be negative with respect to the anode, thus forward biasing the diode. From step 1, you know $\nu_o = \nu_{in}$ when the diode is forward biased.

Similarly, when ν_{in} is greater than V, the cathode is positive with respect to the anode. Consequently, the diode is reverse biased. From step 2, you know $\nu_o = V$ when the diode is reverse biased.

Summarizing our results for step 4:

$\nu_o = \nu_{in}$ when $\nu_{in} < V$
$\nu_o = V$ when $\nu_{in} > V$

5. For a negative input voltage, you can picture the circuit shown in Figure 6-5B. Once again, ν_{in} is initially assumed to be zero and then gradually increases in value. From step 3, you know that when $\nu_{in} = 0$, the diode is forward biased by the battery. Since ν_{in} and V are connected series aiding, the diode remains forward biased regardless of the value of ν_{in}. Thus:

$$\nu_o = \nu_{in} \qquad \nu_{in} < V$$
$$\nu_o = \nu_{in} \qquad \nu_{in} > V$$

Summarizing the results of our five-step procedure, the circuit in Figure 6-3 may be described as follows:

Initially, the diode is forward biased by the battery V. Thus, the output voltage equals the input voltage. On the positive half cycle of the input, ν_{in} and V are connected series opposing. Thus, when ν_{in} exceeds the value of V, the diode becomes reverse biased, in which case, $\nu_o = V$. The diode remains reverse biased until ν_{in} falls below the value of the battery voltage V. Once this occurs, ν_o again equals ν_{in}.

On the negative half cycle of the input, ν_{in} and V are connected series aiding. Since the diode is forward biased, the output voltage equals the input voltage. Figure 6-6 illustrates the input and output waveforms for the circuit.

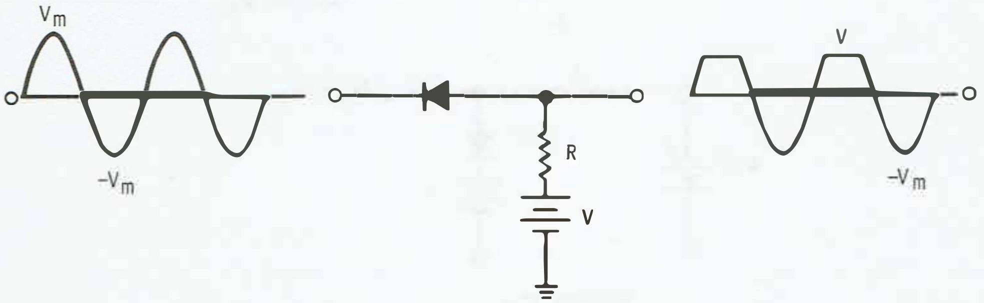


Figure 6-6

Input and output waveforms for Example 6-1.

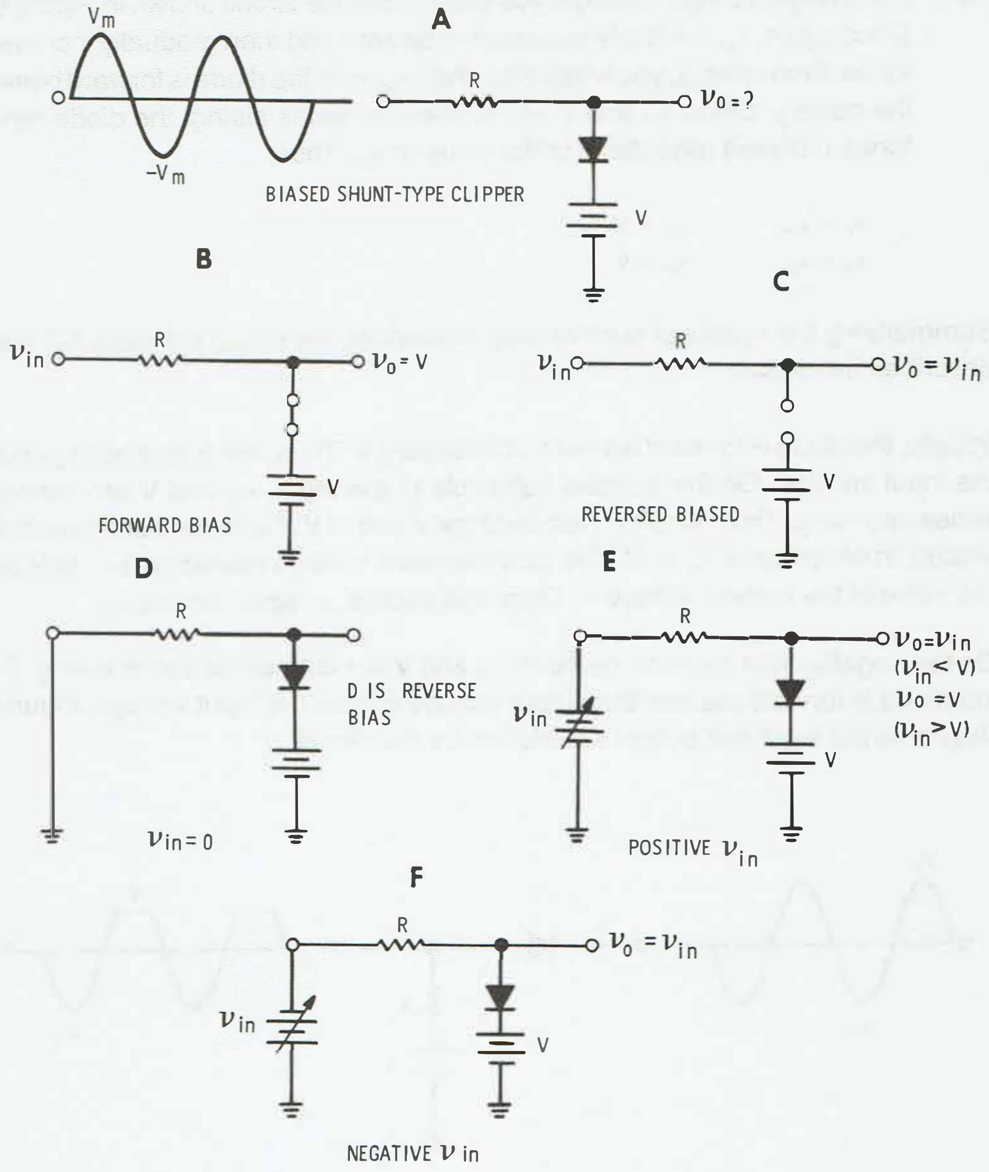


Figure 6-7

Circuits for Example 6-2.

A. Biased shunt-type clipper.
B. Step 1 Forward bias.
C. Step 2 Reverse bias.
D. Step 3 $\nu_{in} = 0V$.
E. Step 4 Positive ν_{in}.
F. Step 5 Negative ν_{in}.

Example 6-2

Sketch the output voltage for the biased shunt-type clipper in Figure 6-7A. Assume the diode is ideal and that $V_m > V$.

Figures 6-7B through 6-7F illustrate the equivalent circuits for the five steps. Referring to these figures, we find:

1. $\nu_o = V$ when the diode is forward biased.
2. $\nu_o = \nu_{in}$ when the diode is reverse biased.
3. When $\nu_{in} = 0V$, the diode is reverse biased by the DC voltage source.
4. For positive input voltages, ν_{in} and V are connected series opposing. Thus, when $\nu_{in} < V$, the diode is reverse biased, and $\nu_o = \nu_{in}$. Similarly, when $\nu_{in} > V$, the diode is forward biased, and $\nu_o = V$.
5. For negative input voltages, ν_{in} and V are connected series aiding. Consequently, the diode remains reverse biased and $\nu_o = \nu_{in}$.

Summarizing the results of our five-step procedure, the circuit in Figure 6-7A may be described as follows:

Initially, the diode is reverse biased by the battery V. Thus, the output voltage equals the input voltage. On the positive half cycle of the input, ν_{in} and V are connected series opposing. Therefore, when ν_{in} exceeds the value of V the diode becomes forward biased, in which case $\nu_o = V$. The diode remains forward biased until ν_{in} falls below the value of the battery voltage V. Once this occurs, ν_o again equals ν_{in}.

On the negative half cycle of the input, ν_{in} and V are connected series aiding. Since the diode is reverse biased, the output voltage equals the input voltage. Figure 6-8 illustrates the input and output waveforms for the circuit.

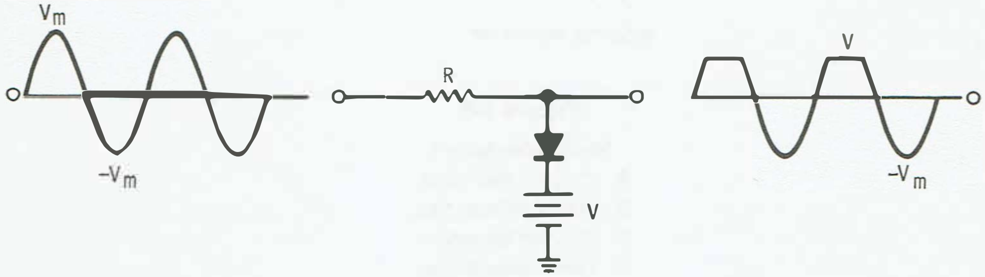


Figure 6-8

Input and output waveforms for Example 6-2.

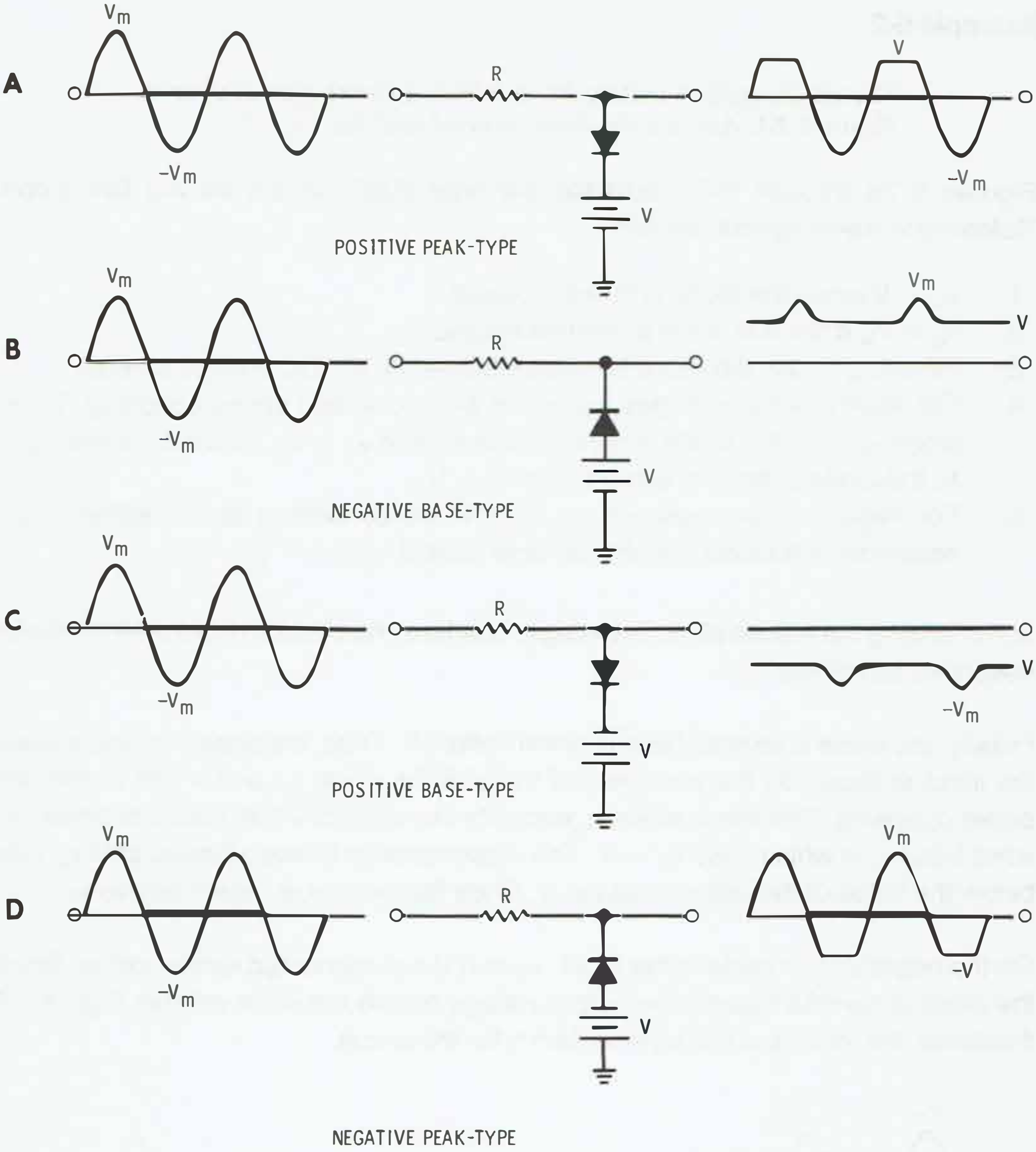


Figure 6-9

Shunt-type clippers.

A. Positive peak-type.
B. Negative base-type.
C. Positive base-type.
D. Negative peak-type.

Example 6-1 and Example 6-2 illustrated how the five-step method can be applied to determine the output waveform for positive peak, series-type and shunt-type clippers. The standard shunt- and series-type clipper configurations are illustrated in Figure 6-9 and Figure 6-10 respectively. In these Figures, note that both the shunt-

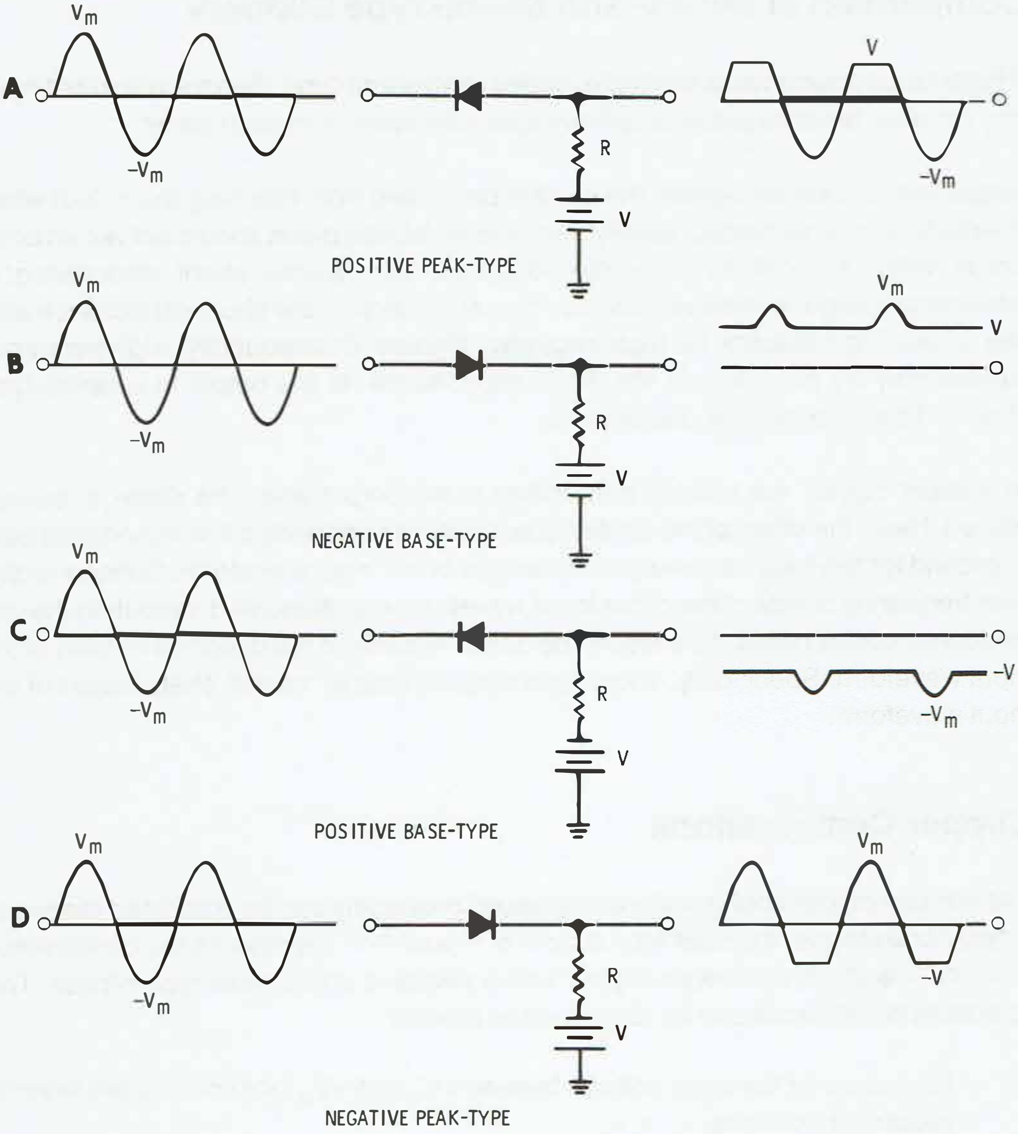


Figure 6-10

Series-type clippers.
A. Positive peak-type.
B. Negative base-type.
C. Positive base-type.
D. Negative peak-type.

and series-type clippers may be further classified as peak- or base-type clippers. The various output waveforms for each configuration can be determined by applying the five-step method illustrated previously.

Comparison of Series- and Shunt-Type Clippers

If ideal components were available, series- and shunt-type clippers would not have any inherent advantages or disadvantages with respect to each other.

Recall that in a series clipper, the input is prevented from reaching the output when the diode is reverse biased. Ideally, the reverse biased diode should act like an open circuit. When a real diode is reverse biased it exhibits a small shunt capacitance, in addition to a large reverse resistance. In a series clipper, the shunt capacitance acts like a coupling capacitor for high frequency signals. Consequently, high frequency signals may be coupled, via the diode capacitance, to the output in a series-type clipper. This, of course, is undesirable.

In a shunt clipper, the input is transmitted to the output when the diode is reverse biased. Here, the effect of the diode capacitance is to provide a low impedance path to ground for the high frequency components of the input waveform. Consequently, high frequency components of the input waveform are attenuated more than the low frequency components. As a result, the output waveform is a distorted version of the input waveform. Specifically, shunt-type clippers tend to "round" sharp edges of the input waveform.

Clipper Combinations

The various clipper configurations discussed previously can be combined in several ways. For example, the dual-type clipper of Figure 6-11 represents the combination of a positive shunt peak-type clipper and a negative shunt peak-type clipper. The operation of the circuit can be described as follows:

1. For values of the input voltage between V_1 and $-V_2$, both diodes are reverse biased. In this case, $\nu_o = \nu_{in}$.
2. When the input voltage is more positive than V_1, diode D_1 is forward biased and diode D_2 is reverse biased. Consequently, $\nu_o = V_1$.
3. When the input voltage is more negative than $-V_2$, diode D_1 is reverse biased, and diode D_2 is forward biased. Therefore, $\nu_o = -V_2$.

Clearly, voltages more positive than V_1 or more negative than V_2 are removed by the circuit. For this reason, the circuit in Figure 6-11 is referred to as a dual-type clipper.

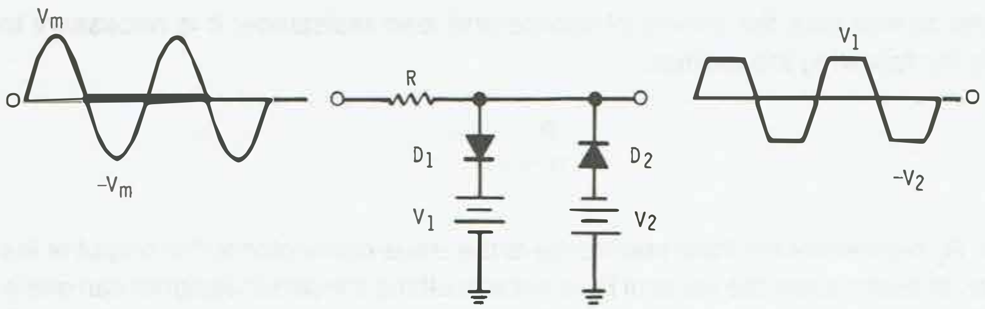


Figure 6-11

A dual-type clipper.

Designing Clipper Circuits

The previous discussion of clippers neglected the effects of source resistance, load resistance, and the nonideal properties of real diodes. Naturally, these effects must be taken into account when you design real circuits.

In order to minimize the effects of a diode's turn-on voltage, forward resistance, and reverse resistance the following inequalities should be satisfied:

$$V_{TH} >> V_T$$
$$R >> r_F$$
$$R << R_R$$

where: V_T, r_F and R_R are, respectively, the diode's turn-on voltage, forward resistance, and reverse resistance.

V_{TH} is the peak value of the Thevenin voltage driving the circuit.

R is the resistance that appears in the clipping circuits discussed previously.

Note that when the diode is forward biased, it is desirable to have a large value of R. Similarly, when the diode is reverse biased, a small value of R is desirable. Since these two requirements are contradictory, a compromise must be made to select an appropriate value for R. Theoretically, the optimum value for R is the geometric average of the diode's forward and reverse resistances. Stated mathematically:

$$R = \sqrt{r_F R_R} \qquad \text{(Eq. 6-1)}$$

By consulting the manufacturer's specification sheet for the diode, estimates for r_F and R_R may be obtained.

In order to minimize the effects of source and load resistance, it is necessary to satisfy the following inequalities:

$$R >> R_{TH}$$
$$R_L >> R$$

Often, R_L represents the input resistance of the stage connected to the output of the clipper. In such cases, the value of R_L is not something the circuit designer can easily vary. Consequently, it may not be possible to satisfy the requirement that $R_L >> R$ with the optimum value of R calculated from Equation 6-1. In such instances, a value of R is chosen that is less than the optimum value. Naturally, whatever value is chosen must still satisfy the requirements that $R >> r_F$ and $R << R_R$.

Example 6-3

Design a clipper that removes voltages more positive than 4V.

The signal source driving the clipper has a Thevenin resistance of 600Ω and provides a peak sinusoidal output voltage of 10V.

The input resistance of the circuit driven by the clipper is 1MΩ. In addition, the silicon diode available for the design has the following specifications:

I_F = 10mA at V_F = 1V
I_R = 25nA at V_R = –20V (25°c)

Based on the given information, design a suitable circuit. Also discuss any significant nonideal effects.

Since voltages more positive than 4V are to be removed, a positive, series or shunt, peak-type clipper is required.

In order to calculate the optimum value of R, you first need to estimate the diode's forward and reverse resistances. Thus, referring to the diode specifications:

$$r_F = \frac{V - V_T}{I_F} = \frac{1V - 0.7V}{10mA} = 30\Omega$$

$$R_R = \frac{V_R}{I_R} = \frac{20V}{25nA} = 800\,M\Omega$$

Substituting the preceding values into Equation 6-1 yields:

$$R = \sqrt{r_F R_R} = \sqrt{30\Omega\,(800\,M\Omega)} = 154.9\ k\Omega\ \text{(optimum value)}$$

The input resistance of the circuit driven by the clipper was given as 1MΩ. This is the effective load resistance for the clipper circuit. In order to satisfy the requirement that $R_L >> R$, you must select a value for R that is relatively small compared to R_L. Generally speaking, for the design of clipper circuits, a 20:1 or 10:1 rule is adequate to define large and small. Using the 10:1 rule then:

$$R = \frac{RL}{10} = \frac{1M\Omega}{10} = 100k\Omega\ \text{(selected value)}$$

In this case, the value chosen for R is reasonably close to the theoretical optimum value. **More importantly**, note that the following inequalities have been satisfied:

$R >> r_F$ since $100k\Omega >> 30\Omega$
$R << R_R$ since $100k\Omega << 800M\Omega$
$R >> R_{TH}$ since $100k\Omega >> 600\Omega$
$R_L >> R$ since $1M\Omega >> 100k\Omega$

In this example, the most significant nonideal effect is the diode's turn-on voltage, since the requirement that $V_{TH} >> V_T$ is not adequately met.

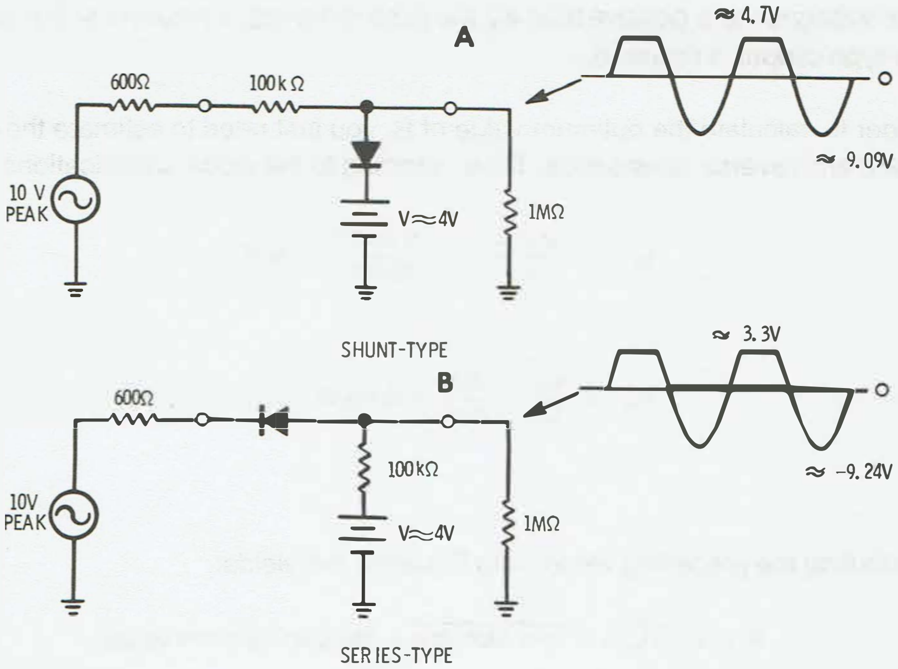


Figure 6-12
Positive peak-type clippers for Example 6-3.
A. Shunt-type.
B. Series-type.

The net effect of the diode's turn-on voltage is to change the clipping levels by an amount equal to V_T. In a shunt-type clipper, the actual clipping level equals $(V + V_T)$, since V and V_T are effectively connected series aiding. Similarly, in a series-type clipper, the actual clipping level equals $(V - V_T)$ since V and V_T are effectively connected series opposing. For these reasons, the actual clipping levels in Figure 6-12A and Figure 6-12B would respectively equal 4.7V and 3.3V, assuming the diodes turn-on voltage is 0.7V.

To obtain clipping levels of exactly 4V, you could change the value of the DC voltage source, V, in each circuit. Specifically, you would decrease V by 0.7V in Figure 6-12A, and increase V by 0.7V in Figure 6-12B.

To examine the effects of source and load resistance, you can construct the AC equivalent circuits shown in Figure 6-13. Note in these circuits that the DC voltage source has been replaced by a short circuit via the superposition principle.

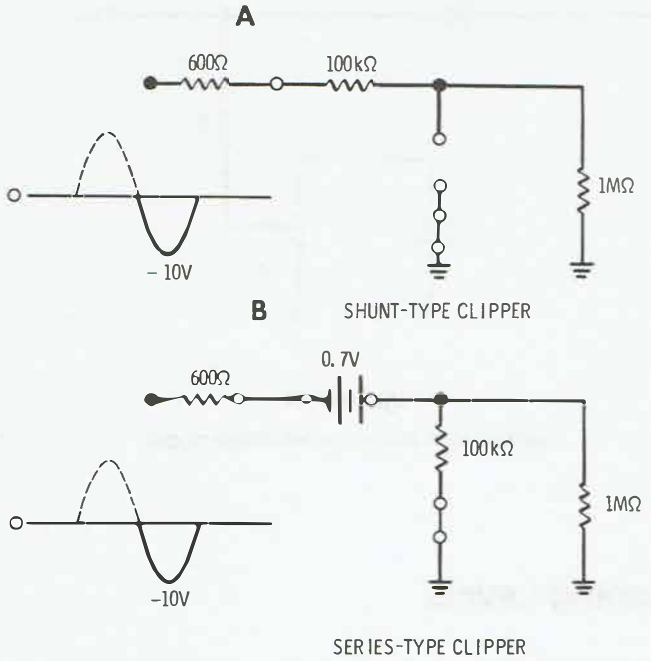


Figure 6-13

AC equivalent circuits for the clippers in Figure 6-12.

A. Shunt-type clipper.
B. Series-type clipper.

For the shunt-type clipper in Figure 6-13A, the peak output voltage is obtained via voltage division. Thus:

$$v_o = \frac{-10\text{V}\,(1\text{M}\Omega)}{600\Omega + 100\text{k}\Omega + 1\text{M}\Omega} = \frac{-10\text{V}\,(1\text{M}\Omega)}{1.1006\text{M}\Omega} \approx -9.09\text{V}$$

Similarly, for the series-type clipper in Figure 6-13B:

$$100\text{k}\Omega \parallel 1\text{M}\Omega \approx 100\text{k}\Omega$$

Therefore:

$$v_o = -(10\text{V} - 0.7\text{V})\,\frac{(100\text{k}\Omega)}{600\Omega + 100\text{k}\Omega}$$

$$v_o = \frac{-9.3\text{V}\,(100\text{k}\Omega)}{100.6\text{k}\Omega} \approx -9.24\text{V}$$

Note that for the series-type clipper, the diode's turn-on voltage has a more significant effect on the peak output voltage than does the source and load resistance.

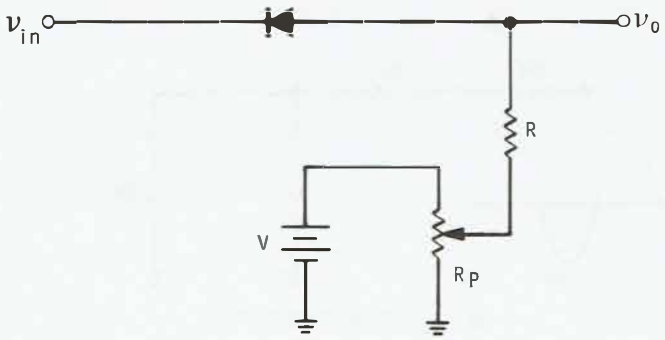


Figure 6-14

A positive peak series-type adjustable clipper.

Adjustable Clipping Levels

It is often desirable to have some means of changing the level at which clipping occurs.

In order to provide adjustable clipping levels, a variable DC voltage source is required. The desired voltage source is easy to obtain by using a fixed DC voltage source and potentiometer as shown in Figure 6-14. Here, the clipping level can be adjusted between 0V and +V volts.

All of the clipping circuits discussed previously can be made adjustable by adding a simple potentiometer to the circuit. However, to obtain satisfactory clipping action, the end-to-end resistance of the potentiometer must be small compared to the value of R. If possible, you should select a potentiometer so that:

$$R_p \leq \frac{R}{100}$$

Self-Test Review

1. Clippers are also referred to as __________ .
2. Shunt-type clippers tend to __________ sharp edges of the input waveform.
3. Shunt diode capacitance can act like a coupling capacitor in __________-type clippers.

In order for real clipping circuits to closely approximate the characteristics of ideal clipping circuits, various inequalities must be satisfied. For questions 4 through 9, insert the symbol >>, much greater than, or <<, much less than, to indicate how component values should be selected to achieve near ideal operation of clipper circuits.

4. V_{TH} ____________ V_T
5. R ____________ r_F
6. R ____________ R_R
7. R_{TH} ____________ R
8. R_L ____________ R
9. R_P ____________ R

10. A silicon diode has a forward current of 20mA for a forward voltage of 1V. When the diode is reverse biased by 40V, the reverse current is 0.2μA. The theoretical optimum value for R is, therefore, __________ kΩ.

Answers

1. limiters
2. round
3. series
4. >>
5. >>
6. <<
7. <<
8. >>
9. <<
10. 54.8kΩ

The solution to question 10 follows:

$$r_F = \frac{V - V_T}{I_F} = \frac{1V - 0.7V}{20mA} = \frac{0.3V}{20mA} = 15\Omega$$

$$R_R = \frac{V_R}{I_R} = \frac{40V}{0.2\mu A} = 200M\Omega$$

Thus:

$$R = \sqrt{r_F R_R} = \sqrt{15\Omega(200M\Omega)} = 54.8k\Omega$$

DIODE CLAMPERS AND VOLTAGE MULTIPLIERS

Clamping circuits change the average, DC, value of the signal waveform, by fixing either the positive or negative peak at a specified DC level. As you will see, the DC level may be zero, positive or negative. Unlike clippers, the output voltage from a clamper, ideally, has the same shape as the input voltage — only the DC levels are different.

By combining clampers and peak-type smoothing filters, it is possible to construct voltage multipliers. These circuits provide a DC output voltage that is a multiple of the peak input voltage. Consequently, voltage multipliers are useful as power supplies designed for high voltage-low current applications.

The Negative Clamper

The concept of a negative clamper is illustrated in Figure 6-15. Here, the input voltage has a DC level of 0V, while the output voltage has a DC level of $-V_m$ volts. Note that the positive peak of the output voltage is fixed or "clamped" to 0V.

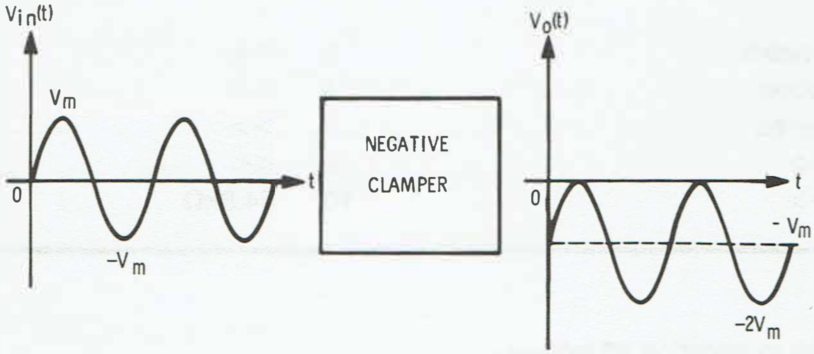


Figure 6-15

Negative clamping action.

To see how the negative clamping function can be accomplished, consider the circuit shown in Figure 6-16A. For simplicity, we will assume the voltage source, capacitor, and diode are ideal. The operation of this circuit may be described as follows:

1. Assuming the capacitor is initially uncharged, the diode becomes forward biased when the input voltage goes positive, as shown in Figure 6-16B. Consequently, during the first quarter cycle of the input voltage, the output voltage is 0V. Note that at the end of the first quarter cycle of the input voltage, the capacitor voltage equals V_m.

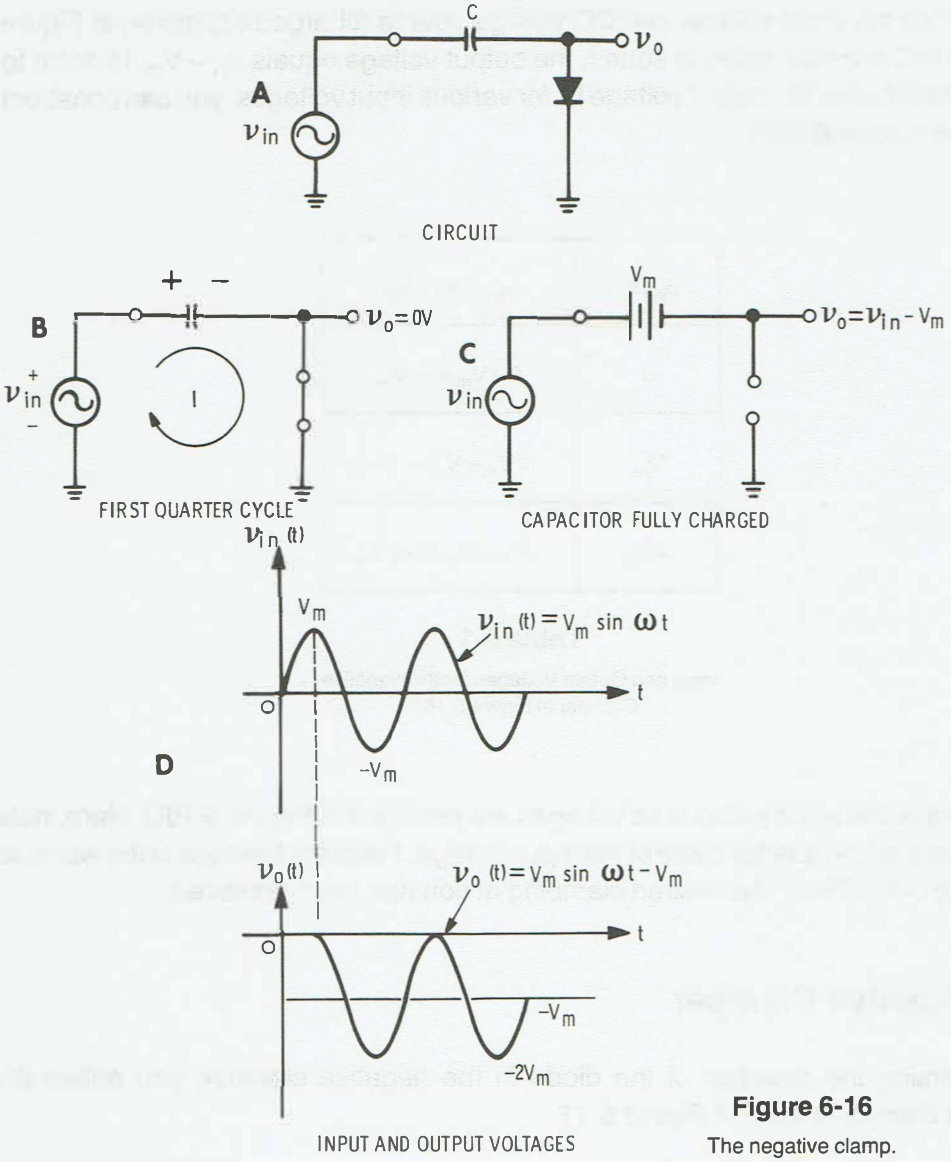


Figure 6-16

The negative clamp.

A. Circuit.
B. First quarter cycle.
C. Capacitor fully charged.
D. Input and output voltages.

2. At the beginning of the second quarter cycle of the input voltage, the input voltage starts to decrease. As soon as the input voltage decreases below V_m, the diode becomes reverse biased as shown in Figure 6-16C. From this point on, the diode remains reverse biased.

3. Since there is no dischange path for the capacitor, the voltage across the capacitor must remain at V_m. Thus, the capacitor acts like a DC voltage source equal to V_m with the polarity as indicated in Figure 6-16C.

4. Since the input voltage and DC voltage source (charged capacitor) in Figure 6-16C are connected in series, the output voltage equals $\nu_{in} - V_m$. In order to predict what the output voltage is, for various input voltages, you can construct the following table:

ν_{in} (V)	$\nu_o = \nu_{in} - V_m$
0	$0 - V_m = -V_m$
V_m	$V_m - V_m = 0$
$-V_m$	$-V_m - V_m = -2V_m$

Table 6-1

Input and Output Voltages for the Negative Clamper in Figure 6-16A.

Sketches of the input and output voltages are provided in Figure 6-16D. Here, note that, after the first quarter cycle of the input voltage, the output voltage is the same as in Figure 6-15. Thus, the desired clamping action has been achieved.

The Positive Clamper

By reversing the direction of the diode in the negative clamper, you obtain the positive clamper shown in Figure 6-17.

A description of the operation of the positive clamper is similar to the one provided for the negative clamper. The essential difference is that, since the diode has been reversed, the polarity of the capacitor voltage is just the opposite of what it was in the negative clamper. For this reason, once the capacitor is fully charged, the output voltage equals $\nu_{in} + V_m$. Selecting values of ν_{in} equal to 0, V_m and $-V_m$ provides the data for Table 6-2.

v_{in} (V)	$v_o = v_{in} + V_m$
0	$0 + V_m = V_m$
V_m	$V_m + V_m = 2V_m$
$-V_m$	$-V_m + V_m = 0$

Table 6-2

Input and Output Voltages for the Positive Clamper in Figure 6-17.

Note that the values in Table 6-2 agree with the sketches of the waveforms in Figure 6-17. For clarity, the waveforms in Figure 6-17 assume the steady-state condition has been reached.

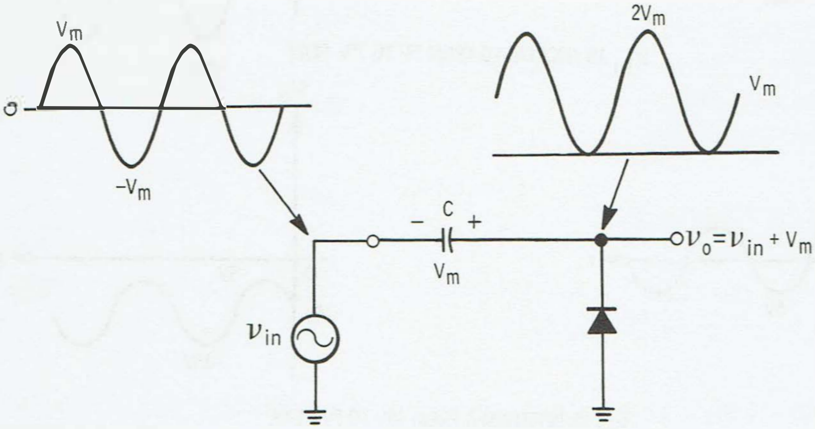


Figure 6-17

The positive clamp.

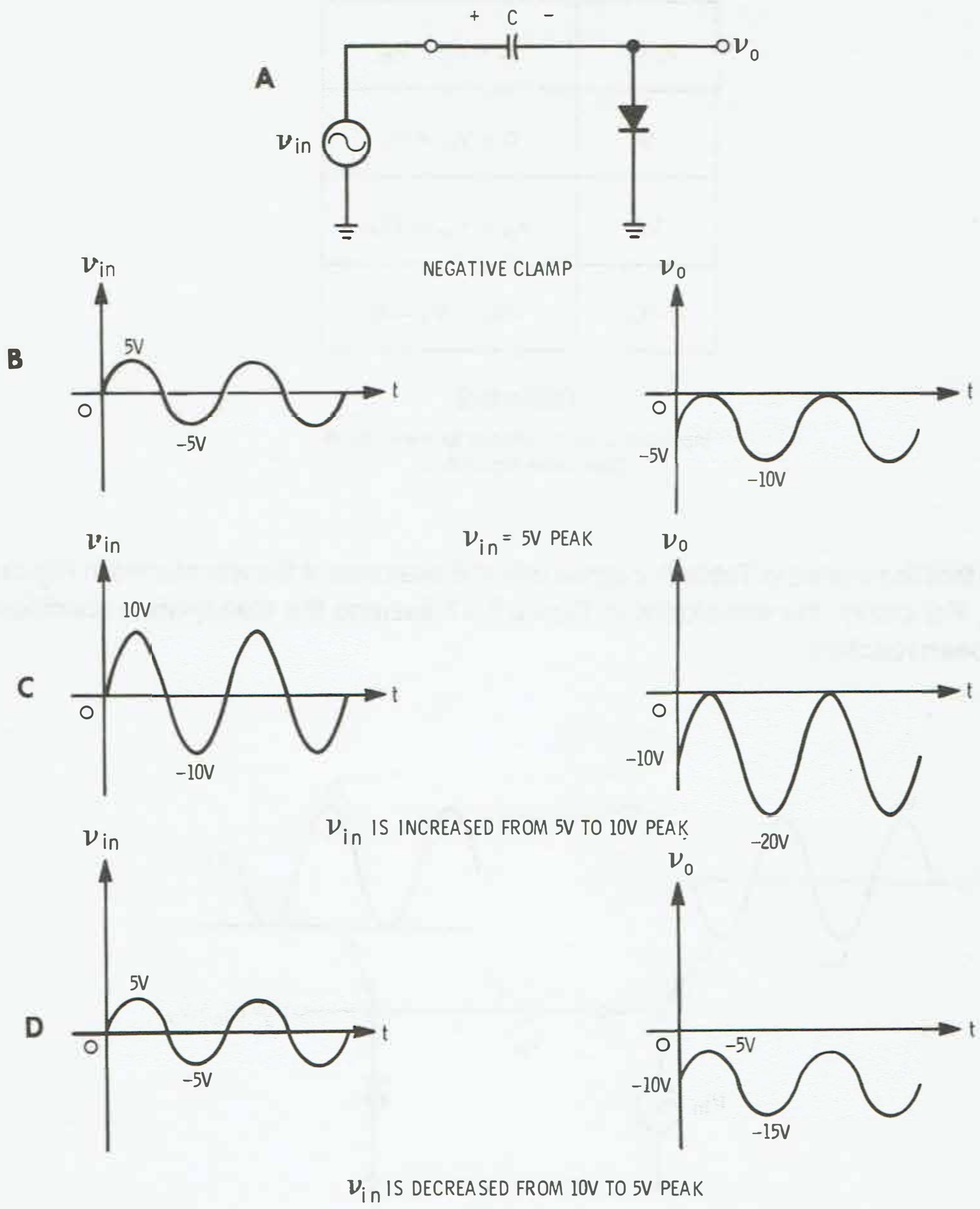


Figure 6-18

A potential problem.

A. Negative clamp.
B. $\nu_{in} = 5V$ peak.
C. ν_{in} is increased from 5V to 10V peak.
D. ν_{in} is decreased from 10V to 5V peak.

Response To Amplitude Changes

Ideally, the output voltage from the negative clamp discussed previously has the positive peak clamped at 0V. Thus, if the input voltage in Figure 6-18A is 5V peak, the input and output voltages should appear as shown in Figure 6-18B.

To illustrate how amplitude changes affect the operation of a clamper, we will assume the input voltage in Figure 6-18A is suddenly increased from 5V to 10V peak. Once this has been discussed, we will then assume the input voltage is suddenly decreased from 10V to 5V peak. As you will see, rapid changes in the amplitude of the input voltage can adversely affect the operation of the clamper.

To begin then, assume the input voltage in Figure 6-18A is suddenly increased from 5V to 10V peak.

Once the input voltage exceeds 5V, the diode becomes forward biased. Therefore, current will flow, for approximately one-quarter of a cycle, until the capacitor charges to the new value of V_m, which is 10V in this example. At this point, the diode once again becomes reverse biased. Everything works as expected. Thus, the input and output voltages now appear as shown in Figure 6-18C.

Now suppose the input voltage is suddenly decreased from 10V to 5V peak. **Since there is no discharge path for the capacitor the voltage across the capacitor must remain at 10V.** For this reason, the output voltage equals $v_{in} - 10V$ as shown by the following table.

v_{in} (V)	$v_o = v_{in} - 10V$
0	$0V - 10V = -10V$
5	$5V - 10V = -5V$
–5	$-5V - 10V = -15V$

Table 6-3

Input and output voltages in Figure 6-18A, once v_{in} has been decreased from 10V to 5V peak.

Consequently, the input and output voltages now appear as shown in Figure 6-18D. Comparing the waveforms in Figure 6-18D with those in Figure 6-18B, you can see that the DC levels of the output voltages are different, even though the input voltages are identical. Specifically, note that the output voltage in Figure 6-18D is clamped at –5V rather than at 0V.

Clearly, the clamper in Figure 6-18 does not function properly when the amplitude of the input signal is suddenly decreased. As you might suppose, a similar problem is encountered in positive clampers. In this case, however, sudden decreases in the amplitude of the input signal produce an output voltage that is clamped at a positive voltage rather than at 0V.

Shunt Diode Resistor

In order for a clamper to respond to sudden amplitude changes in the input voltage, a discharge path must be provided for the capacitor. In real clamper circuits discharge paths exist due to leakage, source, and load resistances. If, as is usually the case, the discharge time constants are relatively long, the response of the clamper to sudden amplitude changes in the input voltage will be very sluggish.

To prevent this situation from occurring in practical clampers, an adequate discharge path must be provided for the capacitor. To accomplish this, a resistor is placed in parallel with the diode, as shown in Figure 6-19. As was the case with the clipper circuits discussed previously, the theoretical optimum value for R is the geometric average of the diode's forward and reverse resistances. Thus, if possible, R is chosen so that:

$$R = \sqrt{r_F R_R}$$

In practice, an acceptable value for R is frequently determined experimentally.

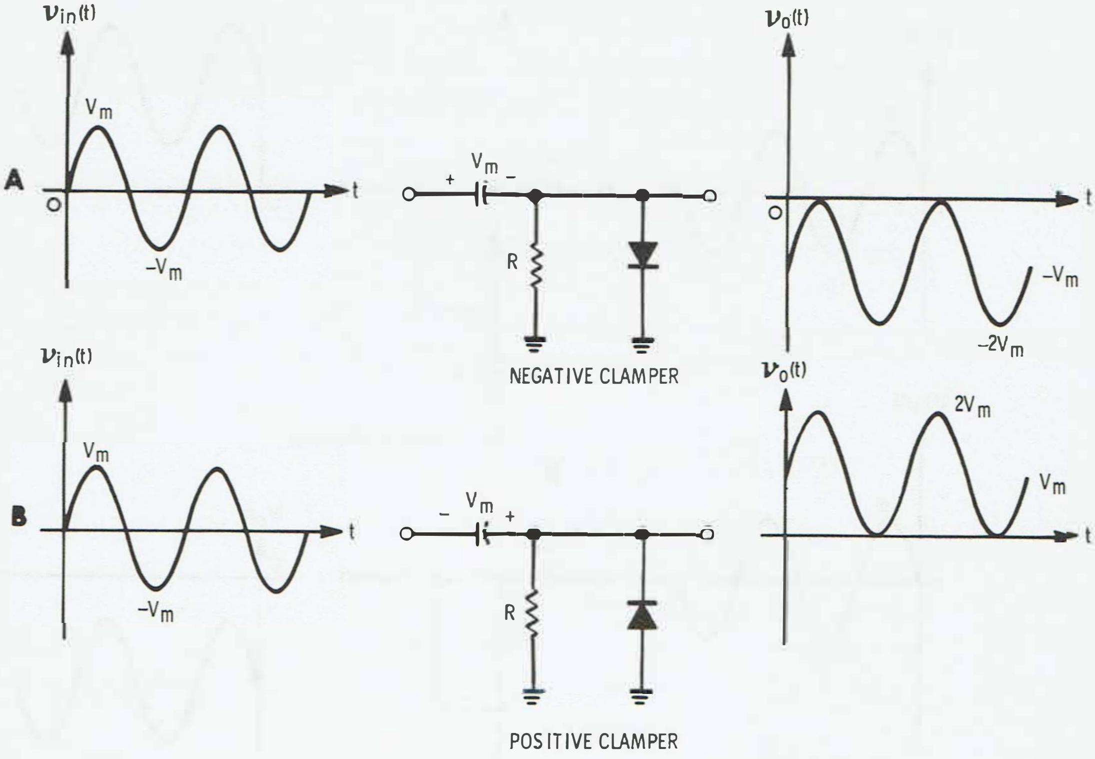


Figure 6-19

Shunt diode resistor.

The addition of the shunt diode resistor R enables the clampers to respond quickly to amplitude changes in the input voltage.

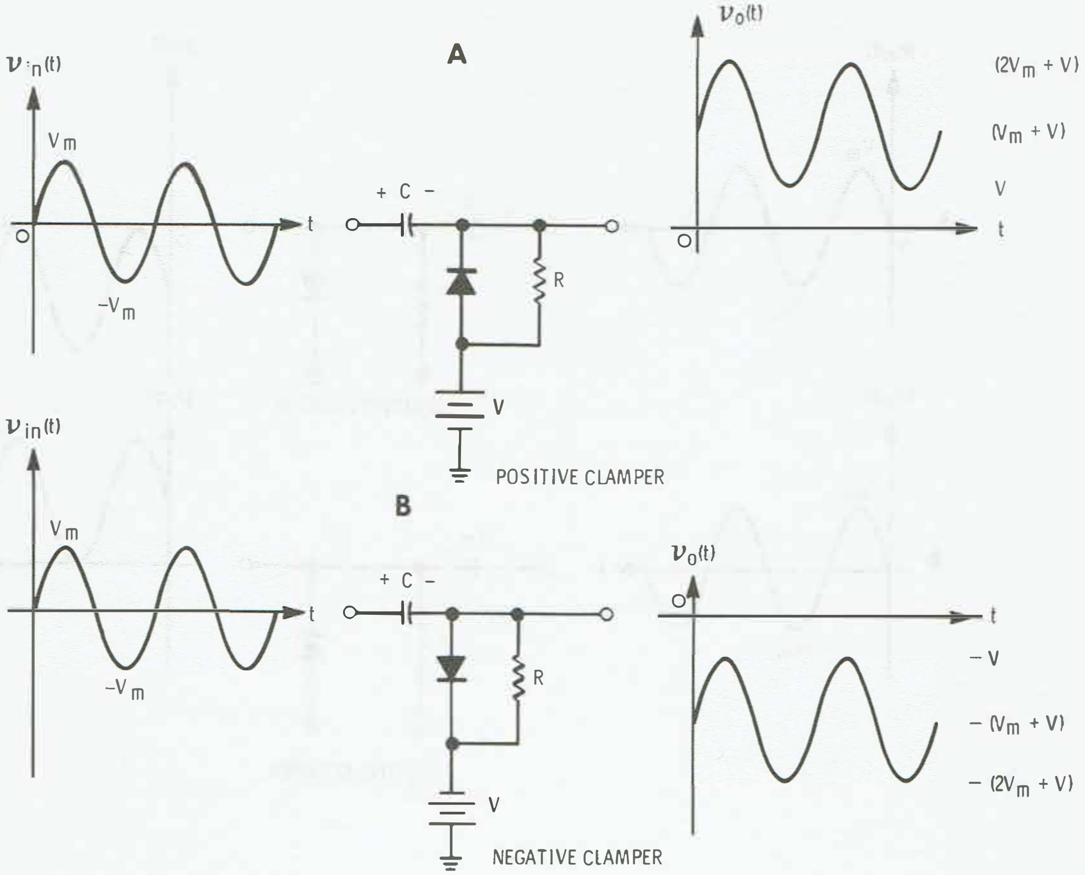


Figure 6-20

Biased clamping circuits.
A. Positive clamper.
B. Negative clamper.

Biased Clampers

In order to clamp the signal voltage to some value other than 0V, a DC voltage source is required. Examples of biased positive and negative clampers are provided in Figure 6-20A and Figure 6-20B respectively.

To illustrate the operation of these circuits, consider the negative clamper in Figure 6-20B. During the positive portion of the input cycle, v_{in} and V are connected series aiding. Note that the polarity of these voltages is correct to forward bias the diode. Consequently, during the first quarter of the input cycle, the capacitor charges to $(V_m + V)$ with the polarity as indicated in Figure 6-20B. Once the input voltage decreases below V_m the diode becomes reverse biased. Since, the output voltage equals the algebraic sum of the capacitor and input voltages, we have:

$$v_o = v_{in} - (V_m + V) \qquad \text{Biased negative clamper}$$

Selecting values of ν_{in} of 0, V_m and $-V_m$ yields:

ν_{in} (V)	$\nu_o = \nu_{in} - (V_m + V)$
0	$0V - (V_m + V) = -(V_m + V)$
V_m	$V_m - (V_m + V) = V$
$-V_m$	$-V_m - (V_m + V) = -(2V_m + V)$

Table 6-4

Input and Output Voltages for the Circuit in Figure 6-20B.

The data in Table 6-4 suggests the output waveform of Figure 6-20B.

The operation of the biased positive clamp in Figure 6-20A is, of course, similar to the operation of the biased negative clamp in Figure 6-20B. In this case, however, the capacitor charges with the opposite polarity. Therefore, once the capacitor is fully charged:

$$\nu_o = \nu_{in} + (V_m + V) \qquad \text{(Biased Positive Clamper)}$$

As usual, selecting values of ν_{in} equal to 0, V_m and $-V_m$, we have:

ν_{in} (V)	$\nu_o = \nu_{in} + (V_m + V)$
0	$0 + (V_m + V) = (V_m + V)$
V_m	$V_m + (V_m + V) = (2V_m + V)$
$-V_m$	$-V_m + (V_m + V) = V$

Table 6-5

Input and Output Voltages for the Circuit in Figure 6-20A.

The data in Table 6-5 suggests the output waveform of Figure 6-20A.

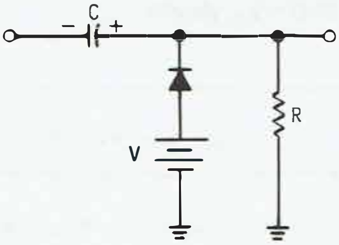


Figure 6-21

A variation of the positive clamper in Figue 6-20A.
Note that the shunt resistor has been returned to ground.

A frequently encountered variation of the clampers in Figure 6-20 is illustrated by the circuit in Figure 6-21. Here, note that the diode shunt resistor is connected across the series combination of the diode and bias voltage V. In some cases, R may represent the input resistance of the circuit being driven by the clamper.

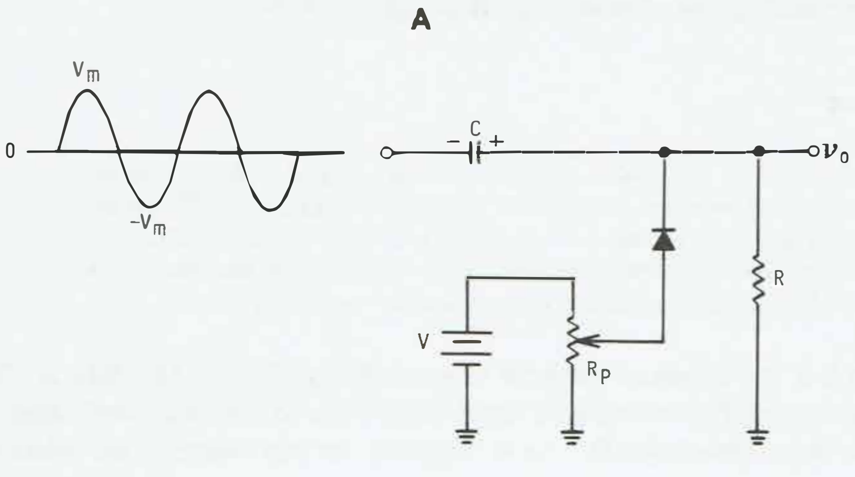
An additional modification of the clampers in Figure 6-20 is shown in Figure 6-22. Here, the clamping level can be varied between V volts and 0 Volts as illustrated in Figure 6-22B. To obtain adjustable clamping levels between –V volts and 0V, you would reverse the direction of the diode and polarity of the DC voltage source in Figure 6-22A.

Designing Clamper Circuits

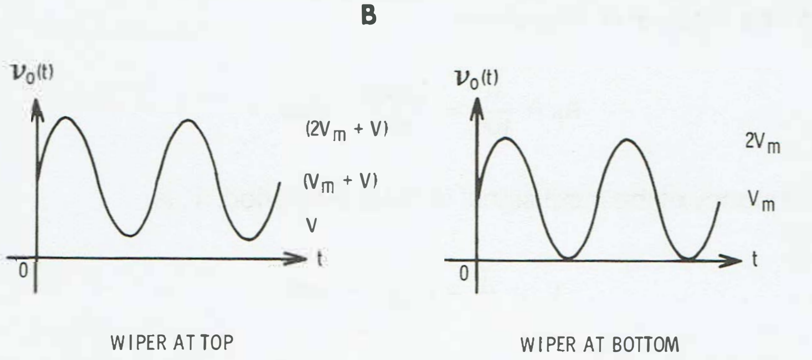
The effects of source resistance, load resistance, and the nonideal properties of real diodes in clampers are similar to those encountered in clippers. For this reason, the inequalities that had to be satisfied to design clippers also must be satisfied when designing clampers. Thus, component values should be chosen so that:

$$V_{TH} >> V_T \qquad R >> R_{TH}$$
$$R >> r_F \qquad R_L >> R$$
$$R << R_R \qquad R_P \leq \frac{R}{100}$$

An additional requirement imposed for clampers is that the discharging time constant must be sufficiently large so that the capacitor will not discharge appreciably during the time the diode is reverse biased. This time constant, however, must not be so large that the clamper fails to respond quickly to amplitude changes in the input voltage.



CIRCUIT



OUTPUT VOLTAGE

Figure 6-22

An adjustable positive clamper.
A. Circuit.
B. Output voltage.

An “exact” analysis to determine the optimum value for the size of the capacitor in a clamper is rather complex. Generally speaking, satisfactory results will be obtained if C is chosen so that:

$$C = \frac{50\,T}{R} \qquad \text{(Eq. 6-2)}$$

where: C = required capacitance in Farads (F)
T = period of the input voltage in seconds (s)
R = shunt resistance in ohms (Ω)

The following example illustrates a typical design problem.

Example 6-4

> *Design an adjustable clamper whose output voltage can be clamped between 0V and 5V. The signal source has a Thevenin voltage and resistance respectively of 10V peak at 1kHz and 600Ω. The input resistance of the circuit driven by the clamper is 1MΩ. Design the circuit, using the diode of Example 6-3.*

In Example 6-3, the optimum value for R was calculated to be 154.9kΩ. A 100kΩ resistor was selected since this was reasonably close to the calculated value, and satisfied the requirement that $R_L >> R$. Similarly, for this example, we select R = 100kΩ.

For effective clamping, or clipping, the resistance of the potentiometer must be small compared to the value of R. Therefore:

$$R_P \approx \frac{R}{100} \approx \frac{100\text{k}\Omega}{100} = 1\text{k}\Omega$$

Since the frequency of the input signal is 1kHz the period, T, is:

$$T = \frac{1}{f} = \frac{1}{1\text{kHz}} = 1\text{ms}$$

Using Equation 6-2 to calculate the required value of capacitance, yields:

$$C = \frac{50\,T}{R} = \frac{50\,(1\text{ms})}{100\text{k}\Omega} = 0.5\mu\text{F}$$

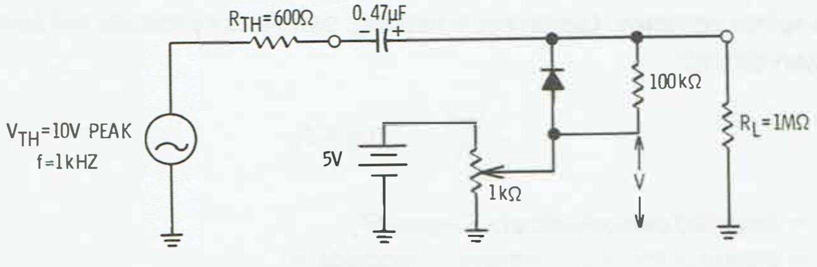


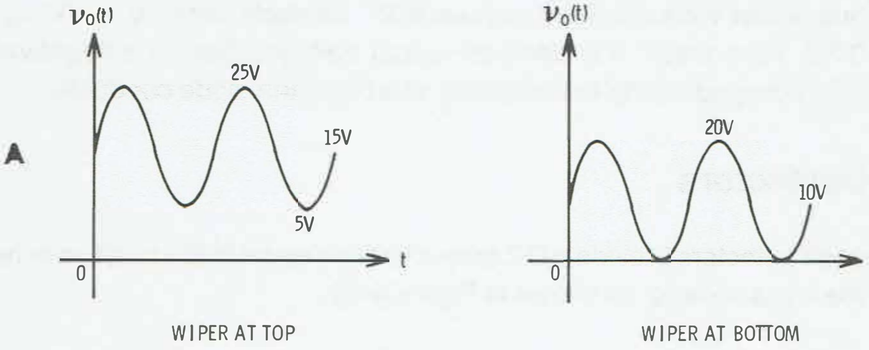
Figure 6-23

Circuit for Example 6-4.

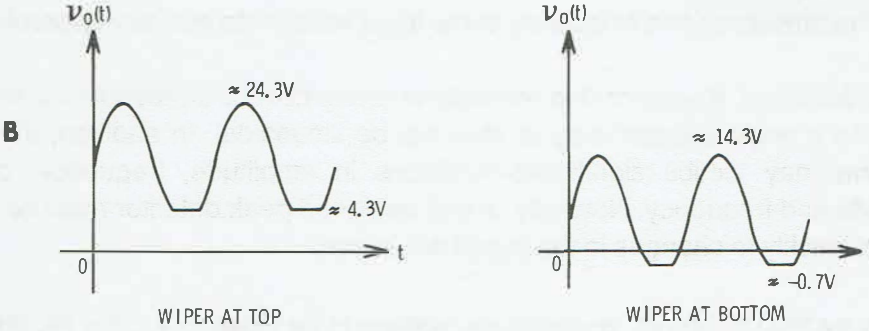
A standard value 0.47μF capacitor is selected for the design since it is quite close to the calculated value of 0.5μF. If you prefer, you could use a somewhat larger capacitance value. The completed circuit is shown in Figure 6-23.

Ideally, the output voltages would appear as shown in Figure 6-24A. In reality, the actual output voltages appear more like those shown in Figure 6-24B. The reasons for this are as follows:

1. The diode's turn-on voltage and the bias voltage are effectively connected series opposing. Consequently, the capacitor charges to a voltage V_T volts less than the ideal values. For this reason, the actual clamping levels are also V_T volts less than the ideal clamping levels.



IDEAL OUTPUT VOLTAGES



TYPICAL OUTPUT VOLTAGES

Figure 6-24

Ideal and typical output voltages for Examples 6-4.
A. Ideal output voltages.
B. Typical output voltages.

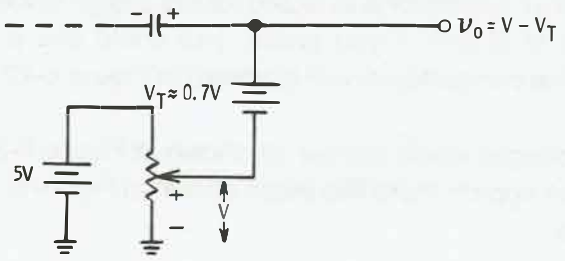


Figure 6-25

Partial schematic for Example 6-4.

2. When the diode conducts, the voltage across the diode is approximately V_T volts as shown by the partial schematic in Figure 6-25. Here, $\nu_o = V - V_T$. Thus, when V equals 5V, ν_o equals 4.3V. Similarly, when V = 0V, ν_o equals −0.7V. As a result, the clamped output voltages have the negative peaks slightly clipped during the relatively short time the diode conducts.

Peak Dectectors

Ideally, peak detectors provide a DC output voltage equal to the positive or negative peak of the input voltage, as shown in Figure 6-26.

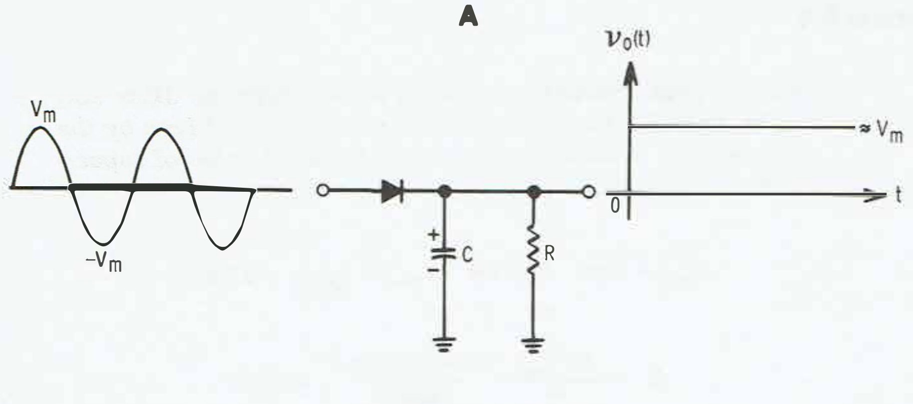
As you can see, the peak detectors in Figure 6-26 are essentially half-wave rectifiers with capacitor input filters. In rectifier applications, however, the following restrictions usually apply:

1. The input voltage is sinusoidal.
2. The amplitude and frequency of the input voltage do not vary appreciably.

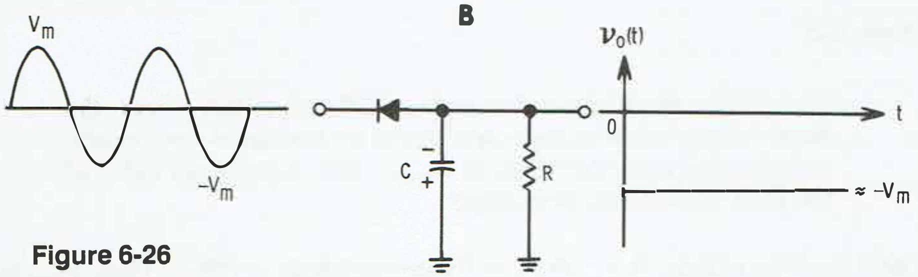
In peak detectors, the preceding restrictions are removed. Concequently, the input voltage to a peak detector may or may not be sinusoidal. In addition, the signal waveform may exhibit significant variations in amplitude, frequency, or both amplitude and frequency. Naturally, a well designed peak detector must be able to respond quickly to changes in the signal waveform.

In order for the DC output, or capacitor, voltage to be close to V_m, the RC time constant must be large compared to the period of the signal waveform. As was the case with clampers, however, if the time constant is too large, the output voltage will not be able to respond quickly to amplitude changes in the input voltage. Generally speaking, satisfactory results are obtained if the capacitor is chosen such that:

$$C = \frac{50\,T_{max}}{R} \qquad \text{(Eq. 6-3)}$$



POSITIVE PEAK DETECTOR



NEGATIVE PEAK DETECTOR

Figure 6-26

Peak detector circuits.

A. Positive peak detector.

B. Negative peak detector.

Note in Equation 6-3 that the **maximum** period of the signal waveform should be used to compute the required value of C. Since period and frequency are reciprocals:

$$T_{max} = \frac{1}{f_{min}} \qquad \text{(Eq. 6-4)}$$

where: f_{min} = lowest frequency input signal to be processed by the peak detector.

Incidentally, Equation 6-3 also applies to clampers that are designed to process a range of signal frequencies.

Designing Peak Detectors

The effects of source resistance, load resistance, and the nonideal properties of real diodes in peak detectors are similar to those encountered in clampers. Consequently, component values should be selected so that the following inequalities are satisfied:

$V_m >> V_T$ $\qquad$ $R << R_R$

$R >> r_F$ $\qquad$ $R >> R_{TH}$

Example 6-5

Design a peak detector to process signals between 3kHz and 25kHz. The input resistance of the circuit being driven by the peak detector is 68kΩ. Calculate the required value of capacitance.

$$f_{min} = 3\text{kHz}. \text{ Hence, } T_{max} = \frac{1}{3\text{kHz}} = 0.33\text{ms}$$

$$C = \frac{50\,T_{max}}{R} = \frac{50\,(0.33\text{ms})}{68\text{k}\Omega} = 0.24\mu\text{F}$$

Thus, you could use a standard value 0.33μF capacitor.

Example 6-6

To achieve near ideal peak detection in Example 6-5, what is the largest Thevenin resistance acceptable for the signal source driving the peak detector? Also, if $V_{TH} = 20V$, what is the value of the peak-inverse diode voltage?

For good peak detection, $R >> R_{TH}$. In Example 6-5, R = 68kΩ. Thus, R_{TH} must be small compared to 68kΩ. Using the 20:1 rule then:

$$R_{TH_{max}} = \frac{R}{20} = \frac{68\text{k}\Omega}{20} = 3.4\text{k}\Omega$$

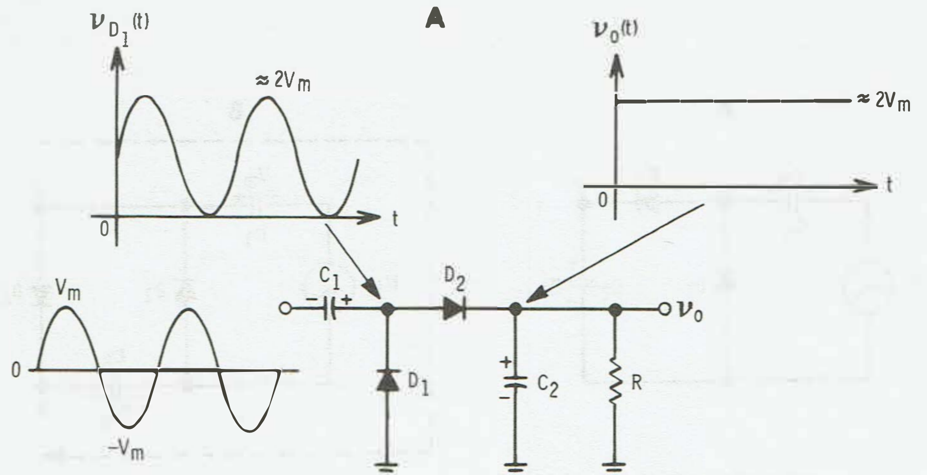
Since a peak detector is essentially a half-wave rectifier with a capacitor input filter, the peak-inverse voltage equals $2V_m$. Thus:

$$\text{PIV} = 2V_m = 2(20V) = 40V$$

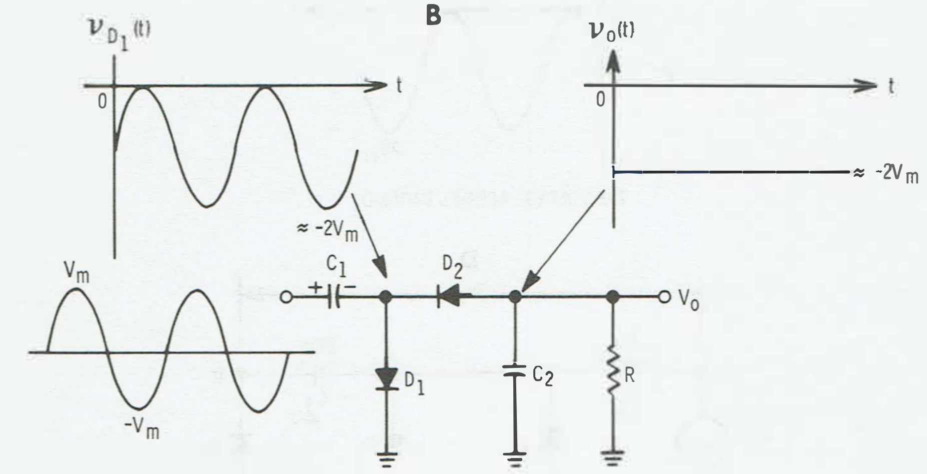
Peak-to-Peak Detectors

Ideally, a peak-to-peak detector provides a DC output voltage equal to the peak-to-peak value of the input voltage. Peak-to-peak detectors are constructed by cascading a clamper and peak detector as shown in Figure 6-27.

In Figure 6-27A, capacitor C_1 and diode D_1 constitute a positive clamper. Thus, the voltage across D_1 essentially varies between 0V and $2V_m$ volts. Similarly, diode D_2 and capacitor C_2 constitute a positive peak detector. Since the input to the peak detector is the clamper output, which has a positive peak of $2V_m$, the DC output voltage from the peak detector will also equal $2V_m$.



POSITIVE PEAK-TO-PEAK DETECTOR



NEGATIVE PEAK-TO-PEAK DETECTOR

Figure 6-27

Peak to peak detector circuits.

A. Positive peak to peak detector.

B. Negative peak to peak detector.

By reversing the direction of the diodes in Figure 6-27A, you obtain the circuit shown in Figure 6-27B. Here, a negative clamper is cascaded with a negative peak detector. The result of course, is a negative peak-to-peak detector.

The design of a peak-to-peak detector is quite similar to the design of the clamper and peak detector circuits discussed previously. In most cases, the capacitors are chosen so that $C_1 = C_2 = C$. Equation 6-3 can then be used to calculate the required value of capacitance.

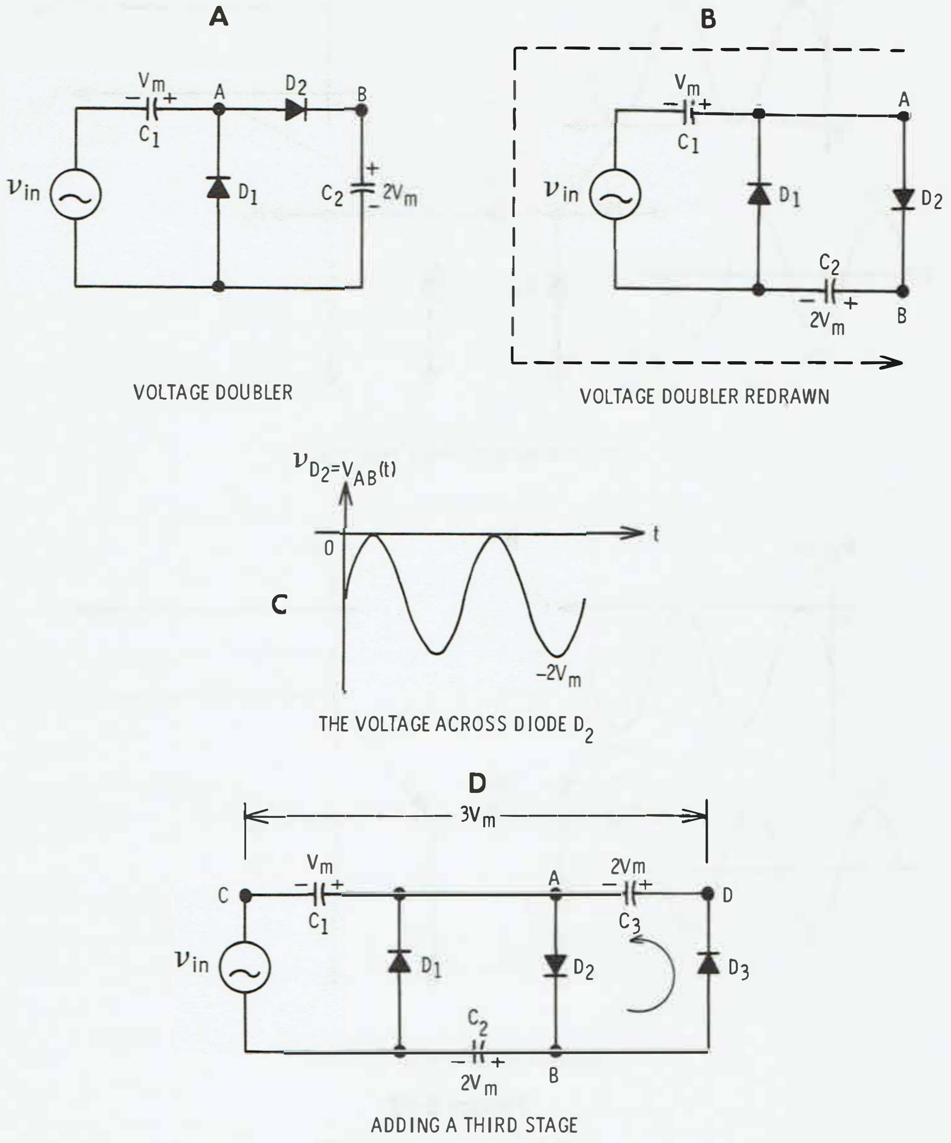


Figure 6-28

Constructing a voltage tripler.

A. Voltage doubler.
B. Voltage doubler redrawn.
C. The voltage across diode D_2.
D. Adding a third stage.

Voltage Multipliers

Voltage multipliers, ideally, provide a DC output voltage such that:

$$V_o = n\,V_m \qquad \text{(Eq. 6-5)}$$

Consequently, a peak detector can be thought of as a voltage multiplier with n = 1. Similarly, a peak-to-peak detector is a voltage multiplier with n = 2. For this reason, peak-to-peak detectors are also referred to as **voltage doublers.**

Clampers and peak detectors are essentially the same circuit. In each case, the input voltage is connected across the series combination of a capacitor and diode. Whether a particular circuit functions as a clamper or peak detector simply depends on where the output voltage is taken. Recall that for both clampers and peak detectors the "circuit action" may be described as follows:

1. The capacitor charges to the peak value of the input voltage.
2. Once the capacitor is fully charged, the diode voltage equals the algebraic sum of the capacitor and input voltages.

By cascading diode capacitor combinations, it is possible to construct voltage multipliers with n = 3, 4, ... and so forth. To see how this can be done, refer to Figure 6-28.

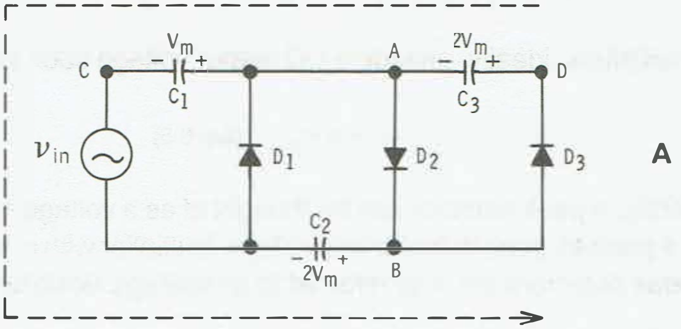
In Figure 6-28A, we have the voltage doubler circuit discussed previously. Here, the DC voltage across C_1 equals V_m, and the DC voltage across C_2 equals $2V_m$.

The voltage doubler in Figure 6-28A can be redrawn as shown in Figure 6-28B. Here, an expression for the voltage across D_2 is obtained by writing an appropriate loop equation. Thus, starting at point A and going around the outside loop until you arrive at point B, yields:

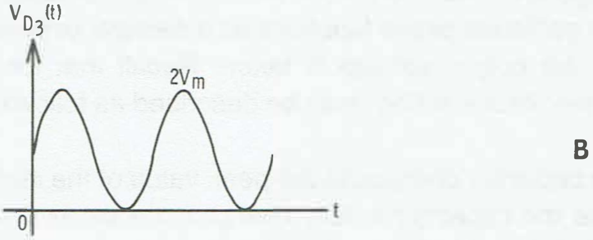
$$\begin{aligned} \nu_{D_2} = \nu_{AB} &= V_{C_1} + \nu_{in} - V_{C_2} \\ &= V_m + \nu_{in} - 2V_m \\ &= \nu_{in} - V_m \end{aligned}$$

Consequently, the voltage across D_2 appears as shown in Figure 6-28C.

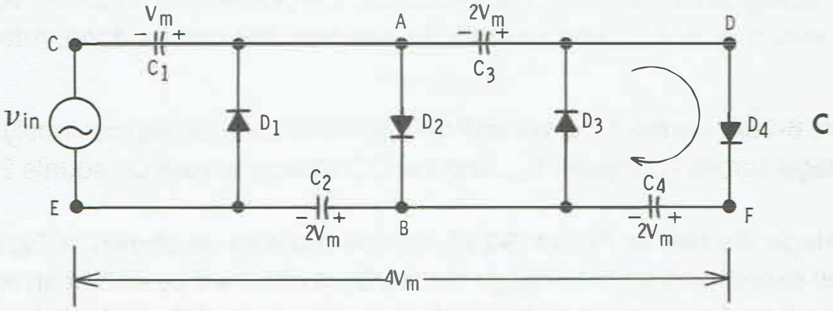
In Figure 6-28D, a third stage has been added to the circuit. Note that the voltage across D_2 is the input voltage for the third stage. Since the voltage across D_2 has a peak value equal to $2V_m$, capacitor C_3 charges through diode D_3 to $2V_m$. As a result, the DC output voltage taken from points C and D equals $3V_m$. This is the sum of the voltages across C_1 and C_3. For this reason, the voltage multiplier in Figure 6-28D is called a voltage tripler.



VOLTAGE TRIPLER



THE VOLTAGE ACROSS DIODE D_3



ADDING A FOURTH STAGE

Figure 6-29

Constructing a voltage quadrupler.

A. Voltage tripler.

B. The voltage across diode D_3.

C. Adding a fourth stage.

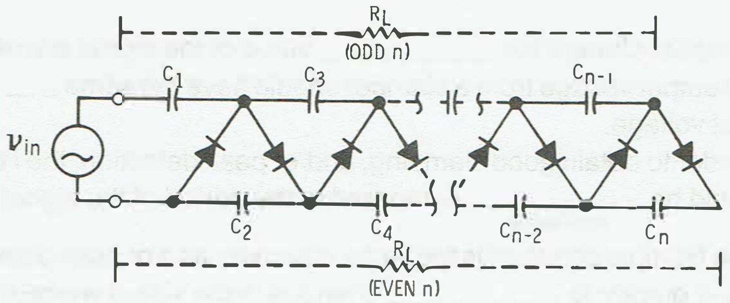


Figure 6-30

An n section voltage multiplier.

To construct higher-order voltage multipliers, additional stages are added to the voltage tripler. This concept is illustrated in Figure 6-29.

In Figure 6-29A, you can determine the voltage across D_3 by writing the following loop equation:

$$\begin{aligned} \nu_{D_3} &= V_{C_3} + V_{C_1} + \nu_{in} - V_{C_2} \\ &= 2V_m + V_m + \nu_{in} - 2V_m \\ &= \nu_{in} + V_m \end{aligned}$$

Thus, the voltage across D_3 appears as shown in Figure 6-29B.

In Figure 6-29C, a fourth stage has been added to the voltage tripler. The input voltage to the fourth stage is the voltage across D_3. Since the voltage across D_3 has a peak value of $2V_m$, capacitor C_4 charges through diode D_4 to $2V_m$. Consequently, the output voltage across points E and F equals $4V_m$. This is the sum of the voltages across C_2 and C_4. For this reason, the voltage multiplier in Figure 6-29C is called a voltage quadrupler.

Theoretically, as shown in Figure 6-30, any number of sections can be added to the original voltage doubler. In practice however, the percent voltage regulation quickly increases as more sections are added. For this reason, voltage multipliers with n greater than 5 are uncommon.

Self-Test Review

11. Clampers change the __________ value of the signal waveform.
12. The output voltage from a clamper should have the same __________ as the input voltage.
13. In order to obtain good clamping, and/or peak detection, the RC time constant should be __________ compared to the period of the signal waveform.
    large/small
14. If the RC time constant is too large, clampers and/or peak detectors, do not respond quickly to __________ changes in the signal waveform.
15. The ____________ frequency of the signal waveform is used to compute the
    maximum/minimum
    required size for the capacitor in clampers and peak detectors.
16. Another name for a peak-to-peak detector is a ________________ ________________.

Refer to Figure 6-31 for questions 17 and 18.

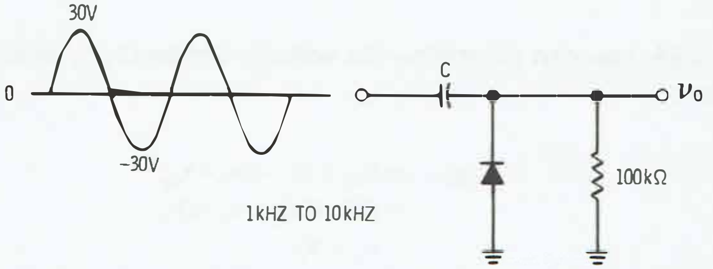


Figure 6-31

Circuit for questions 17 and 18.

17. The capacitor should have a value of approximately __________ μF.
18. The average value of the voltage across the 100kΩ resistor equals ____ V.

Refer to Figure 6-32 for questions 19 and 20.

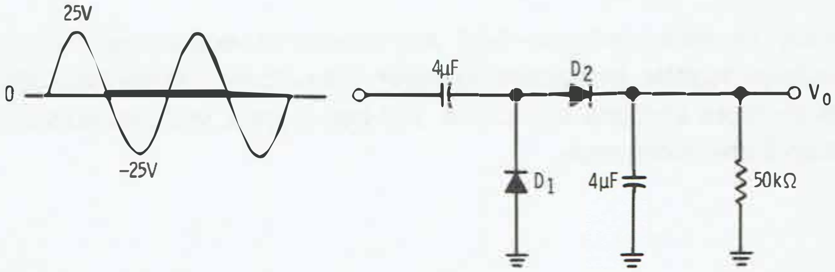


Figure 6-32

Circuit for questions 19 and 20.

19. For proper operation, the input signal should have a frequency no lower than __________ Hz.

20. The DC output voltage is approximately __________ V.

Answers

11. average or DC
12. shape
13. large
14. amplitude
15. minimum
16. voltage doubler
17. 0.5μF
18. 30V
19. 250Hz
20. 50V

The solution to appropriate questions follow:

17.

$$T_{max} = \frac{1}{f_{min}} = \frac{1}{1\text{kHz}} = 1\text{ms}$$

$$C = \frac{50\,T_{max}}{R} = \frac{50\,(1\text{ms})}{100\text{k}\Omega} = 0.5\mu\text{F}$$

18. The circuit in Figure 6-31 is a positive clamper. For the given input voltage, the output voltage is sinusoidal — with a positive peak clamped at 60V and a negative peak clamped at 0V. Thus, the average, DC, value of the output voltage is 30V.

19. The RC time constant for the peak-to-peak detector in Figure 6-32 is:

$$\tau = RC = 50\text{k}\Omega\,(4\mu\text{F}) = 200\text{ms}$$

From Equation 6-3 you can see that the time constant, RC, equals 50 T_{max}. Therefore,

$$T_{max} = \frac{\tau}{50} = \frac{200\text{ms}}{50} = 4\text{ms}$$

Thus

$$f_{min} = \frac{1}{T_{max}} = \frac{1}{4\text{ms}} = 250\text{Hz}$$

20. A peak-to-peak detector provides a DC output voltage equal to the peak-to-peak value of the input voltage. For a symmetrical input voltage, $\nu_o = 2V_m$, or 50V in this case.

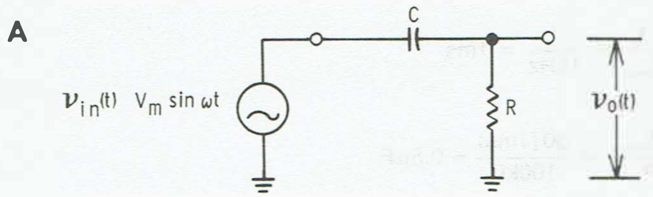
THE AMAZING RC CIRCUIT

Units 1, 2, and 3 stressed the techniques used to analyze electronic circuits. In this section, you will use these techniques to analyze a series RC circuit as the frequency of the AC source is varied from 0Hz to ∞ Hz.

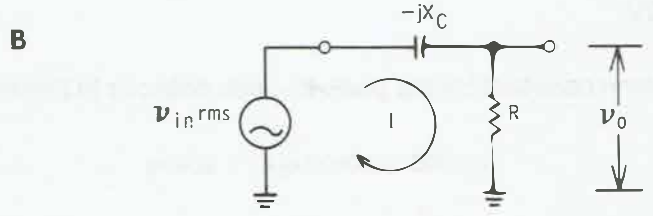
Although relatively simple, the series RC circuit is surprisingly versatile. As you will see, this circuit can be used to perform any one of eight different functions. For a given circuit, the specific function performed is determined by the relationship that exists between the frequency of the signal waveform and the RC time constant.

Case I — Output Taken Across Resistor

Consider the circuit shown in Figure 6-33A. Here, we wish to determine how the output voltage varies as the frequency of the input voltage is varied from 0Hz to ∞ Hz.



TIME DOMAIN CIRCUIT



FREQUENCY DOMAIN CIRCUIT

Figure 6-33

A series RC circuit in which the output voltage is taken across the resistor.

A. Time domain circuit.

B. Frequency domain circuit.

To begin, you convert the time domain circuit to its frequency domain equivalent circuit as shown in Figure 6-33B.

By inspection the input impedance, Z, at any frequency is:

$$Z = R - jX_c = R - \frac{j}{\omega C} = \frac{\omega RC - j}{\omega C}$$

Therefore, the current, I, is:

$$I = \frac{V}{Z} = \frac{V}{\frac{\omega RC - j}{\omega C}} = \frac{V\omega C}{\omega RC - j}$$

Since $V_o = V_R$, we have:

$$V_o = IR = \frac{V\,\omega RC}{\omega RC - j}$$

Dividing numerator and denominator by ωRC:

$$V_o = \frac{V}{1 - j\frac{1}{\omega RC}}$$

The voltage gain of the circuit is defined as the ratio of the output voltage to the input voltage. Thus:

$$A_v = \frac{V_o}{V_{in}} = \frac{1}{1 - j\frac{1}{\omega RC}}$$

Converting from rectangular to polar notation:

$$A_v = \frac{1\angle 0°}{\sqrt{1 + \frac{1}{(\omega RC)^2}} - \angle \arctan \frac{1}{\omega RC}}$$

$$A_v = \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}} \angle \arctan \frac{1}{\omega RC}$$

Letting $(\omega RC) = n$ yields:

$$A_v = \frac{1}{\sqrt{1 + \frac{1}{n^2}}} \angle \arctan \frac{1}{n} \qquad \text{(Eq. 6-6)}$$

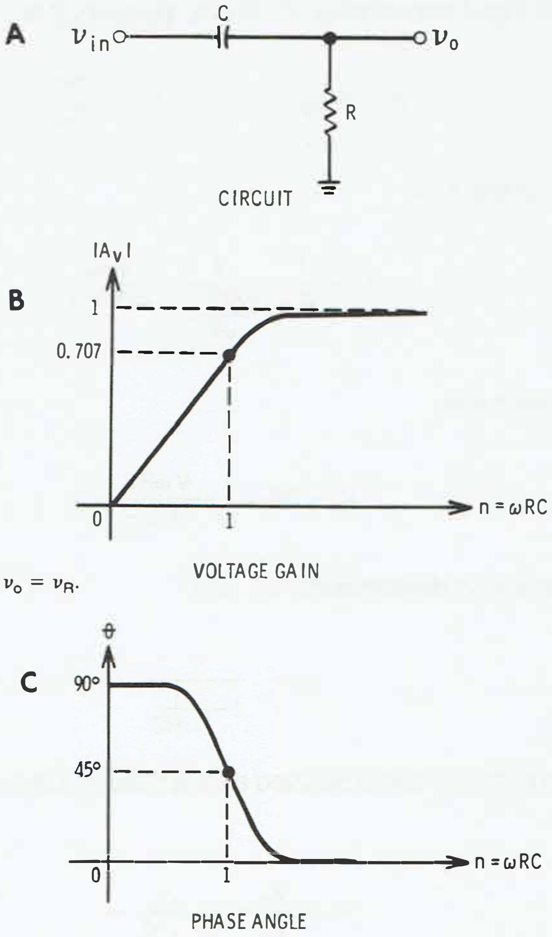


Figure 6-34

Frequency response of the series RC circuit when $v_o = v_R$.

A. Circuit.
B. Voltage gain.
C. Phase angle.

Equation 6-6 indicates that the voltage gain is described by a complex number. For this reason, the voltage gain consists of two parts — a **magnitude**, $|A_v|$, and a **phase angle**, θ.

In some circuits, you are primarily interested in the magnitude of the gain. In other circuits, phase information, or both magnitude and phase information are important. Consequently, it is useful to split Equation 6-6 into two parts as follows:

$$|A_v| = \frac{1}{\sqrt{1 + \frac{1}{n^2}}} \qquad \text{(Eq. 6-6a)}$$

$$\theta = \arctan \frac{1}{n} \qquad \text{(Eq. 6-6b)}$$

By substituting different values of n into Equation 6-6a and Equation 6-6b, you can obtain the data required to plot the curves sketched in Figure 6-34B and Figure 6-34C respectively. Note that when $n = 1$, the voltage gain has a value of 0.707, and the phase angle equals 45°. The frequency at which this occurs is called the **cutoff frequency**, f_{co}, and may be found as follows:

$$n = \omega RC = 1$$

$$2\pi f_{co} RC = 1$$

or

$$f_{co} = \frac{1}{2\pi RC} \qquad \text{(Eq. 6-7)}$$

Finally, the ratio of any frequency, f, to the cutoff frequency, f_{co}, is:

$$\frac{f}{f_{co}} = \frac{f}{\frac{1}{2\pi RC}} = 2\pi fRC = \omega RC$$

Since $\omega RC = n$, and the ratio of f to f_{co} equals ωRC, it follows that the ratio of f to f_{co} also equals n.

The following table summarizes the results of our analysis.

Parameter	Formula
Voltage gain	$\lvert A_v \rvert = \frac{1}{\sqrt{1 + \frac{1}{n^2}}}$
Phase angle	$\theta = \arctan \frac{1}{n}$
cutoff frequency	$f_{co} = \frac{1}{2\pi RC}$
n	$n = \frac{f}{f_{co}} = \omega RC$

Table 6-6

Characteristics of the Series RC Circuit When $V_o = V_R$.

Example 6-7

In Figure 6-35, $v_{in}(t) = 10 \sin 3{,}768t$. Determine the cutoff frequency of the circuit. Also, sketch the input and output voltages.

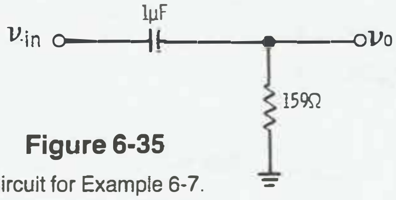


Figure 6-35

Circuit for Example 6-7.

Referring to Table 6-6:

$$f_{co} = \frac{1}{2\pi RC} = \frac{1}{2\pi(159\Omega)(1\mu F)} = 1\text{kHz}$$

Since the radian frequency of the input voltage is 3 768 rad/s, the cyclic frequency is:

$$f = \frac{\omega}{2\pi} = \frac{3{,}768}{2\pi} = 0.6\text{kHz}$$

Again, referring to Table 6-6:

$$n = \frac{f}{f_{co}} = \frac{0.6\text{kHz}}{1\text{kHz}} = 0.6$$

Therefore:

$$|Av| = \frac{1}{\sqrt{1 + \frac{1}{n^2}}} = \frac{1}{\sqrt{1 + \frac{1}{(0.6)^2}}} = \frac{1}{\sqrt{3.78}} = 0.514$$

Since $v_{in} = 10V$ peak, and $|A_v| = v_o/v_{in}$ the output voltage, v_o, is:

$$v_o = |A_v|\, v_{in}$$
$$v_o = 0.514\,(10V) = 5.14V \text{ peak.}$$

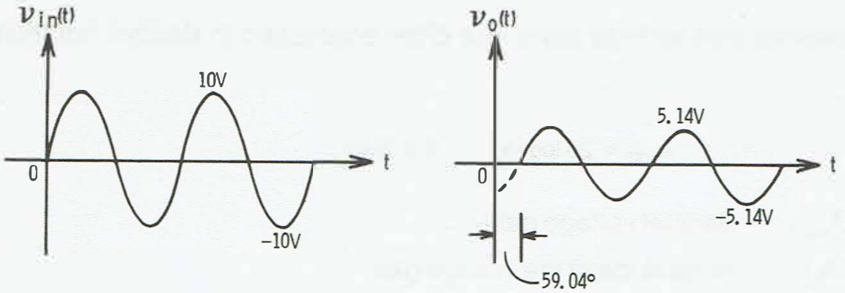


Figure 6-36

Input and output voltages for Example 6-7.

From Table 6-6, the phase angle, θ, is:

$$\theta = \arctan \frac{1}{n}$$

$$\theta = \arctan \frac{1}{0.6} = \arctan 1.67 = 59.04°$$

Sketches of the input and output voltages are provided in Figure 6-36. Here, note that the output voltage differs from the input voltage in both amplitude and phase. By using Equation 6-6a and Equation 6-6b, you can calculate the amplitude and phase angle of the output voltage for any signal frequency.

High-Pass Filter

Filters are frequency selective circuits. A high-pass filter tends to pass signals whose frequencies are above the filter's cutoff frequency, and attenuate those below the filter's cutoff frequency. In most cases, a signal that is processed by a filter experiences changes in both amplitude and phase.

The RC circuit analyzed previously meets all the requirements for a high-pass filter. Therefore, Equation 6-6a and Equation 6-6b can be used to predict the magnitude of the voltage gain and phase angle for any signal frequency.

Bode Plots

In Unit 2, you learned that voltage gains are often expressed in decibel notation. Thus:

$$A_{vdB} = 20 \log |A_v| \qquad \text{(Eq. 6-8)}$$

where: A_{vdB} = decibel voltage gain

$|A_v|$ = magnitude of the voltage gain

Recall that Equation 6-6a can be used to compute the magnitude of the voltage gain for any signal frequency. Specifically:

$$|A_v| = \frac{1}{\sqrt{1 + \frac{1}{n^2}}}$$

where: $|A_v|$ = magnitude of the voltage gain at any frequency, f.

n = f/f_c

For large values of n, the term $1/n^2$ approaches zero, and $|A_v|$ approaches 1. Similarly, for small values of n, the term $1/n^2$ is large compared to 1. In this case, $|A_v|$ approaches $1/\sqrt{\frac{1}{n^2}}$ or n.

Consequently, the following approximations are quite accurate for frequencies well above and well below the cutoff frequency:

Above cutoff ($f >> f_c$)	$\lvert A_v\rvert \approx 1$ $A_{vdB} = 20 \log 1 = 0$
Below cutoff ($f << f_c$)	$\lvert A_v\rvert \approx n$ $A_{vdB} = 20 \log n$

Example 6-8

Use the approximate formulas to predict the dB voltage gain of a high-pass RC filter for the following values of n: 0.01, 0.1, 0.25, 0.5, 1, 2, 4, 10, and 100. Compare the approximate values with those predicted by Equation 6-6a.

APPROXIMATE VALUES:

For $n \geqslant 1$, you assume $|A_v| \approx 1$ and $A_{vdB} = 0$.

For $n < 1$, $A_{vdB} \approx 20 \log n$. For example, when $n = 0.5$, $A_{vdB} \approx 20 \log 0.5 = -6.02$ dB.

VALUES BASED ON EQUATION 6-6A:

Here, you simply substitute the value of n into Equation 6-6a. For example, when $n = 0.5$.

$$A_v = \frac{1}{\sqrt{1 + \frac{1}{(0.5)^2}}} = \frac{1}{\sqrt{5}} = 0.447$$

Thus: $$A_{vdB} = 20 \log 0.447 = -6.99 \text{ dB}$$

The results of similar calculations for the remaining values of n are provided in Table 6-7.

n	(Approximate) A_{vdB}	(Eq. 6-6a) A_{vdB}
0.01	−40	−40
0.1	−20	−20
0.25	−12	−12.3
0.5	−6.02	−6.99
1	0	−3.01
2	0	−0.97
4	0	−0.26
10	0	−0.04
100	0	≈0

Table 6-7

Characteristics of the Series RC Circuit When $V_o = V_c$.

Referring to Table 6-7, you can draw the following conclusions:

1. For frequencies above the cutoff frequency, the approximate dB voltage gain is 0.
2. For frequencies below the cutoff frequency, the voltage gain changes by 6dB/octave and 20dB/decade. An octave is a 2:1 frequency ratio, and a decade is a 10:1 frequency ratio.
3. The approximate dB voltage gains agree closely with those based on Equation 6-6a for frequencies well below and well above the cutoff frequency. Note that the largest error, –3.01dB, occurs at the cutoff frequency.

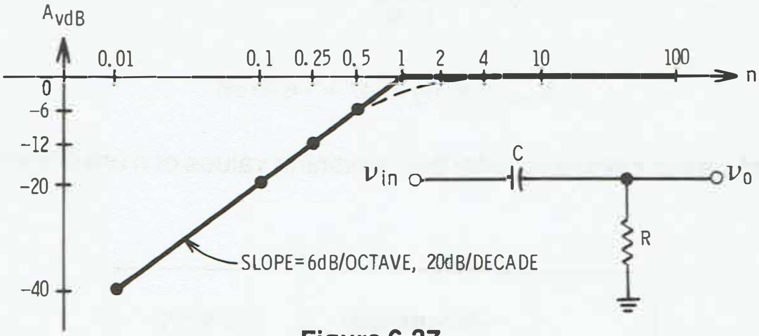


Figure 6-37

Bode plot for the high-pass filter.

A graph of decibel voltage gain versus frequency, based on the approximate formulas, is called a **Bode plot.** The Bode plot of the RC high-pass filter is provided in Figure 6-37. Here, note that, for frequencies below the cutoff frequency, each time the frequency changes by a factor of 2 (an octave), the dB gain changes by 6dB. Similarly, each time the frequency changes by a factor of 10 (a decade), the dB gain changes by 20dB. The dashed line in Figure 6-37 illustrates how the actual response differs from the approximate response near cutoff.

Example 6-9

Design a high-pass filter to pass frequencies above 10kHz. Predict the peak output voltage, assuming a 10V peak input voltage, for frequencies of 0.1kHz, 5kHz, 10kHz, and 100kHz. Also, sketch a Bode plot for the filter.

The filter's cutoff frequency is:

$$f_{co} = \frac{1}{2\pi RC}$$

Consequently, you select a standard value capacitor and then calculate the required value of resistance. Selecting $C = 0.01\mu F$, we have:

$$R = \frac{1}{2\pi f_{co} C} = \frac{1}{2\pi(10 \times 10^3)(.01 \times 10^{-6})} = 1.59k\Omega$$

0.1kHz is two decades below the 10kHz cutoff frequency. Thus, $A_{vdB} \approx -40dB$. Since $A_{vdB} = 20 \log |A_v|$ we have:

$$20 \log |A_v| = -40$$
$$\log |A_v| = -40/20 = -2$$
$$|A_v| = 10^{-2} = 0.01$$

Therefore, $\nu_o = |A_v| \nu_{in} = 0.01\,(10V) = 0.1V$ peak.

Similarly, 5kHz is one octave below the 10kHz cutoff frequency. Thus, $A_{vdB} \approx -6dB$.

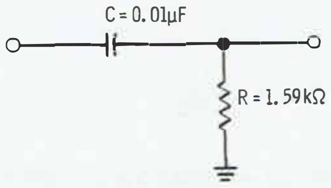
$$20 \log |A_v| = -6$$
$$\log |A_v| = -6/20 = -0.3$$
$$|A_v| = 10^{-0.3} \approx 0.5$$

Therefore, $\nu_o = |A_v| \nu_{in} = 0.5\,(10V) = 5V$ peak.

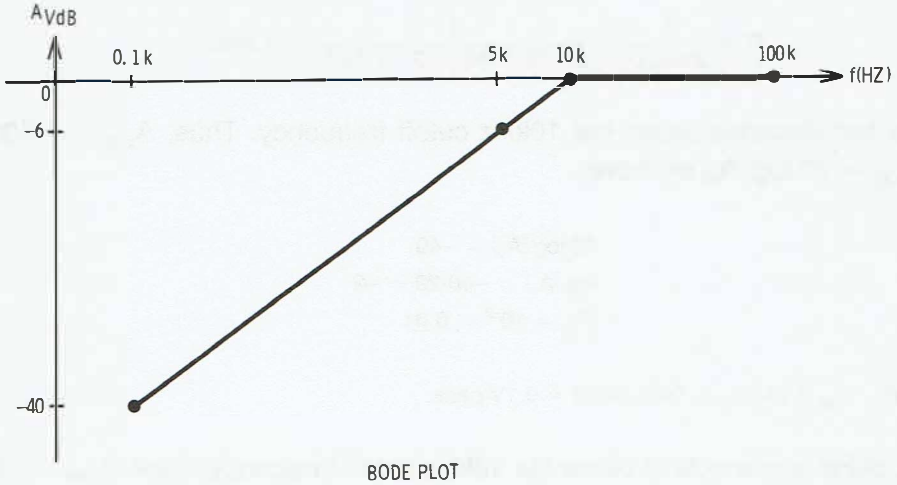
At the 10kHz cutoff frequency, the Bode plot predicts a dB gain of 0. However, you know that the actual dB gain at cutoff is about –3dB, which corresponds to a voltage gain of 0.707. Thus, when f = 10kHz:

$$\nu_o = |A_v| \nu_{in} = 0.707\,(10V) = 7.07V \text{ peak.}$$

Since 100kHz is well above the cutoff frequency $A_{vdB} \approx 0$, which corresponds to a voltage gain of 1. Thus, $\nu_o \approx 10V$ peak.



CIRCUIT



BODE PLOT

Figure 6-38

Circuit and Bode plot for Example 6-9.

The circuit and Bode plot are illustrated in Figure 6-38. Due to their simplicity, Bode plots are widely used in industry.

Coupling Circuit.

Recall, from the discussion in Unit 4, that the function of a coupling capacitor is to pass, or couple, an AC signal from one point to another.

Ideally, the output voltage from a coupling circuit has the same amplitude and phase angle as the input voltage. By examining the Bode plot of the high-pass filter, it is clear that the filter functions as a coupling circuit for signal frequencies well above cutoff.

In Unit 4, coupling capacitors were selected so that at the lowest frequency to be coupled:

$$|X_c| = \frac{R}{20}$$

since

$$|X_c| = \frac{1}{2\pi fC},$$

we have:

$$\frac{1}{2\pi fC} = \frac{R}{20}$$

Solving for f yields:

$$f = \frac{20}{2\pi RC}$$

Since

$$f_{co} = \frac{1}{2\pi RC},$$

it follows that:

$$f = 20\, f_{co}$$

Thus, the ratio of $f/f_{co} = 20$ when C is selected so that $|X_c| = R/20$.

Substituting $n = 20$ into Equation 6-6a and Equation 6-6b, respectively, yields:

$$|A_v| = \frac{1}{\sqrt{1 + \frac{1}{n^2}}} = \frac{1}{\sqrt{1 + \frac{1}{(20)^2}}} \approx 0.999$$

$$\theta = \arctan \frac{1}{n} = \arctan \frac{1}{20} = 2.86°$$

Thus, by selecting C so that $|X_c| = R/20$, the voltage gain is very close to 1, and the phase angle is close to 0°. This is **why** the rule of thumb provided in Unit 4 is effective in selecting appropriate values for coupling capacitors.

0 to 90° Phase Shifter

In many applications, it is necessary to introduce a precise phase shift, at a particular point in the circuit. For angles between 0 and 90°, an RC circuit is often adequate for this purpose.

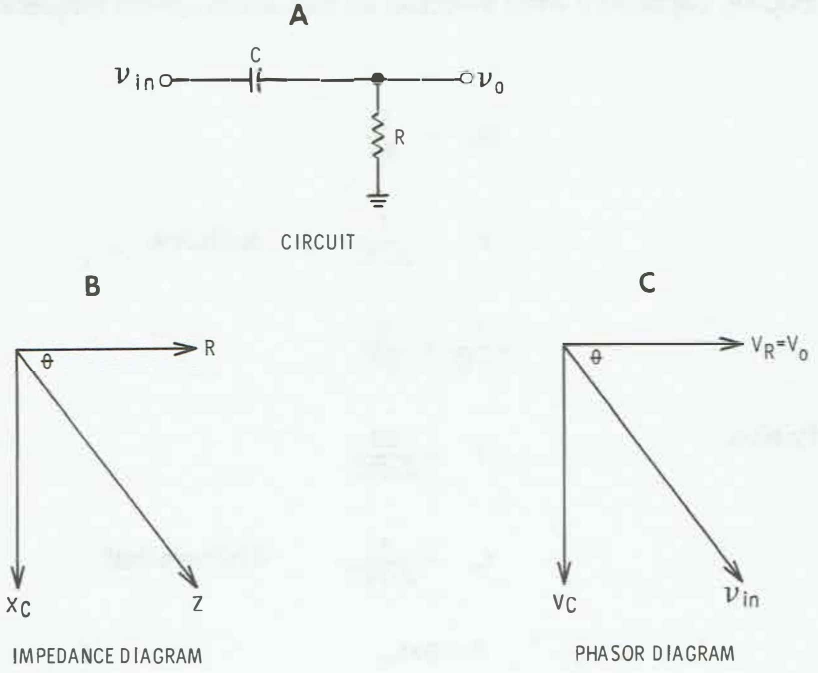


Figure 6-39

A 0 to 90° phase shifter.

Note that V_R leads V_{in} by the angle Θ.

A. Circuit.

B. Impedance diagram.

C. Phasor diagram.

Figure 6-39B and Figure 6-39C illustrate the respective impedance and phasor diagrams for the RC circuit in Figure 6-39A. Here, note that the voltage across the resistor leads the input voltage by the angle θ. For this reason, the circuit in Figure 6-39A is often referred to as a **lead circuit.**

From Figure 6-39B it is obvious that the angle θ is:

$$\theta = \arctan \frac{|X_c|}{R} \qquad \text{(Eq. 6-9)}$$

By using Equation 6-9, you can select component values to produce phase angles between, approximately, 0° and 90°.

Example 6-10

Design a circuit so that the output voltage leads the input voltage by 30°, at 1kHz. What is the peak output voltage for a 10V peak input voltage?

As usual, you select a standard value capacitor, and then calculate the required value of resistance. Selecting C = 0.01μF, you proceed as follows:

$$|X_c| = \frac{1}{2\pi fC} = \frac{1}{2\pi(10^3)(0.01 \times 10^{-6})} = 1.59\text{k}\Omega$$

Since $\theta = \arctan \dfrac{|X_c|}{R}$, the required value of R is:

$$R = \frac{|X_c|}{\tan\theta} = \frac{15.9\text{k}\Omega}{\tan 30°} = 27.6\text{k}\Omega$$

With C = 0.01μF and R = 27.6kΩ the cutoff frequency, f_{co}, is:

$$f_{co} = \frac{1}{2\pi RC} = \frac{1}{2\pi(27.6\text{k}\Omega)(0.01\mu\text{F})} = 0.577\text{kHz}$$

Therefore, $n = f/f_{co} = 1\text{kHz}/0.577\text{kHz} = 1.73$

With n = 1.73 the voltage gain is:

$$|A_v| = \frac{1}{\sqrt{1 + \dfrac{1}{(1.73)^2}}} = \frac{1}{\sqrt{1.33}} = 0.866$$

Thus, when $\nu_{in} = 10\text{V}$ peak:

$$\nu_o = 0.866\,(10\text{V}) = 8.66\text{V}.$$

Differentiator

A high-pass filter passes frequencies above the cutoff frequency, and attenuates frequencies below the cutoff frequency. Fourier's theorem indicates that a complex wave consists of numerous frequency components. Consequently, a high-pass filter attenuates the harmonic components of a complex wave whose frequencies are below the filter's cutoff frequency. As a result, when complex waves are applied to filters, the resulting output voltages may be quite different from the input voltages.

In a given RC circuit, the function performed by the circuit is determined by the relationship that exists between the period of the input signal and the RC time constant. For example, when the time constant is large compared to the period of the input signal, the high-pass filter acts like a coupling circuit.

With advanced mathematics, it can be shown that a high-pass filter acts like a differentiator when the time constant is small compared to the period of the input signal. **A differentiator provides an output voltage that is directly proportional to the slope of the input voltage.** As a guide, good differentiation is obtained if component values are chosen such that:

$$RC \leqslant \frac{T}{20} \qquad \text{(Eq. 6-10)}$$

The following example illustrates the concept of a differentiator.

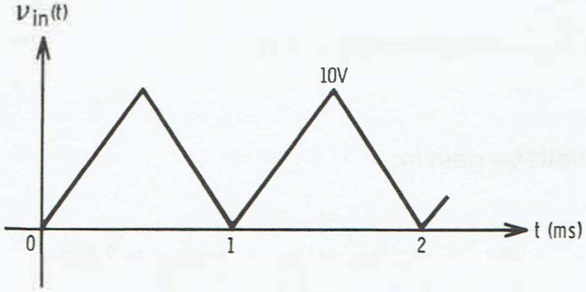


Figure 6-40

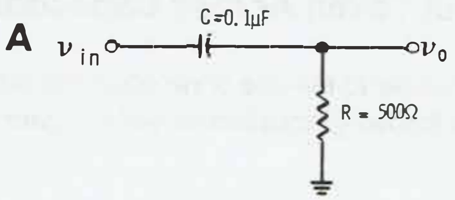
Voltage for Example 6-40.

Example 6-11

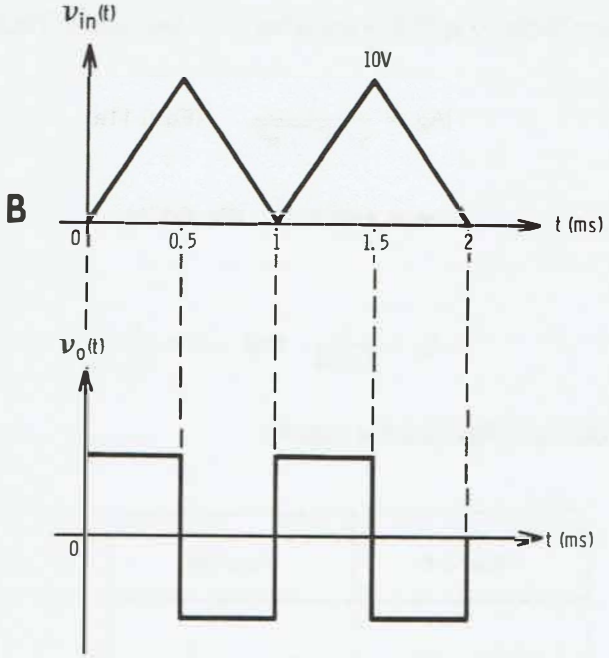
Design a differentiator to process the voltage in Figure 6-40. Also, sketch the expected "shape" of the output voltage.

In Figure 6-40, $T = 1ms$. Thus:

$$RC = \frac{T}{20} = \frac{1ms}{20} = 0.05ms$$



DIFFERENTIATOR



INPUT AND OUTPUT VOLTAGES

Figure 6-41

Solutions to Example 6-11.

A. Differentiator.

B. Input and output voltages.

As usual, you select a standard value capacitor and then calculate the required resistance. Thus, selecting C = $0.1\mu F$, we have:

$$R\,(0.1\mu F) = 0.05\text{ms}$$

$$R = \frac{0.05\text{ms}}{0.1\mu F} = 500\Omega$$

The circuit for the differentiator is illustrated in Figure 6-41A.

Since the input voltage is composed of straight-line segments, the slope of the input voltage is constant for a particular segment. Note however, that some segments have positive slopes while other segments have negative slopes.

Since the output from a differentiator is directly proportional to the slope of the input voltage, the differentiator effectively converts the triangular input voltage into a square wave output voltage as shown in Figure 6-41B. Obviously, one application for differentiators is as waveform converters.

Case II — Output Taken Across Capacitor

Analyzing this case, similar to the one done when the output was taken across the resistor, results in the following equation for voltage gain:

$$A_v = \frac{1}{\sqrt{1 + n^2}} -\angle \arctan n \qquad \text{(Eq. 6-11)}$$

As before, it is desirable to split the equation into two parts. Thus:

$$|A_v| = \frac{1}{\sqrt{1 + n^2}} \qquad \text{(Eq. 6-11a)}$$

$$-\alpha = \arctan n \qquad \text{(Eq. 6-11b)}$$

Also, as before:

$$f_{co} = \frac{1}{2\pi RC} \quad \text{and} \quad n = \frac{f}{f_{co}}$$

The following table summarizes the results.

Parameter	Formula
Voltage gain	$\lvert A_v \rvert = \frac{1}{\sqrt{1 + n^2}}$
Phase angle	$-\alpha = \arctan n$
cutoff frequency	$f_{co} = \frac{1}{2\pi RC}$
n	$n = \frac{f}{f_{co}} = \omega RC$

Table 6-7

Characteristics of the Series RC Circuit When $V_o = V_c$.

The voltage gain and phase characteristics of the circuit are illustrated in Figure 6-42. Here, note that the circuit has the characteristics of a low-pass filter, since frequencies below f_{co} are passed, and frequencies above f_{co} are attenuated.

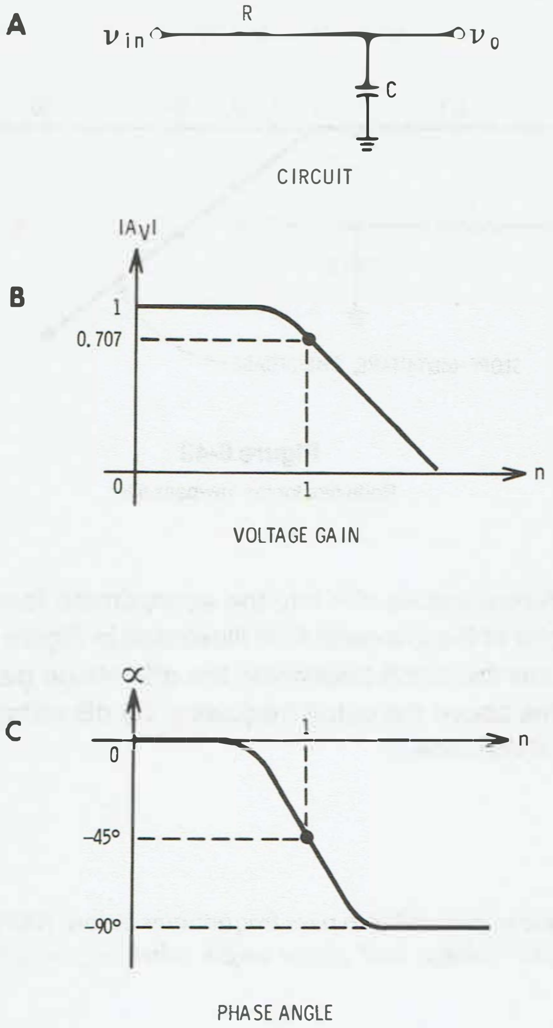


Figure 6-42

Frequency response of the series RC circuit when $\nu_o = \nu_c$.

A. Circuit.
B. Voltage gain.
C. Phase angle.

Low-Pass Filter

For large and small values of n, you can use the following approximations to analyze the low-pass filter.

Below cutoff	$\lvert A_v \rvert$	≈ 1	small n's
$(f << f_c)$	$A_{v_{dB}}$	≈ 0	
Above cutoff	$\lvert A_v \rvert$	$\approx \frac{1}{n}$	large n's
$(f >> f_c)$	$A_{v_{dB}}$	$\approx 20 \log \frac{1}{n}$	

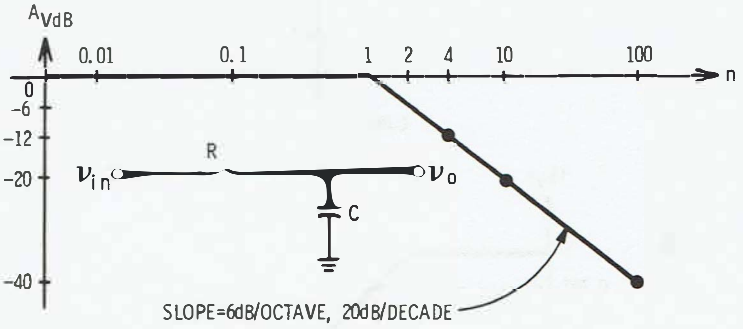


Figure 6-43

Bode plot for the low-pass filter.

By substituting different values of n into the approximate formulas, you obtain the data for the Bode plot of the low-pass filter illustrated in Figure 6-43. Here, note that for frequencies below the cutoff frequency, the dB voltage gain is essentially zero. Also, for frequencies above the cutoff frequency the dB voltage gain "rolls off" at 6 dB/octave and 20 dB/decade.

Example 6-12

Design a low-pass filter to pass frequencies below 100Hz. Predict the output voltage and phase angle when v_{in} = 10V peak at 800Hz.

Selecting C = 10μF and solving for R:

$$R = \frac{1}{2\pi f_o C} = \frac{1}{2\pi(100)(10 \times 10^{-6})} = 159\Omega$$

For f = 800Hz:

$$n = \frac{f}{f_o} = \frac{800\text{Hz}}{100\text{Hz}} = 8$$

Therefore:

$$|A_v| \approx \frac{1}{n} = \frac{1}{8} = 0.125$$

Thus:

$$\nu_o = 10\text{V}\,(0.125) = 1.25\text{V peak}$$

Similarly:

$$-\alpha = \arctan n = \arctan 8$$
$$\text{or } \alpha = -82.87°$$

Bypass Circuit

For frequencies well above cutoff, the output voltage of a low-pass filter is essentially zero. For this reason, the low-pass filter functions as a bypass circuit when $f >> f_o$.

In Unit 4, bypass capacitors were chosen so that $|X_c| = R/20$, at the lowest frequency to be bypassed. Using this rule of thumb ensures that $f >> f_o$, and hence, that good bypassing is achieved.

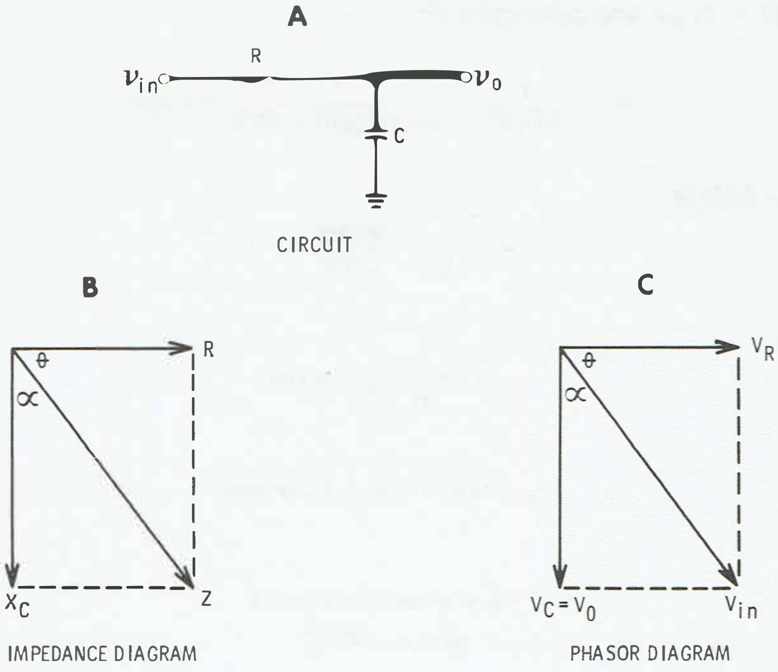


Figure 6-44

A 0 to 90° phase shifter.
Note that V_c lags V_{in} by the angle α.

A. Circuit.
B. Impedance diagram.
C. Phasor diagram.

0 to –90° Phase Shifter

When the output voltage in an RC circuit is taken across the capacitor, the output voltage lags the input voltage by the angle α as shown in Figure 6-44. For this reason, the circuit in Figure 6-44A may be referred to as a **lag circuit**.

In Figure 6-44B, it is clear that the angle α is:

$$\alpha = \arctan \frac{R}{|X_c|} \qquad \text{(Eq. 6-12)}$$

By using Equation 6-12, you can select component values to produce phase angles between, approximately, 0 and –90°.

Integrator

An integrator provides an output voltage that is directly proportional to the area under the input voltage waveform. With advanced mathematics, it can be shown that a low-pass filter acts like an integrator when the time constant is relatively large compared to the period of the input signal. As a guide, good integration results when component values are chosen such that:

$$RC \geqslant 20T \qquad \text{(Eq. 6-13)}$$

Integration is the opposite of differentiation. Recall that when a triangular input voltage was applied to a differentiator the resulting output was a square wave. Thus, if an integrator has a square wave input, the output will be a triangular-type voltage. As was the case with the differentiator, an integrator is often used as a waveform converter.

Source and Load Resistance

For all of the RC circuits discussed in this section to work as described, component values should be chosen to mask out the effects of source and load resistance. Therefore, the value of R should be selected so that:

$R >> R_{TH}$ and $R_L >> R$

where R_{TH} = resistance of the signal source driving the circuit.

Self-Test Review

21. In a high-pass filter the output voltage $\underset{\text{leads/lags}}{\underline{\qquad\qquad}}$ the input voltage.
22. Bode plots have the largest error when f = ___________.
23. The slope of a Bode plot equals ___________ /decade.
24. A ___________ provides an output voltage that is directly proportional to the slope of the input voltage.
25. For effective coupling and/or bypassing, $|X_c|$ should be $\underset{\text{large/small}}{\underline{\qquad\qquad}}$ compared to R.

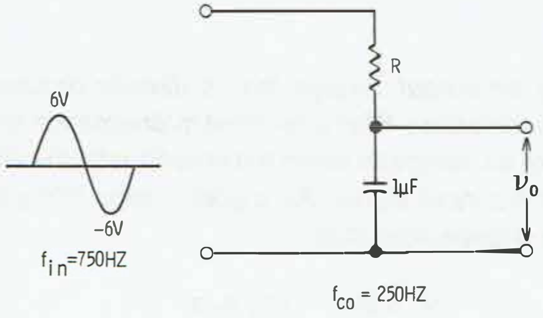


Figure 6-45

Circuit for Questions 26-29.

Refer to Figure 6-45 for questions 26 through 29.

26. The value of R equals __________ ohms.
27. The peak output voltage equals __________ V.
28. ν_o lags ν_{in} by __________ degrees.
29. The dB voltage gain equals __________ dB.
30. A low-pass filter has a cutoff frequency of 300Hz. The **approximate** dB voltage gains for the following frequencies equal:

    A. f = 3Hz A_{vdB} = ________ dB
    B. f = 300Hz A_{vdB} = ________ dB
    C. f = 3kHz A_{vdB} = ________ dB
    D. f = 6kHz A_{vdB} = ________ dB

Answers

21. leads
22. f_{co}
23. −20dB/decade
24. differentiator
25. small
26. 636Ω
27. 1.9V
28. −71.56°
29. −10dB
30. A. 0
    B. 0
    C. −20dB
    D. −26dB

The solutions to questions 26-30 follow.

26. $f_{co} = \dfrac{1}{2\pi RC} = 250\text{Hz}$

Thus, $R = \dfrac{1}{2\pi f_{co}C} = \dfrac{1}{2\pi(250)(10^{-6})} = 636\Omega$

27. $n = f/f_{co} = 750\text{Hz}/250\text{Hz} = 3$

Employing Equation 6-11a, you have:

$$|A_v| = \frac{1}{\sqrt{1 + n^2}} = \frac{1}{\sqrt{10}} = 0.316$$

Since $\nu_{in} = 6\text{V}$, $\nu_o = 0.316\,(6\text{V}) = 1.9\text{V}$ peak

28. Employing Equation 6-11b yields:

$$-\alpha = \arctan n = \arctan 3$$

$$\text{or } \alpha = -71.56°$$

29. $A_{vdB} = 20 \log |A_v| = 20 \log 0.316 = -10\text{dB}$

30. A. Since 3Hz << f_{co}, $A_{vdB} \approx 0$

B. 300Hz = f_{co}. Thus, $A_{vdB} \approx 0$. Recall that the largest error occurs when $f = f_{co}$ on a Bode plot. The actual dB voltage gain at $f_{co} \approx -3.01$ dB.

C. 3kHz is one decade above f_{co}. Thus, $A_{vdB} \approx -20$ dB.

D. 6kHz is one octave above 3kHz. Thus, $A_{vdB} \approx -26$ dB.

SUMMARY

The following points summarize the key ideas of this unit.

Clippers are used to remove voltages above or below a reference level. In both series- and shunt-type clippers, the shunt diode capacitance introduces nonideal effects. Thus, in series clippers, high frequency signals are coupled to the output via the shunt diode capacitance. Similarly, in shunt clippers, sharp edges of the input waveform tend to get rounded, due to the fact that the high frequency components are bypassed to ground.

Dual-type clippers are combinations of positive and negative clippers. By using a potentiometer and DC voltage source, adjustable clipping levels may be obtained. When designing clipper circuits, you should select component values that satisfy the various inequalities discussed in this unit.

Clampers fix either the positive or negative peak of the signal waveform at a specified DC level. Ideally, the output voltage from a clamper should have the same shape as the input voltage. Like clippers, clampers can be biased to obtain non-zero reference levels.

Shunt diode resistors are used with clampers so that the circuit responds quickly to amplitude changes in the input voltage. The value of this resistor is often determined experimentally.

Ideally, peak detectors provide a DC output voltage equal to the positive or negative peak of the input voltage. As was the case with clampers, the RC time constant should not be so large that the circuit response to amplitude changes in the input voltage becomes sluggish.

By combining clampers and peak detectors, you can create voltage multipliers. These circuits provide a DC output voltage equal to n V_m. Due to the poor regulation characteristics inherent in multiplier circuits, values of n greater than 5 are uncommon.

The series RC circuit functions as a high-pass filter when the output voltage is taken across the resistor. Examples were provided that illustrated how the high-pass filter could be used to couple signals, provide a phase shift between 0 and 90°, and act as a differentiator.

Similarly, by taking the output across the capacitor in a series RC circuit, you obtain a low-pass filter. The low-pass filter can be used to bypass signals, provide a phase shift between 0 and –90°, and act as an integrator.

Bode plots are used to display the dB voltage gain versus frequency characteristics of both high- and low-pass filters. These curves are quite accurate for frequencies well above, and well below the filter's cutoff frequency. For signal frequencies near the cutoff frequency of the filter, you can use the equations provided in the unit to accurately predict the voltage gain and phase angle.

UNIT EXAMINATION

The following multiple choice examination is designed to test your understanding of the material presented in this unit. Place a check beside the multiple choice answer (A, B, C, or D) that you feel is most correct. When you have completed the examination, compare your answers with the correct ones that appear after the exam.

1. Clippers perform the same function as:
   A. Clampers.
   B. Limiters.
   C. Peak detectors.
   D. Voltage multipliers.
2. For signal frequencies $>> f_{co}$, a high-pass filter acts like a/an:
   A. Coupling circuit.
   B. Bypass circuit.
   C. Integrator.
   D. Differentiator.
3. A positive clamper cascaded with a positive peak detector results in:
   A. An output of 0V.
   B. A voltage multiplier with $n = 3$.
   C. A dual-type clipper.
   D. A voltage doubler.
4. For adjustable clippers and clampers, R_p should:
   A. $= \sqrt{r_F R_R}$
   B. $>> R$
   C. $<< R$
   D. $= r_F + R_R$
5. Clampers change the:
   A. Average value of the signal waveform.
   B. Shape of the signal waveform.
   C. Frequency of the signal waveform.
   D. Period of the signal waveform.
6. An example of a lag circuit is the:
   A. Differentiator.
   B. Coupling circuit.
   C. High-pass filter.
   D. Low-pass filter.
7. An octave is a ratio of:
   A. 2:1
   B. 8:1
   C. 1:8
   D. 10:1

8. A circuit that provides an output voltage that is directly proportional to the slope of the input voltage is the:
    - A. Integrator.
    - B. Differentiator.
    - C. Bypass circuit.
    - D. Biased clamper.
9. The output voltage from a negative clamper has the positive peak clamped at approximately:
    - A. 0V.
    - B. V_m V.
    - C. $-V_m$ V.
    - D. $2V_m$ V.
10. By cascading positive and negative, shunt peak-type clippers, you obtain a:
    - A. Voltage doubler.
    - B. Dual-type clipper.
    - C. Peak-to-peak detector.
    - D. Biased clamper.

Refer to Figure 6-46 for questions 11 through 13.

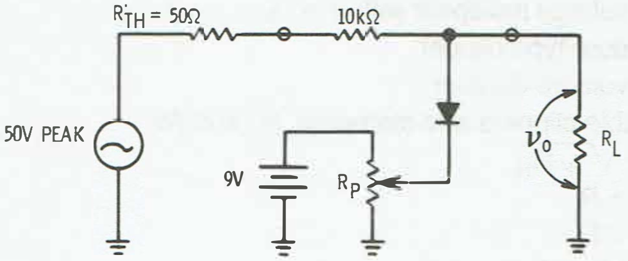


Figure 6-46

Circuit for Questions 11-13.

11. A reasonable value for R_p would be:
    - A. 1kΩ.
    - B. 10kΩ.
    - C. 100Ω.
    - D. 100kΩ.
12. A reasonable value for R_L would be:
    - A. 50Ω.
    - B. 100Ω.
    - C. 10kΩ.
    - D. None of the above.

13. When the wiper is in the middle of the pot, the output would be clipped at:
    - A. +4.5V.
    - B. –9V.
    - C. –4.5V.
    - D. 0V.

Refer to Figure 6-47 for questions 14 through 16.

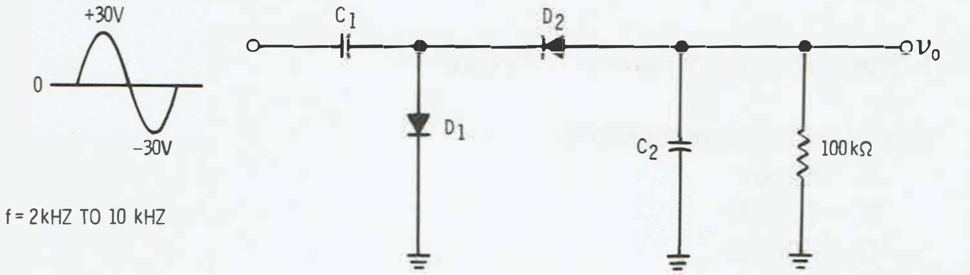


Figure 6-47

Circuit for Questions 14-16.

14. Assuming $C_1 = C_2 = C$, the minimum value for C is:
    - A. 0.25μF.
    - B. 2.5μF.
    - C. 10μF.
    - D. 12.76μF.

15. The maximum voltage across the 100kΩ resistor should be approximately:
    - A. +30V.
    - B. 0V.
    - C. –60V.
    - D. +60V.

16. If the direction of D_2 is reversed, the DC output voltage is:
    - A. +30V.
    - B. 0V.
    - C. –60V.
    - D. +60V.

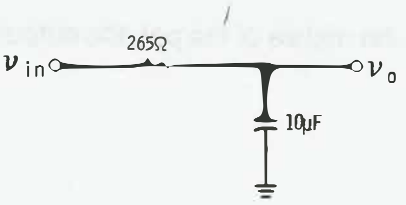


Figure 6-48

Circuit for Questions 17-20.

Refer to Figure 6-48 for questions 17 through 20.

17. The filters cutoff frequency is:
    - A. 60Hz.
    - B. 0.6kHz.
    - C. 600Hz.
    - D. 6kHz.
18. A 10V peak 420Hz input signal would produce a peak output voltage of:
    - A. 10V.
    - B. 8.75V.
    - C. 1.41V.
    - D. 0V.
19. In question 18, ν_o would:
    - A. Lag ν_{in} by 8.13°.
    - B. Lead ν_{in} by 8.13°.
    - C. Lag ν_{in} by 81.87°.
    - D. Lead ν_{in} by 81.87°.
20. When $f = 600$Hz, the dB voltage gain is approximately:
    - A. 20dB.
    - B. –20dB.
    - C. 6dB.
    - D. –6dB.

EXAMINATION ANSWERS

1. B — The terms clippers and limiters are used interchangeably.
2. A — When $f >> f_{co}$, $|A_v| \approx 1$ in the high-pass filter. Thus, $\nu_o \approx \nu_{in}$, and the filter functions as a coupling circuit.
3. D — A voltage doubler, or peak-to-peak detector is constructed by cascading a positive clamper and positive peak detector.
4. C — The potentiometer resistance must be small compared to R for proper circuit operation.
5. A — Clampers change the average values, DC level, of the signal waveform.
6. D — Since $V_o = V_c$ in a low-pass filter, the output voltage lags the input voltage.
7. A — An octave is a 2:1 ratio.
8. B — The output voltage of a differentiator is directly proportional to the slope of the input voltage.
9. A — The output of a negative clamper has the positive peak clamped at approximately 0V.
10. B — A dual-type clipper can be constructed by cascading positive and negative, shunt peak-type clippers.
11. C — $R_p \leqslant \frac{R}{100}$ Since $R = 10k\Omega$ $R_p \leqslant \frac{10k\Omega}{100}$

    or approximately 100Ω.

12. D — $R_L >> R$ None of the choices is large compared to 10kΩ.
13. A — When the wiper is in the middle of the pot, the voltage from the cathode of the diode to ground equals 9V/2 or 4.5V. When $\nu_{in} > 4.5V$ the diode becomes forward biased. This limits ν_o to +4.5V.
14. A —

$$T_{max} = 1/f_{min} = 1/2kHz \text{ or } 0.5ms$$

$$C = \frac{50\,T_{max}}{R} = \frac{50(0.5ms)}{100k\Omega} = 0.25\mu F$$

15. C — The circuit in Figure 6-47 is a negative peak-to-peak detector. Thus, $\nu_o = -2V_m$ or $-60V$.
16. B — Reversing D_2 results in a negative clamper cascaded with a positive peak detector. Consequently, the positive peak into the peak detector is 0V, which makes the output equal to 0V.

17. A — $f_{co} = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi (265)(10)(10^{-6})} = 60\text{Hz}$

18. C — $n = f/f_{co} = 420\text{Hz}/60\text{Hz} = 7$

$$|A_v| = \frac{1}{\sqrt{1 + n^2}} = \frac{1}{\sqrt{50}} = 0.141$$

Thus, $\nu_o = 10\text{V}\,(0.141) = 1.41\text{V}$ peak.

19. C — $-\alpha = \arctan\ n = \arctan 7$

Therefore, $\alpha = -81.87°$.

20. B — 600Hz is one decade above 60Hz. Thus:

$$A_{v_{dB}} \approx -20\text{dB}.$$

Unit 7

SELECTED APPLICATIONS

CONTENTS

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INTRODUCTION

Units 1 through 3 discussed the mathematics and analytical methods used to analyze and design electronic circuits. Unit 4 examined the construction details, and nonideal properties of real circuit components. Finally, units 5 and 6 introduced a number of useful passive circuits, and illustrated how these circuits are designed.

At this point, you should have a good idea how to select component values so that the following circuits approximate the characteristics of ideal circuits:

Coupling circuits
Bypass circuits
Rectifiers
Smoothing Filters
High-Pass Filter
Differentiators
0 to 90° Phase Shifter
Clippers
Clampers
Peak Detectors
Peak-to-Peak Detectors
Low-Pass Filter
Integrators

In this unit, we will examine some typical applications for selected circuits.

UNIT OBJECTIVES

When you have completed this unit you should be able to:

1. Describe seven general applications for passive circuits.
2. Describe nine measurement applications for passive circuits.

UNIT ACTIVITY GUIDE

☐ Read section on “General Applications”.

☐ Answer Self-Test Review Questions 1-10.

☐ Read section on “Measurement Applications”.

☐ Answer Self-Test Review Questions 11-20.

☐ Perform Experiment 10 in Unit 8.

☐ Study Summary.

☐ Complete “Unit Examination”.

☐ Check “Examination Answers”.

GENERAL APPLICATIONS

Diodes are often used to keep the components in sensitive electronic equipment from being damaged. This damage could be caused, for example, by an input signal whose amplitude is excessive. In addition, many electronic components will be permanently damaged if the power supply leads are accidentally reversed.

In unit two you learned that the opening or closing of a switch in an inductive circuit often causes arcing across the switch contacts. In addition to numerous other applications, diodes are also used to suppress this switch contact arcing.

In this section we will examine several diode circuits designed to protect electronic components from polarity reversal, over voltage, and transient-type failures. Additional examples of simple but very useful diode circuits will also be provided, as well as some typical applications for series RC circuits.

Reversed Polarity Protection

In many cases, equipment designed to operate from a DC voltage source may be permanently damaged if the power supply connections are inadvertently reversed.

If the equipment is designed to operate from a single polarity supply, you can get reversed polarity protection by connecting a diode in series with one of the supply leads, as shown in Figure 7-1 A. Here, the diode is forward biased only if the DC supply voltage is such that terminal A is positive with respect to terminal B. If the polarity of the DC supply voltage is reversed, the diode is reverse biased. Since a reverse biased diode acts like an open circuit, the equipment is effectively disconnected from the DC supply voltage.

Integrated circuits, IC's, often require dual supply voltages. An example of reversed polarity protection popular with IC's is illustrated in Figure 7-1 B. Here, a diode is required for both the positive and negative supply leads. In both single and dual supply reversed polarity protection circuits, the foward voltage drop across the diodes should be as small as possible. For this reason, Schottky-type diodes are recommended for reverse polarity protection applications.

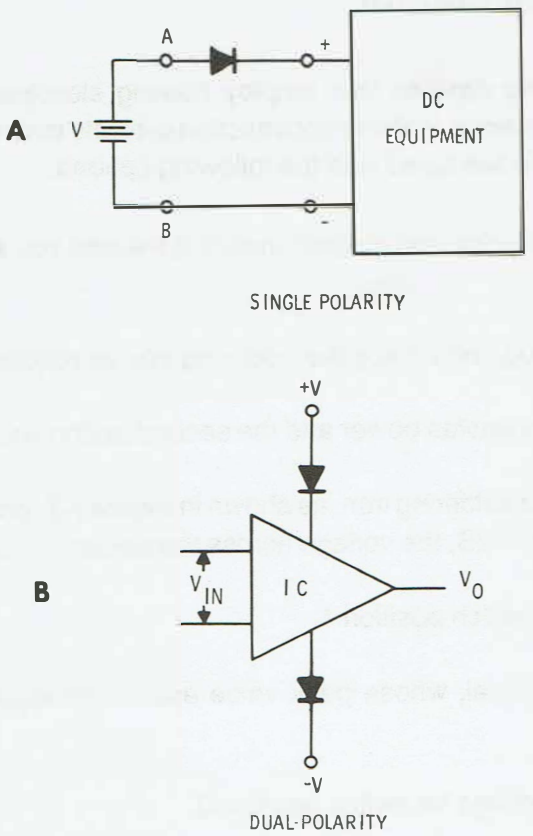


Figure 7-1

Reversed polarity protection for single-and dual-type power supplies.

Elementary Heating Control

Soldering irons, and other devices that employ heating elements, have definite "warm up times". Consequently, in those construction projects that require soldering as intermediate steps, you are faced with the following options:

1. Leave the soldering iron plugged in during the time you are working on the project.
2. Alternately plug and unplug the soldering iron as required.

Obviously, the first option wastes power and the second option wastes time.

A slight modification of the soldering iron, as shown in Figure 7-2, provides an attractive third option. In Figure 7-2B, the voltage across the soldering iron, R_L is:

1. Zero volts for switch position 1.
2. A half-wave signal, whose peak value essentially equals V_m for switch position 2.
3. The full-line voltage for switch position 3.

With advanced mathematics, it can be shown that the rms value of a half-wave signal is:

$$V_{rms} = 0.5\ V_m \qquad \text{(Eq. 7-1)}$$

Recall that the rms value is the equivalent amount of DC with respect to the heating effect produced. Since the rms value of a sine wave is 0.707 V_m, the soldering iron will dissipate less power when the switch is in position 2 than when the switch is in position 3. Specifically:

Position 2

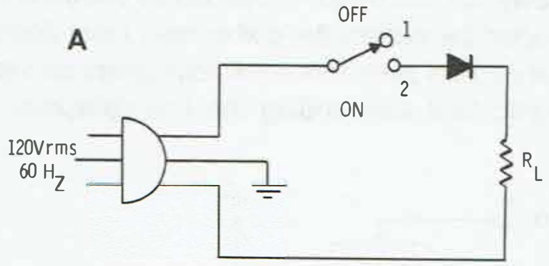
$$P_{RL} = \frac{(V_{rms})^2}{R_L} = \frac{(0.5\ V_m)^2}{R_L} = \frac{0.25\ V_m^2}{R_L}$$

Position 3

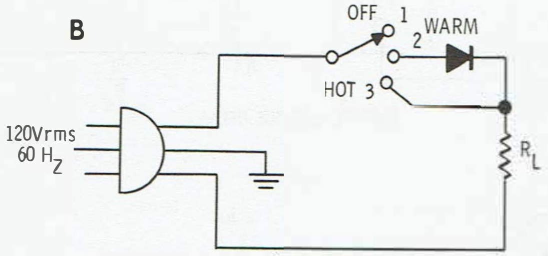
$$P_{RL} = \frac{(V_{rms})^2}{R_L} = \frac{(0.707\ V_m)^2}{R_L} = \frac{0.5\ V_m^2}{R_L}$$

Thus, the soldering iron dissipates approximately half the power for switch position 2 as compared to switch position 3. The power calculations are only "approximate" because the value of R_L changes with temperature. Nevertheless, it is clear that the soldering iron dissipates considerably less power for the half-wave signal, position 2, than for the full line voltage, position 3.

The addition of an appropriate diode, and rotary switch to your soldering iron permits you to utilize the warm option on projects requiring soldering as intermediate steps. This reduces the power consumption of the soldering iron. Also, the time required for the soldering iron to heat up from WARM to HOT is considerably less than the original warm up time. Thus, you save both time and money. This, of course, is desirable!



ORIGINAL CIRCUIT



MODIFIED CIRCUIT

Figure 7-2

Rectifying the line voltage to provide a "WARM" option for a soldering iron.

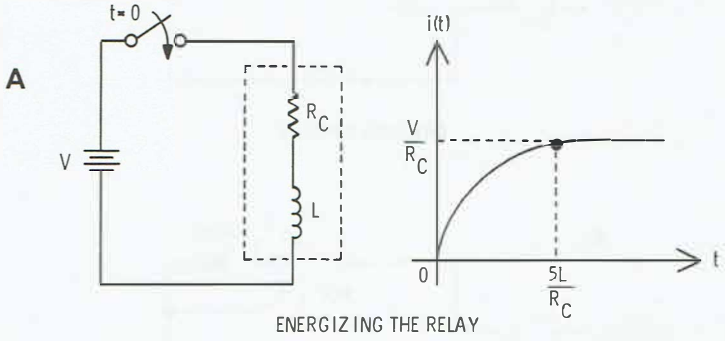
Diode Arc Suppressors

Figure 7-3 shows a relay coil that is energized from a DC voltage source. Once the switch in Figure 7-3A is closed, the coil current exponentially rises to the Ohm's law value of V/R_c. From the discussion of R_L switching circuits in Unit two, it should be clear that the charging current is described by:

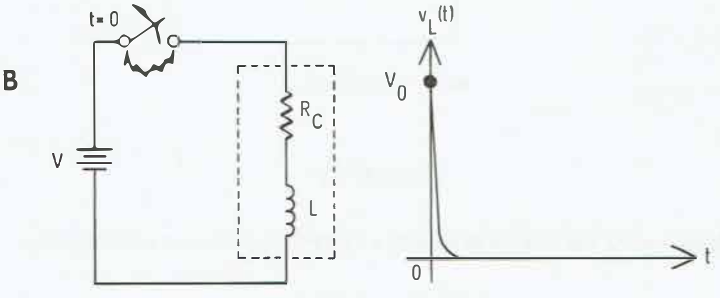
$$i(t) = \frac{V}{R_c}\left(1 - e^{-t/\tau}\right) \qquad \text{(Eq. 7-2)}$$

where V = source voltage in volts (V)
R_c = coil resistance in ohms (Ω)
τ = time constant, $\frac{L}{R_c}$, in seconds (S)

To de-energize the relay coil, the switch is opened as shown in Figure 7-3B. During the time required to open the switch, the coil current must decrease from an initial value of V/R_c to a final value of zero. Since the coil current decreases to zero in very little time, the magnetic field surrounding the coil collapses very quickly. Con-



ENERGIZING THE RELAY



DE-ENERGIZING THE RELAY

Figure 7-3

An example of a switching circuit with an inductive load.

sequently, a large voltage is induced in the coil once the switch is opened. Stated mathematically, the coil voltage at the instant the switch is opened is:

$$V_o = L\frac{\Delta I}{\Delta t} \qquad \text{(Eq. 7-3)}$$

where V_o = $\nu_L(0)$ in volts (V)
L = inductance in Henries (H)
ΔI = change in current in amperes (A)
Δt = time required for the coil current to decrease to zero in seconds (S)

Example 7-1

In Figure 7-3 $V = 15V$, $R_c = 10\Omega$, and $L = 0.25H$.

Determine:

- *(a) The steady-state coil current once the switch is closed.*
- *(b) The time required to reach the value in (a).*
- *(c) The coil voltage at the instant the switch is opened, assuming it takes $10\mu S$ to open the switch.*

$$\text{(a) } I = \frac{V}{R} = \frac{15V}{10\Omega} = 1.5A$$

$$\text{(b) } \tau = \frac{L}{R} = \frac{0.25H}{10\Omega} = 25mS$$

The steady-state current is reached after a period of time equal to five time constants has elapsed. Thus:

$$t = 5\tau = 5\,(25mS) = 0.125\ S$$

(c) Employing Equation 7-3 yields:

$$V_o = L\frac{\Delta I}{\Delta t}$$

$$V_o = 0.25\,H\ \frac{1.5A}{10\mu S} = 37\,500V!$$

Even though the source voltage is only 15V, 37.5kV is induced in the coil at the instant the switch is opened. The induced voltage is large enough to produce arcing across the switch contacts as shown in Figure 7-3B.

A simple method to suppress arcing in inductive circuits is illustrated in Figure 7-4. Note that, when the switch is closed, the DC voltage source reverse biases the diode as shown in Figure 7-4B. Since $R_R >> R_c$, the diode effectively acts like an open circuit. Consequently, the circuit in Figure 7-4B is essentially equivalent to the RL charging circuit in Figure 7-3A. For this reason, after five time constants have elapsed, the coil current equals the steady-state value of V/R_c.

If the switch is now opened, the voltage induced in the coil forward biases the diode as shown in Figure 7-4C. Here, the forward resistance of the diode provides a discharge path for the coil current. Thus, the discharging coil current is now described by:

$$i(t) = \frac{V}{R_c} e^{-t/\tau} \qquad \text{(Eq. 7-4)}$$

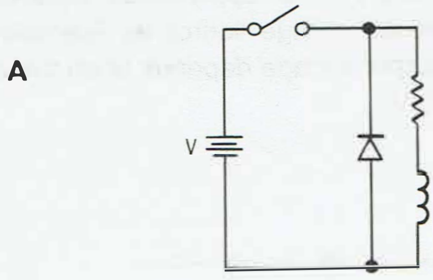
$$\text{where } \tau = \frac{L}{R_c + r_F}$$

Since i(t) flows through r_F, the voltage across the diode and coil equals:

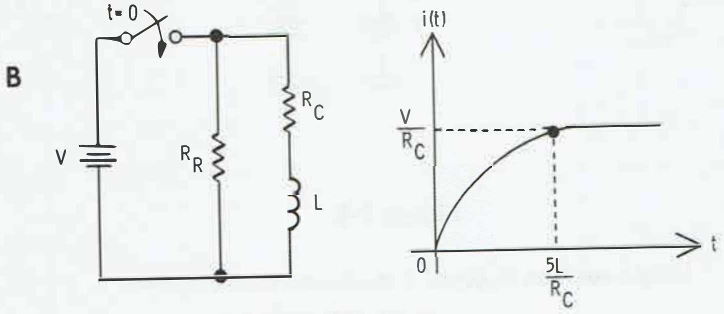
$$V_{r_F}(t) = \frac{V r_F e^{-t/\tau}}{R_c} \qquad \text{(Eq. 7-5)}$$

Since r_F typically is $\leqslant R_c$, the magnitude of the coil voltage at the instant the switch is opened is $\leqslant V$. For this reason, the induced coil voltage is not large enough to produce arcing across the switch contacts.

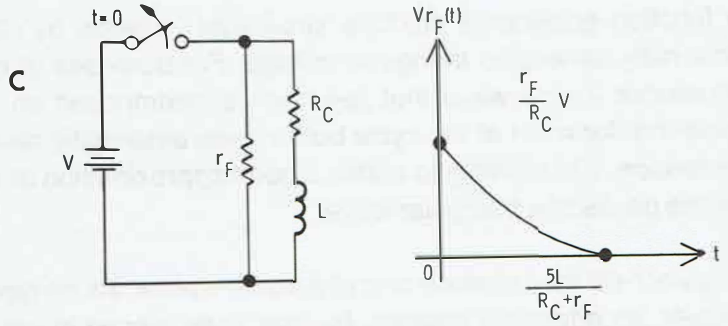
Due to their excellent switching characteristics, Schottky barrier-type diodes are recommended for arc suppression circuits. For arc suppression applications, the diodes PIV rating should be at least twice the value of the DC source voltage. In addition, the diodes peak current rating should be greater than the steady-state coil current, V/R_c.



CIRCUIT



CLOSING THE SWITCH



OPENING THE SWITCH

Figure 7-4

A diode arc suppressor.

Wave Shapers

Various clipper combinations can be used to provide numerous wave-shaping functions. For example, you can easily get an approximate square wave by driving a dual-type clipper from a sinusoidal voltage source as illustrated in Figure 7-5. Here, the "squareness" of the output voltage depends upon the amplitude of the sine wave and the clipping level, V.

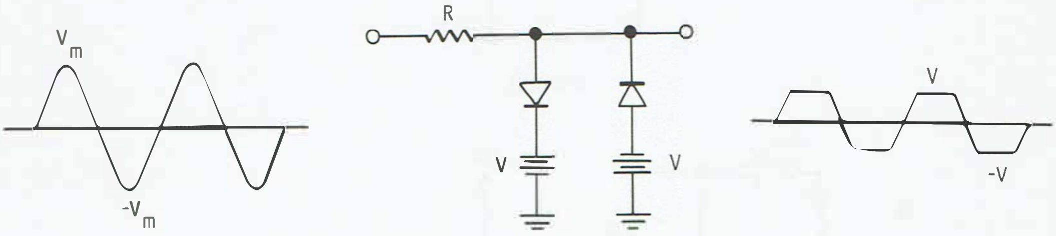


Figure 7-5

Using a dual-type clipper as a sine to square wave converter.

Most modern function generators produce sinusoidal voltages by changing the shape of an internally generated triangular voltage. For purposes of comparison, Figure 7-6A illustrates a sine wave that has been superimposed on a triangular wave. Here, note that for most of the cycle both waves essentially have the same shape. For this reason, it is possible to obtain a good approximation of a sine wave by rounding off the peaks of a triangular wave.

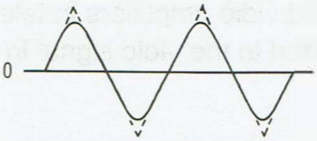
The circuit in Figure 7-6B is essentially that of a positive peak, shunt-type clipper. In this case, however, an additional resistor, R_2, has been placed in series with the diode. The function of R_2 is to introduce a bowing effect similar to the one you encountered in Experiment 8. For this reason, the positive peaks of the output voltage appear slightly rounded as shown in Figure 7-6B.

In Figure 7-6B, the point at which the slope of the output voltage departs from the slope of the triangular voltage is called a breakpoint. The value of the breakpoint is determined by the voltage V_B. Consequently, you can adjust the breakpoint value by varying the value of R_p. Similarly, the slope change above the breakpoint is determined by the value of the bowing resistor, R_2. Thus, by varying both R_1 and R_2, you can adjust the shape of the positive peaks of the output voltage to obtain the best fit.

To shape both positive and negative peaks, a modified dual-type clipper is required. In addition, by adding more diodes and resistors, you can obtain virtually any number of breakpoints and slope changes. In this way, it is possible to generate sinusoids with less than one percent total harmonic distortion.

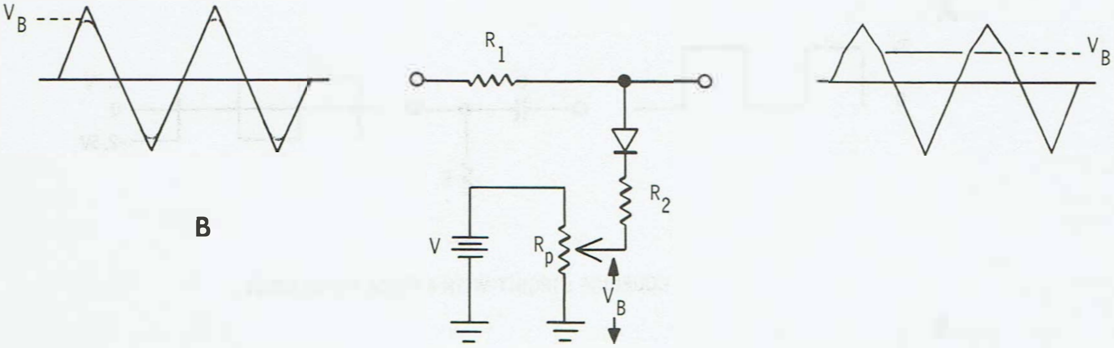
A typical triangular to sine converter is illustrated in Figure 7-6C. Here, the upper portion of the circuit shapes the positive peaks and the lower portion of the circuit shapes the negative peaks. Frequently, wave shapers are used with active devices, such as Op-Amps, to amplify and buffer the shaped waveform.

A



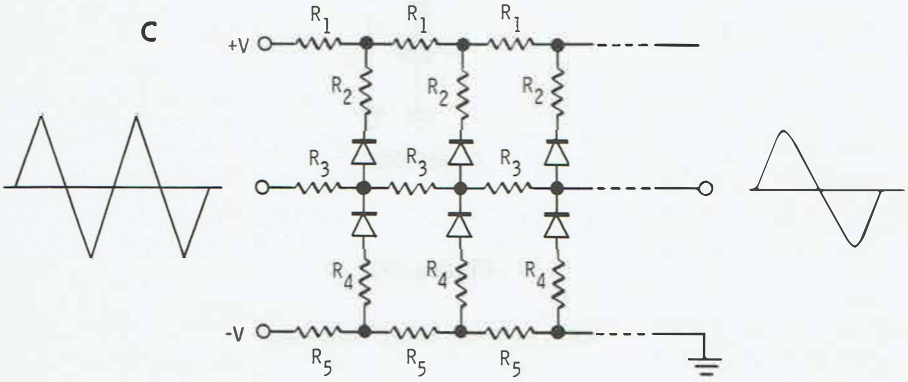
COMPARISON OF TRIANGULAR AND SINE WAVES

B



MODIFIED POSITIVE PEAK SHUNT-TYPE CLIPPER

C



CASCADED DUAL-TYPE CLIPPERS

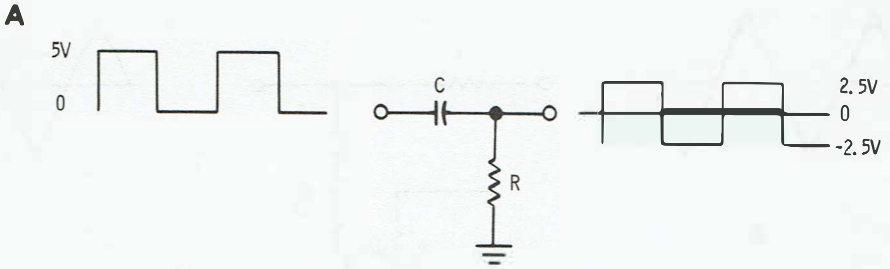
Figure 7-6

Triangular to sine conversion.

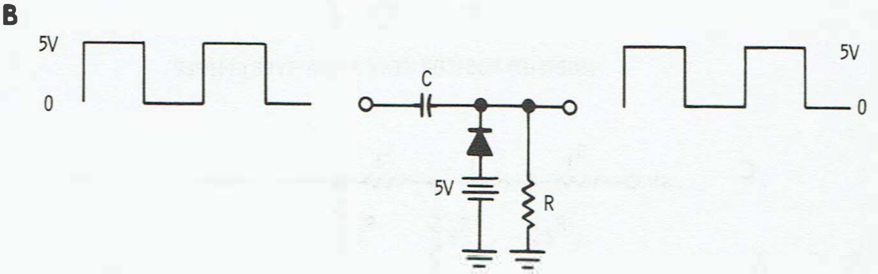
DC Restoration

The essential difference between a pulse train and square wave is that a pulse train has an average value that is not zero. Consequently, if a pulse train or another asymmetrical waveform passes through a transformer or coupling circuit, the DC component is removed as shown in Figure 7-7A.

In order to restore the DC level of the signal waveform, a clamper may be employed as shown in Figure 7-7B. Here, both the input and output waveforms have an average value of 2.5V. Clampers employed for the purpose of restoring the DC level of a signal waveform are generally referred to as DC restorers. DC restorers are used in capacitively coupled vidio amplifiers in television receivers. In this case, a negative DC voltage is added to the vidio signal in order to cut off the picture tube during blanking periods.



COUPLING CIRCUIT WITH A PULSE TRAIN INPUT



DC RESTORER

Figure 7-7

Using a clamper for DC restoration.

Catching Diodes

Transistors and other semiconductor devices are often used to perform a switching function. In such applications, the semiconductor device can be modeled by a switch as shown in Figure 7-8A. Note that the switch drives a capacitive load, C.

Without the diodes, the capacitor will charge through R_c to the source voltage, V_{cc}, when the switch is open. Similarly, the capacitor will discharge through the switch resistance once the switch is closed. Consequently, in the absence of the diodes, the capacitor voltage would appear as shown by the dashed lines in Figure 7-8C, as the switch is alternately opened and closed.

In Figure 7-8B, the circuit of Figure 7-8A has been redrawn to emphasize the purpose of the diodes. D_1 and V_1 constitute a positive peak-type clipper, while D_2 and V_2 function as a negative base-type clipper. Therefore, D_1 and V_1 remove voltages above V_1, while D_2 and V_2 remove voltages below V_2. For these reasons, the capacitor voltage appears as shown by the solid line in Figure 7-8C.

Catching diodes are often used in digital circuits to reduce the rise and fall times of digital waveforms.

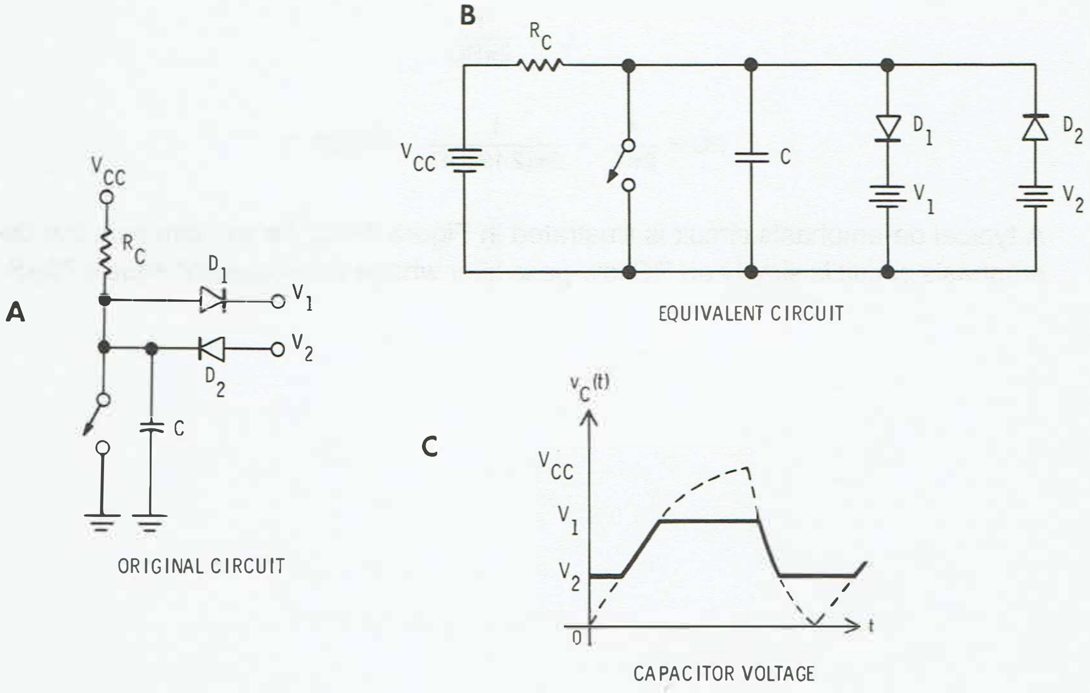


Figure 7-8

Catching diodes.

De-emphasis Circuit

The ability of a frequency modulated, FM, signal to suppress noise is less for high frequency components than for low frequency components. To minimize this effect, the higher frequency audio components are amplified, or emphasized prior to transmission. Naturally, the use of a pre-emphasis circuit alters the normal relationship between the high and low frequency audio components of the transmitted signal.

To restore the natural balance between the high and low frequency audio components, a de-emphasis circuit is required in the FM receiver. Figure 7-9A and Figure 7-9B respectively illustrate the frequency characteristics of the pre-emphasis and de-emphasis circuits. Here, the de-emphasis curve is just the opposite of the pre-emphasis curve. The imbalance between the high and low frequency audio components introduced at the transmitter is precisely compensated for at the receiver.

You should recognize the de-emphasis curve in Figure 7-9B as the frequency response curve of a low-pass RC filter. Furthermore, note that the cutoff frequency is 2.12kHz. Thus, you can calculate the required time constant for the de-emphasis circuit as folows:

$$f_c = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_c} = \frac{1}{2\pi(2.12\text{kHz})} = 75\mu S$$

A typical de-emphasis circuit is illustrated in Figure 7-9C. As you can see, the de-emphasis circuit is simply an RC low-pass filter whose time constant equals 75μS.

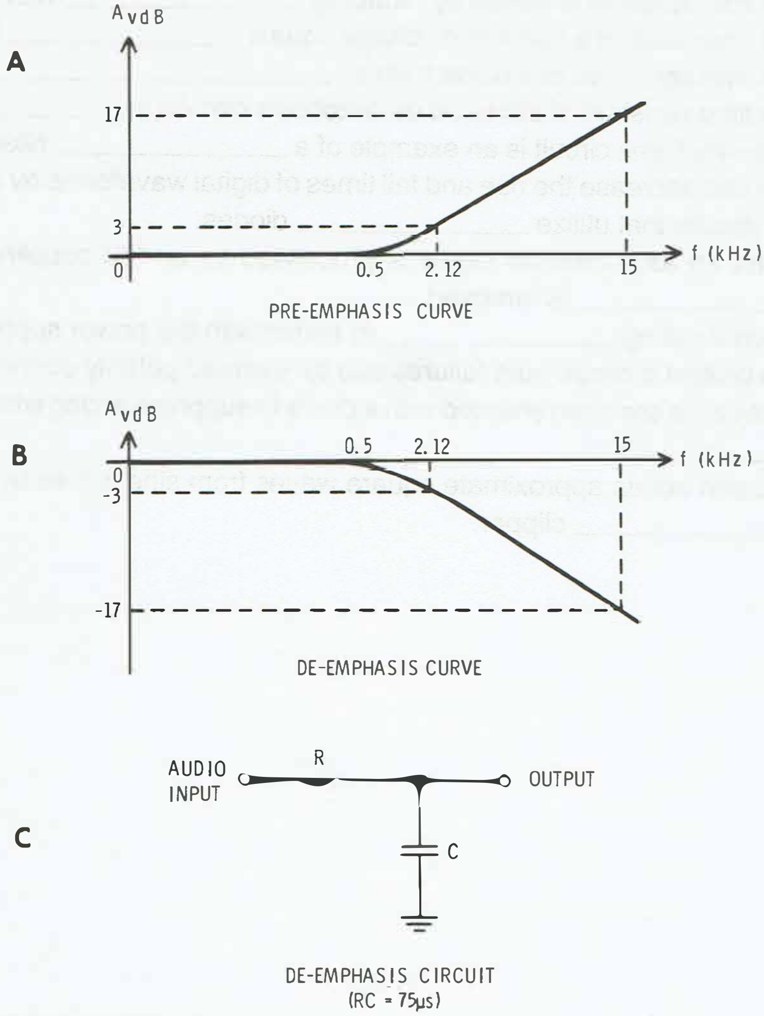


Figure 7-9

Application for a low-pass filter.

Self-Test Review

1. You can obtain sine waves by "shaping" ______________ waves.
2. The rms value of a half-wave voltage equals ______________ times V_m.
3. The average value of a pulse train is ______________ ______________.
4. The time constant of standard de-emphasis circuits is ______________.
5. A de-emphasis circuit is an example of a ______________ filter.
6. You can decrease the rise and fall times of digital waveforms by using switching circuits that utilize ______________ diodes.
7. When an asymmetrical signal is processed by an RC coupling circuit, the ______________ is removed.
8. By connecting ______________ in series with the power supply leads, you can protect a circuit from failures due to reversed polarity connections.
9. Relay coils are often shunted with a diode to suppress arcing when the relay is ______________ .
10. You can obtain approximate square waves from sine waves by employing a ______________ clipper.

Answers

1. triangular
2. 0.5
3. not zero
4. 75µS
5. low-pass
6. catching
7. DC level
8. diodes
9. de-energized
10. dual-type

MEASUREMENT APPLICATIONS

Diodes, resistors, and capacitors have numerous applications in basic measurement circuits. By using these components with a DC ammeter or DC voltmeter, you can construct a surprisingly large number of simple, yet very useful, measurement circuits.

Meter Protection

The moving coil mechanism of an ammeter is wound with very fine wire. For this reason, the ammeter has a very definite resistance. If excessive current flows through the ammeter, the moving coil mechanism can easily be destroyed. Consequently, it is desirable to provide some form of meter protection circuit.

You can get the simplest form of protection by shunting the meter with two diodes as shown in Figure 7-10B. Here, the voltage across each diode equals the voltage across the meter, $I_m R_m$. In the absence of an overload, the turn-on voltage of the diodes is greater than the voltage across the meter. For this reason, the diodes are normally reverse biased and, therefore, act like open circuits.

When the meter is overloaded, the voltage across it exceeds the turn-on voltage of the diodes. Depending upon the direction of the meter current, either D_1 or D_2 will become forward biased. In this case the meter is protected, since most of the current flows through the forward-biased diode rather than through the meter.

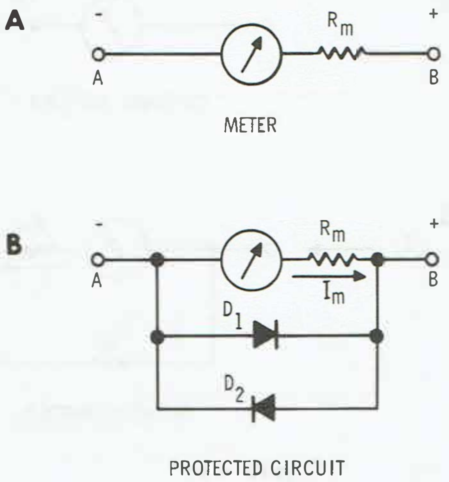


Figure 7-10

A DC meter protection circuit.

Increasing Current Ranges

The moving coil mechanism, by itself, is capable of measuring relatively small currents; typically in the µA to mA ranges. To extend the current handling ability of the circuit, a resistor is placed in parallel with the moving coil mechanism as shown in Figure 7-11B. Here, the current through the moving coil mechanism via the current division principle is:

$$I_1 = \frac{I_2 R_{SH}}{R_m + R_{SH}}$$

Solving for the value of the shunt resistor, R_{SH} yields:

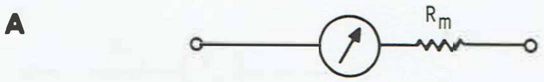
$$I_1 R_m + I_1 R_{SH} = I_2 R_{SH}$$

$$I_1 R_m = I_2 R_{SH} - I_1 R_{SH}$$

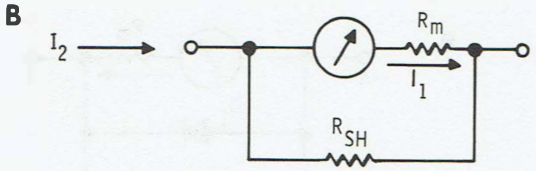
$$I_1 R_m = R_{SH} (I_2 - I_1)$$

$$R_{SH} = \frac{I_1 R_m}{I_2 - I_1} \qquad \text{(Eq. 7-7)}$$

where R_{SH} = required shunt resistance in ohms (Ω)
R_m = resistance of the moving coil mechanism in ohms (Ω)
I_1 = full-scale current of the moving coil mechanism in amperes (A)
I_2 = full-scale current of the shunted ammeter in amperes (A)



ORIGINAL AMMETER



SHUNTED AMMETER

Figure 7-11

Increasing the current range of an ammeter.

Example 7-2

A moving coil mechanism has a resistance of 3kΩ, and a full-scale current rating of 100μA. Calculate the shunt resistance required to measure currents up to 10mA.

$$R_{SH} = \frac{I_1 R_m}{I_2 - I_1} = \frac{100\mu A\,(3k\Omega)}{10mA - 100\mu A}$$

$$R_{SH} = \frac{100\mu A\,(3k\Omega)}{9\,900\mu A} = 30.3\Omega$$

In the shunted ammeter, 100μA flows through the moving coil mechanism and 9.9mA flows through the shunt resistor when the total current equals 10mA. Also, since $R_{SH} \parallel R_m$, the effective resistance of the shunted ammeter is $3k\Omega \parallel 30.3\Omega$, or approximately 30Ω.

DC Ammeter to DC Voltmeter Conversion

A DC ammeter is easily converted to a DC voltmeter with the addition of a series resistor as shown in Figure 7-12. Here, series resistor R_s is used to limit the current through the moving coil mechanism to the rated full-scale value, I_1, when the full-scale voltage, V_T is being measured.

Since R_s and R_m are connected in series, current I_1 is, from Ohm's law:

$$I_1 = \frac{V_T}{R_s + R_m}$$

Solving for R_s yields:

$$I_1 R_s + I_1 R_m = V_T$$

$$I_1 R_s = V_T - I_1 R_m$$

$$R_s = \frac{V_T - I_1 R_m}{I_1}$$

$$R_s = \frac{V_T}{I_1} - R_m \qquad \text{(Eq. 7-8)}$$

where R_s = required series resistance in ohms (Ω)
V_T = full-scale voltage in volts (V)
I_1 = full-scale current of the moving coil mechanism in amperes (A)
R_m = resistance of the moving coil mechanism in ohms (Ω)

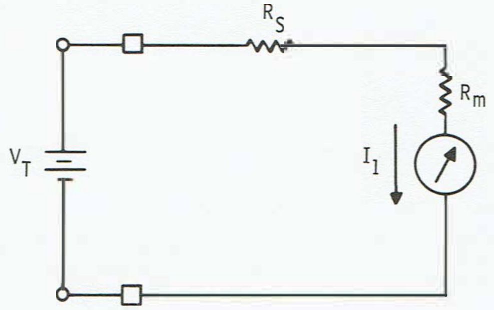


Figure 7-12

Converting a DC ammeter to a DC voltmeter.

Example 7-3

Determine the series resistance required to convert the moving coil mechanism of example 7-2 into a 0-10V DC voltmeter.

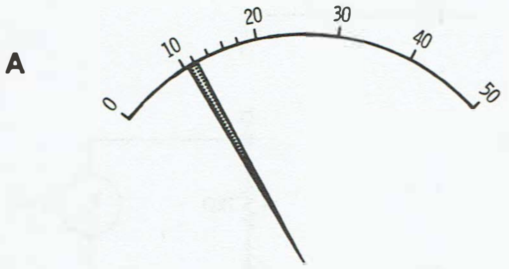
$$R_s = \frac{V_T}{I_1} - R_m = \frac{10\text{V}}{100\mu\text{A}} - 3\text{k}\Omega = 97\text{k}\Omega.$$

Scale Expanders

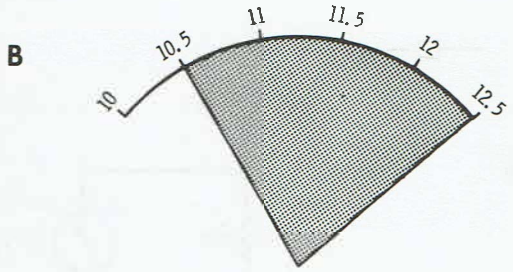
Commercial voltmeters usually employ several series resistors, and an appropriate switching arrangement to provide a variety of DC voltage ranges. A typical multimeter, for example, provides DC voltage ranges of 2.5, 10, 50, 250, and 1000 V.

Let's assume that you wish to measure DC voltages between 10.5V and 12.5V with a typical multimeter. In this case, you select the 0-50V range for the measurements as shown in Figure 7-13A. Because the range of measured values is a small portion of the 0-50V scale, it would be very difficult to accurately read the measured values.

By "expanding" the portion of the scale you are interested in, you can obtain much more accurate readings, as shown in Figure 7-13B. Here, note that the lower range of the scale corresponds to 10V, while the upper range of the scale corresponds to 12.5V.



ORIGINAL 0-50V SCALE



EXPANDED SCALE

Figure 7-13

The concept of scale expansion.

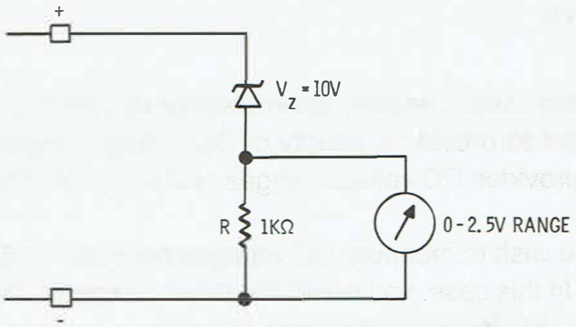


Figure 7-14

A 10V to 12.5V expanded-scale DC voltmeter.

An elementary expanded-scale voltmeter can be constructed as shown in Figure 7-14. Here, note that a 10V zener diode is connected in series with the positive test probe. Consequently, for input voltages less than 10V, the reverse biased diode acts like an open circuit as shown in Figure 7-15A. In this case, the voltage across resistor R equals zero volts, and there is no deflection of the voltmeter.

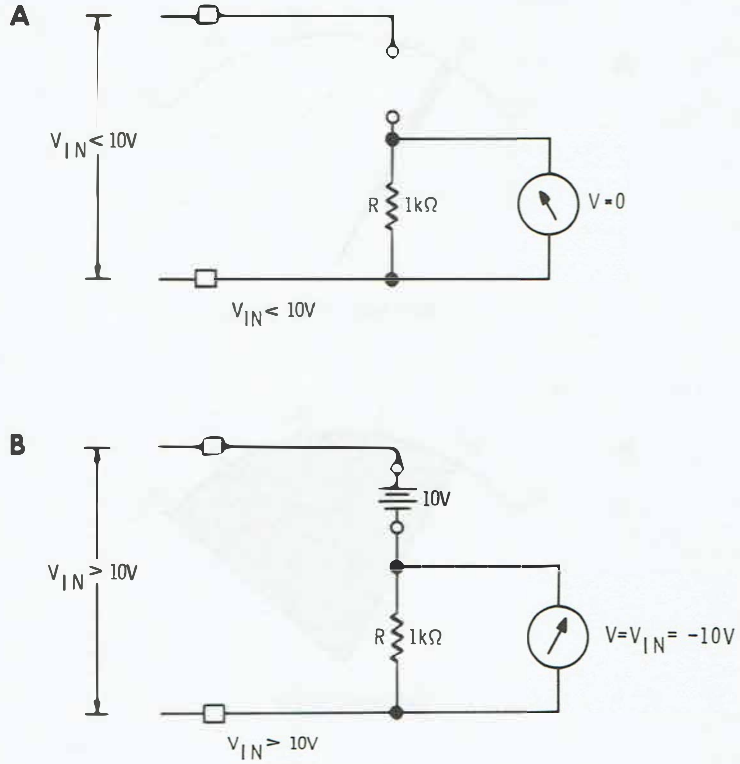


Figure 7-15

Equivalent circuits for the expanded-scale DC voltmeter in Figure 7-14.

When the input voltage exceeds 10V, the zener diode acts like a 10V DC voltage source as shown in Figure 7-15B. Now, the voltage developed across resistor R equals $V_{in} - 10V$. By using this relationship, the data in Table 7-1 was obtained.

V_{in}	$\upsilon_R = V_{IN} - 10V$
10V	0V
10.5V	0.5V
11V	1V
11.5V	1.5V
12V	2V
12.5V	2.5V

Table 7-1

Resistor voltages for the expanded-scale voltmeter in Figure 7-14.

In Figure 7-14, note that the 0-2.5V DC voltmeter measures the voltage developed across resistor R. Consequently, when V_{in} is 11V, the voltmeter reads 1V. Similarly, when V_{in} is 12.5V, the voltmeter reads 2.5V. Thus, by mentally adding 10V to the voltmeter reading, you effectively have the expanded scale illustrated in Figure 7-13B!

For proper operation, resistor R must be small enough to ensure that the diode conducts sufficient current to remain in the zener region when V_{in} exceeds 10V. Also, to minimize loading, the DC voltmeter R must be small compared to the voltmeters input resistance. For these reasons the expanded-scale voltmeter has a relatively low input resistance — which is undesirable. You can increase the input resistance substantially by employing active components, such as Op-Amps.

Peak-Type AC Voltmeter

How can you construct a circuit to measure an AC voltage? To begin, you can convert the AC voltage into a relatively pure DC voltage. The resulting DC voltage can then be measured with a DC voltmeter.

The peak-type AC voltmeter in Figure 7-16 works in the manner just described. Here, the diode and capacitor form a positive peak detector. Thus, the DC capacitor voltage is essentially equal to the positive peak value of the AC input voltage. As you can see, the DC capacitor voltage is measured with the DC voltmeter.

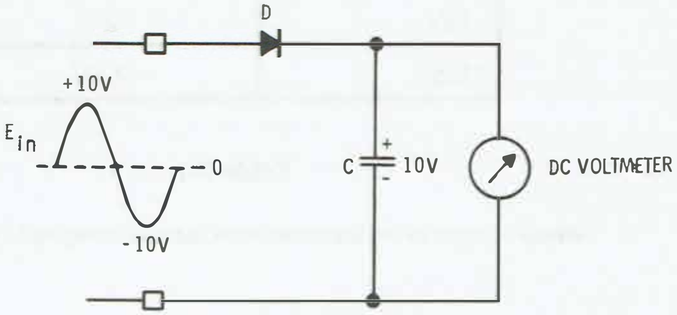


Figure 7-16

A peak-type AC voltmeter.

Example 7-4

A multimeter has an "ohms per volt" rating of 20kΩ/V. The input resistance of the multimeter, when used as a DC voltmeter, equals the ohms per volt rating times the voltage required to produce full-scale deflection. For example, on the 0-10V range, R_{in} *equals 200kΩ. Similarly, on the 0-50V range,* R_{in} *equals 1MΩ.*

Assuming that you are using the 0-50V range, sketch a circuit for a 0-50V peak-type AC voltmeter. Assume that the frequency of the AC input voltage is between 100Hz and 10kHz.

In this case, $f_{min} = 100\text{Hz}$; and $R = \frac{20\text{k}\Omega}{\text{V}} \times 50\text{V}$, or 1MΩ. Thus:

$$T_{max} = \frac{1}{f_{min}} = \frac{1}{100\text{Hz}} = 0.01\text{S}$$

$$C = \frac{50\,T_{max}}{R} = \frac{50\,(0.01\text{S})}{1\text{M}\Omega} = 0.5\mu\text{F}$$

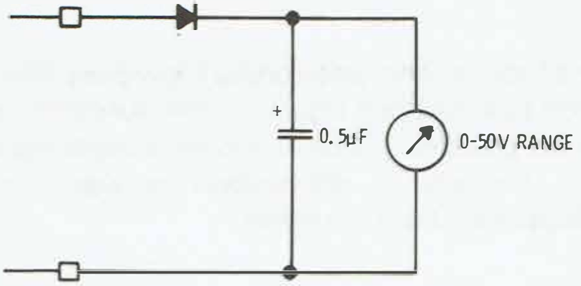


Figure 7-17

Circuit for Example 7-4.

Figure 7-17 illustrates a sketch of the resulting circuit. Here, the reading of the DC voltmeter corresponds to the positive peak value of the AC input voltage. Thus, when V_{in} equals 50V peak, the voltmeter reads 50V. Since the voltmeter is calibrated to indicate the peak value of V_{in}, the input voltage need not be sinusoidal to obtain accurate measurements.

Peak-To-Peak-Type AC Voltmeter

A peak-to-peak type AC voltmeter can be used to measure asymmetrical signal voltages as shown in Figure 7-18. Here, D_1 and C_1 form a positive clamper, while D_2 and C_2 constitute a positive peak detector. The result, of course, is a peak-to-peak detector whose output voltage is measured by the DC voltmeter. As was the case with the peak-type AC voltmeter, the input resistance of the DC voltmeter is in parallel with the peak detector capacitor, C_2.

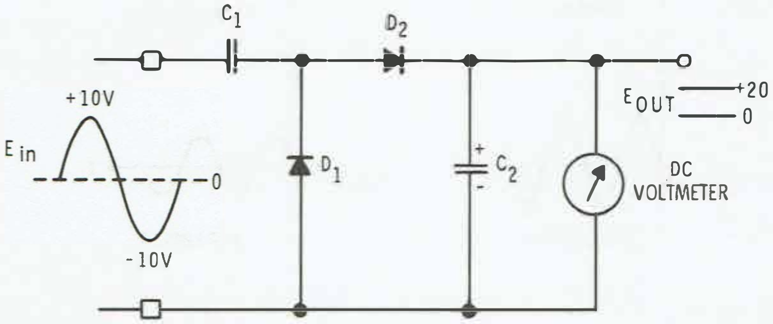


Figure 7-18

A peak-to-peak type AC voltmeter.

Waveform Converters

In Unit 6, you learned that a long time-constant low-pass filter functions as an integrator, and a short time-constant high-pass filter functions as a differentiator. Recall that an integrator provides an output voltage proportional to the area under the input waveform. Similarly, a differentiator provides an output voltage proportional to the slope of the input waveform.

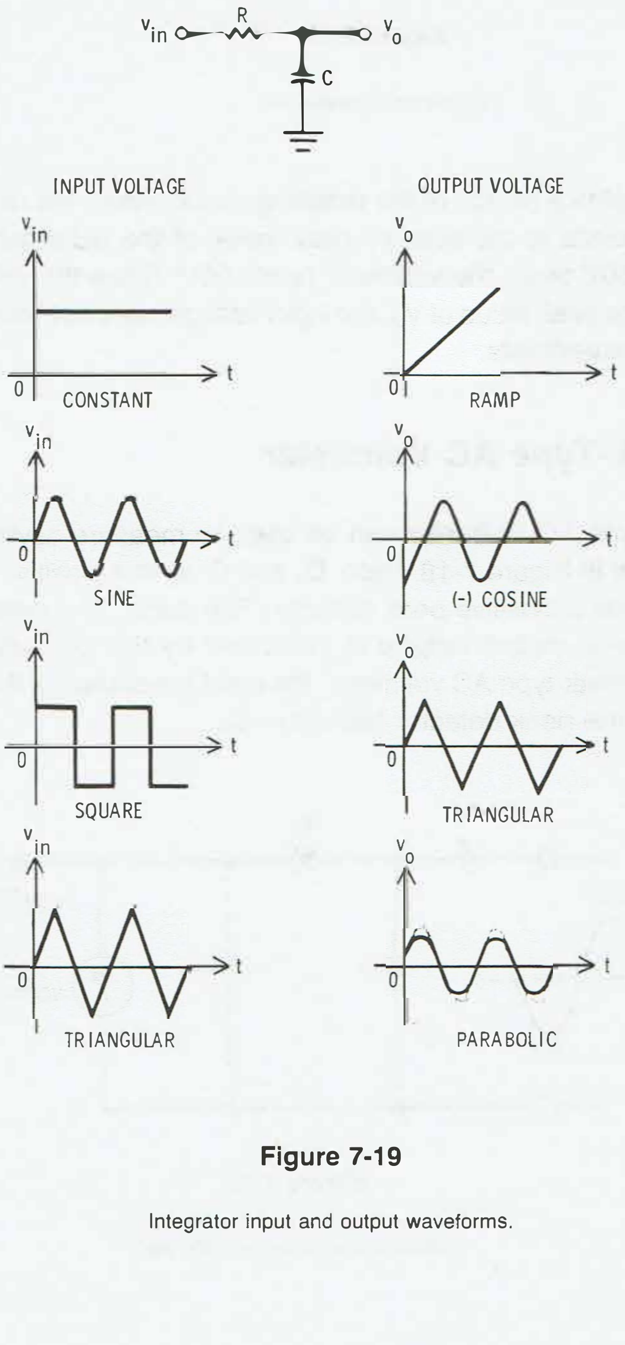


Figure 7-19

Integrator input and output waveforms.

Both integrators and differentiators can be used as simple "waveform converters". Figure 7-19 and Figure 7-20 respectively, illustrate how integrators and differentiators change the shape of frequently encountered signal waveforms. Obviously, there are numerous applications for these waveform converters. For example, the spikes you obtain by differentiating a square-wave make excellent trigger pulses for pulse and digital circuits.

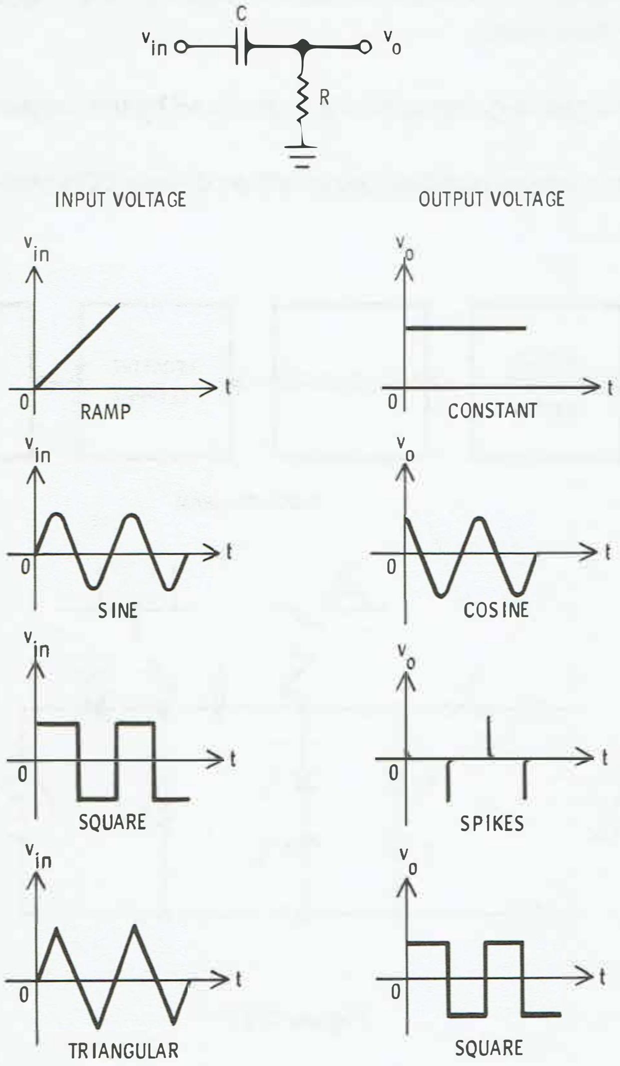


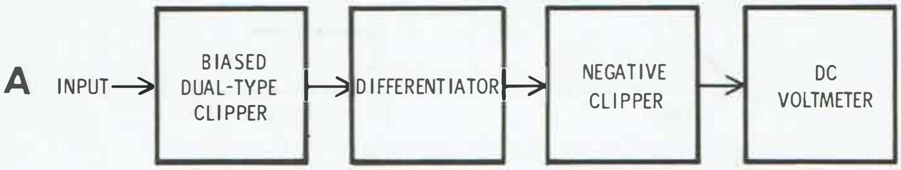
Figure 7-20

Differentiator input and output waveforms.

Rate Meter

The various sections of a basic frequency, or rate meter are indicated by the block diagram in Figure 7-21A. The operation of the circuit may be described as follows:

1. The Ac input voltage is converted to a square-wave by the biased dual-type clipper.
2. The square-wave is converted into positive- and negative-going spikes by the differentiator.
3. The negative-going spikes are removed by the negative clipper.
4. The remaining positive-going spikes drive a DC voltmeter.



BLOCK DIAGRAM

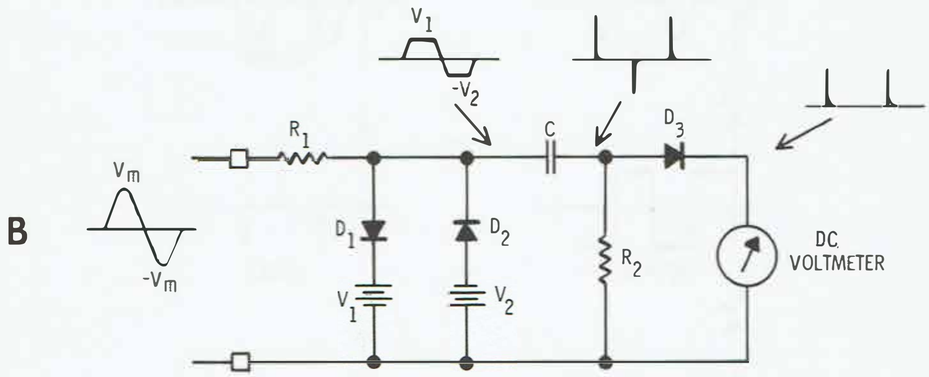


Figure 7-21

An elementary rate meter.

When a DC voltmeter measures an AC waveform, the voltmeter deflects to the average DC value of the waveform. In this case, the average value of the positive spike train is proportional to the frequency, or number of spikes per second. Consequently, the reading of the DC voltmeter varies directly with the frequency of the AC input voltage, from which the positive spike train is derived.

A simple circuit and appropriate waveforms for the rate meter are illustrated in Figure 7-21B. Here, R_1, D_1, D_2, and the two DC voltage sources comprise the biased dual-type clipper. Similarly, C and R_2 make up the differentiator portion of the circuit. Diode D_3 serves as a negative clipper so that only positive spikes are fed to the DC voltmeter.

In order for the rate meter in Figure 7-21B to properly track changes in the signal frequency, the amplitude of the positive spikes should be approximately 40V or more. For this reason, it may be necessary to amplify the AC input signal prior to employing the rate meter circuit.

Diode Checker

Since many of the circuits discussed in this course use diodes, we will conclude this applications section by considering a simple diode checker.

The light-emitting diode, or LED, is a special type of diode. As the name implies, it emits light when it is forward biased. Similarly, no light is emitted when an LED is reverse biased. For purposes of this discussion, we are primarily concerned with what an LED does, rather than how it does it. If you would like to know more about LED's, consult the Heathkit/Zenith Optoelectronics course, EE-105, for appropriate details.

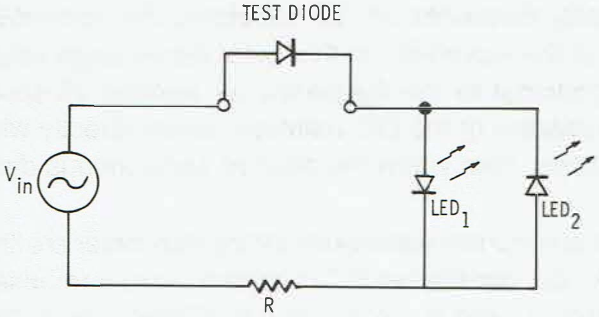


Figure 7-22

Diode checker.

The circuit for the diode checker is illustrated in Figure 7-22. Here, the value of R is chosen to limit the peak current through either LED to a safe value, generally 5 to 10mA. Assuming that the test diode is "good," the circuit operates as follows:

On the positive half cycle of the input voltage, the test diode and LED 1 are forward biased. Consequently, LED 1 lights. During this time, LED 2 is reverse biased, and therefore remains off.

On the negative half cycle of the input voltage, the test diode is reverse biased. Since a reverse biased diode acts like an open circuit, both LED's are off. Thus, for a good diode, LED 1 is turned on once per cycle. If the frequency of the input signal is approximately 30Hz or more, LED 1 will appear to be continuously lit for a good diode.

If the test diode is open, no current flows on either the positive or negative half cycle. Consequently, both LED's remain off. In the case of a shorted test diode, each LED is forward biased on alternate half cycles of the input voltage. For this reason, both LED's will appear to be continuously lit.

The operation of the diode checker is summarized in Table 7-2.

CONDITION	LED STATUS
Good diode	LED 1 lit
Open diode	Neither LED lit
Shorted diode	Both LEDs lit

Table 7-2

Diode checker.

Self-Test Review

11. You can achieve a type of overload protection by shunting an ammeter with two back-to-back __________.

12. A moving coil mechanism has a resistance of 100Ω and a full-scale current rating of 1mA. The shunt resistance required to measure currents up to 10mA is therefore __________ohms.

13. The series resistance required to convert the moving coil mechanism in question 12 to a 0-20V DC voltmeter is __________ Ω.

14. The effective resistance of the shunted ammeter in question 12 is __________ohms.

15. A disadvantage of the expanded-scale voltmeter is the fact that the __________resistance is relatively low.

16. By cascading a peak detector and DC voltmeter, you obtain a __________ type AC voltmeter.

17. If you integrate a square wave, the result is a __________wave.

18. If you differentiate a triangular wave, the result is a __________wave.

19. By cascading a biased dual-type clipper, a differentiator, a negative clipper, and a DC voltmeter, you obtain a simple __________meter.

20. In Figure 7-22, the test diode is __________if both LEDs appear lit.

Answers

11. diodes
12. 11.1
13. 19.9k
14. 10
15. input
16. peak
17. triangular
18. square
19. rate, or frequency
20. shorted

The solutions to the appropriate questions follow:

12. $R_{SH} = \dfrac{I_1 R_m}{I_2 - I_1} = \dfrac{\text{ImA } (100\Omega)}{10\text{mA} - 1\text{mA}} = 11.1\Omega$

13. $R_s = \dfrac{V_T}{I_1} - R_m = \dfrac{20\text{V}}{1\text{mA}} - 100\Omega = 19.9\text{k}\Omega$

14. The effective resistance of the shunted ammeter is $R_{SH} \parallel R_m$. Thus:

$$R = 11.1\Omega \| 100\Omega \approx 10\Omega$$

17. 18. Refer to Figure 7-19 and Figure 7-20 for the appropriate waveforms.

19. Refer to Figure 7-21 for the appropriate waveforms.

20. Refer to Table 7-2 on Page 7-34.

SUMMARY

The following points summarize the key ideas of this unit.

Numerous applications for passive circuits were discussed in this unit. For simplicity, the various applications were grouped into general and measurement catagories as follows:

General Applications

Reversed Polarity Protection
Elementary Heating Control
Diode Arc Suppressors
Wave Shapers
DC Restoration.
Catching Diodes.
De-emphasis Circuit.

Measurement Applications

Meter Protection
DC Ammeters
DC Voltmeters
Expanded-Scale Voltmeters
Peak-Type AC Voltmeters
Peak-to-Peak Type AC.
Voltmeters.
Waveform Converters.
Rate Meter.
Diode Checker.

Obviously, there are many more applications for passive circuits than those considered here. However, the representative sample considered should give you a good idea how passive circuits can be used to perform a number of very useful functions.

UNIT EXAMINATION

The following multiple choice examination is designed to test your understanding of the material presented in this unit. Place a check beside the multiple choice answer (A, B, C, or D) that you feel is most correct. When you have completed the examination, compare your answers with the correct ones that appear after the exam.

1. The time constant of a de-emphasis circuit is:
   - A. 75mS.
   - B. 75μS.
   - C. R/C.
   - D. A function of the transmitted frequency.

2. Modern function generators usually produce sine waves by shaping:
   - A. Square waves.
   - B. Positive spikes.
   - C. Triangular waves.
   - D. Cosine waves.

3. A differentiator with a square wave input has an output that is:
   - A. Triangular.
   - B. Sinusoidal.
   - C. A pulse train.
   - D. Positive and negative spikes.

4. The input resistance of a DC voltmeter equals the ohms per volt rating times:
   - A. The value of V_{in} required for full-scale deflection.
   - B. The peak value of V_{in}.
   - C. The DC value of V_{in}.
   - D. The rms value of V_{in}.

Refer to Figure 7-23 for questions 5 to 7.

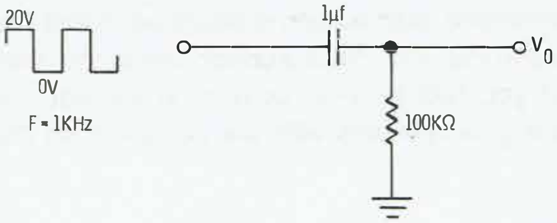


Figure 7-23

Circuit for Questions 5 to 7.

5. The circuit illustrates a:
   - A. Differentiator.
   - B. Integrator.
   - C. Coupling circuit.
   - D. Bypass circuit.

6. V_o would be a:
   - A. Square wave.
   - B. Triangular wave.
   - C. Series of positive and negative spikes.
   - D. Positive ramp.

7. The average value of V_o is:
   - A. 20V.
   - B. 10V.
   - C. 0V.
   - D. 6.67V.

8. By cascading a voltage doubler and DC voltmeter, you obtain a:
   - A. Peak-type AC voltmeter.
   - B. Expanded-scale voltmeter.
   - C. Peak-to-peak type AC voltmeter.
   - D. Rate meter.

9. An LED emits light when:
    - A. Forward biased.
    - B. Reverse biased.
    - C. $r_F = R_R$.
    - D. Power is first applied.

10. DC restoration can be accomplished if you use:
    - A. Clippers.
    - B. Clampers.
    - C. Peak Detectors.
    - D. Pre-emphasis.

11. A sine to square wave converter uses a:
    - A. Biased clamper.
    - B. Dual-type clipper.
    - C. Rate meter.
    - D. Integrator.

12. An expanded-scale voltmeter employs a 20V zener diode and a 0-50V DC voltmeter. The expanded scale is therefore:
    - A. 0-20V.
    - B. 20V-50V.
    - C. 20V-70V.
    - D. 50V-70V.

13. Relay coils are shunted with a diode to:
    - A. Decrease rise and fall times.
    - B. Restore the DC level.
    - C. Minimize arcing.
    - D. Lower the coil impedance.

14. An ammeter can be converted to a voltmeter with an appropriate:
    - A. Shunt resistor.
    - B. Catching diode.
    - C. Series resistor.
    - D. LED.

EXAMINATION ANSWERS

1. B — De-emphasis circuits have a time constant of 75µS.

2. C — By employing diode wave shapers, triangular waves are converted into sine waves.

3. D — The derivative of a square wave consists of positive- and negative-going spikes.

4. A — The product of the ohms per volt rating and full-scale voltage equals the input resistance of a DC voltmeter.

5. C — The circuit is a coupling circuit for the 1kHz input signal since at this frequency $|X_c| << R$. For the circuit to act like a differentiator, the time constant would have to be small compared to the period of the input signal. In this case, the time constant is large compared to the period of the input signal. Consequently, the circuit does **not** function as a differentiator.

6. A — Since this is a coupling circuit, the output voltage has the same shape as the input voltage. Thus, the output voltage would be a square wave.

7. C — When an asymmetrical waveform is passed through a coupling circuit the DC level is removed. Thus, the average value of V_o is 0V, which is what a DC voltmeter would read.

8. C — A peak-to-peak type AC voltmeter consists of a peak-to-peak detector and a voltage doubler cascaded with a DC voltmeter.

9. A — An LED emits light when it is forward biased.

10. B — Clampers are used to restore the DC level of a signal that has passed through a coupling circuit or transformer.

11. B — A dual-type clipper provides an approximate square-wave output for a sine wave input, assuming $V_m > V$.

12. C — The zener diode acts like an open circuit until V_{in} exceeds V_z. Thus, V_z determines the lower limit of the scale. Once V_{in} exceeds V_z, the DC voltmeter reads a voltage equal to $V_{in} - V_z$. For this reason, the upper limit of the scale equals $V_{FS} + V_z$. In this case, $V_z = 20V$, and $V_{FS} = 50V$. Thus, the range is 20V to 70V.

13. C — You can minimize arcing across the relay contacts by employing a shunt diode across the relay coil.

14. C — Series resistors are used to convert a DC ammeter to a DC voltmeter.

Unit 8

EXPERIMENTS

CONTENTS

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Experiment 10: Measurement Circuits .... Page 8-91

INTRODUCTION

The experiments that follow have been designed to complement the material presented in units one through seven. Before building the experimental circuits you should first read each experiment to make sure you understand the purpose of each step.

Important Notice

Newer Heathkit Electronic Design Experimenters employ a 3-terminal power plug, rather than the conventional 2-terminal power plug. If you are using one of these newer Trainers, and the test equipment you are using (oscilloscope, voltmeter, etc.) also employs a 3-terminal power plug, it will be necessary to install a 3-wire to 2-wire adapter on the **Trainer** power plug. This will preclude the possibility of blowing a fuse in your Trainer or obtaining invalid results from some of the experiments. These adapters are inexpensive and readily available at most stores.

The use of this adapter is necessary **only** when using test equipment having 3-terminal power connectors.

CAUTION

Care must be taken to insure that all pieces of equipment used in your experiments are connected to the same power source.

EXPERIMENT 1
DC CIRCUIT ANALYSIS

Objectives:

To verify the superposition theorem.
To verify Thevenin's theorem.
To deduce the values of V_{TH} and R_{TH} experimentally.
To verify the maximum power transfer theorem.

Introduction

In this experiment you will calculate the voltage across a 10kΩ load resistor by the superposition theorem, and by Thevenin's theorem. Naturally, both methods should predict the same response. In addition, you will calculate the load voltage and power for ten different values of load resistance. Once your calculations are complete, you will build the appropriate circuits and make the measurements necessary to verify your calculations. Finally, you will plot a curve of load power versus load resistance to experimentally verify the maximum power transfer theorem. Before you build the circuits discussed in this experiment, be sure to check your calculations so that you know what to look for when you make the appropriate measurements.

Material Required

Heathkit Engineering Design Trainer ET-1000
Volt-ohmmeter (Heathkit SM-1210 or equivalent)

1—1 kΩ, 1/4-watt, 5% resistor
1—2.2 kΩ, 1/4-watt, 5% resistor
1—3.3 kΩ, 1/4-watt, 5% resistor
1—4.7 kΩ, 1/4-watt, 5% resistor
1—6.8 kΩ, 1/4-watt, 5% resistor
2—10 kΩ, 1/4-watt, 5% resistor
1—15 kΩ, 1/4-watt, 5% resistor
1—22 kΩ, 1/4-watt, 5% resistor
1—33 kΩ, 1/4-watt, 5% resistor
1—56 kΩ, 1/4-watt, 5% resistor

Procedure

1. Calculate the voltage across R_L in Figure 8-1A due to the 12V source.
   $V_{ab} = V_{R_L} =$ ________V.

2. Calculate the voltage across R_L in Figure 8-1A due to the 5V source.
   $V_{ab} = V_{R_L} =$ ________V.

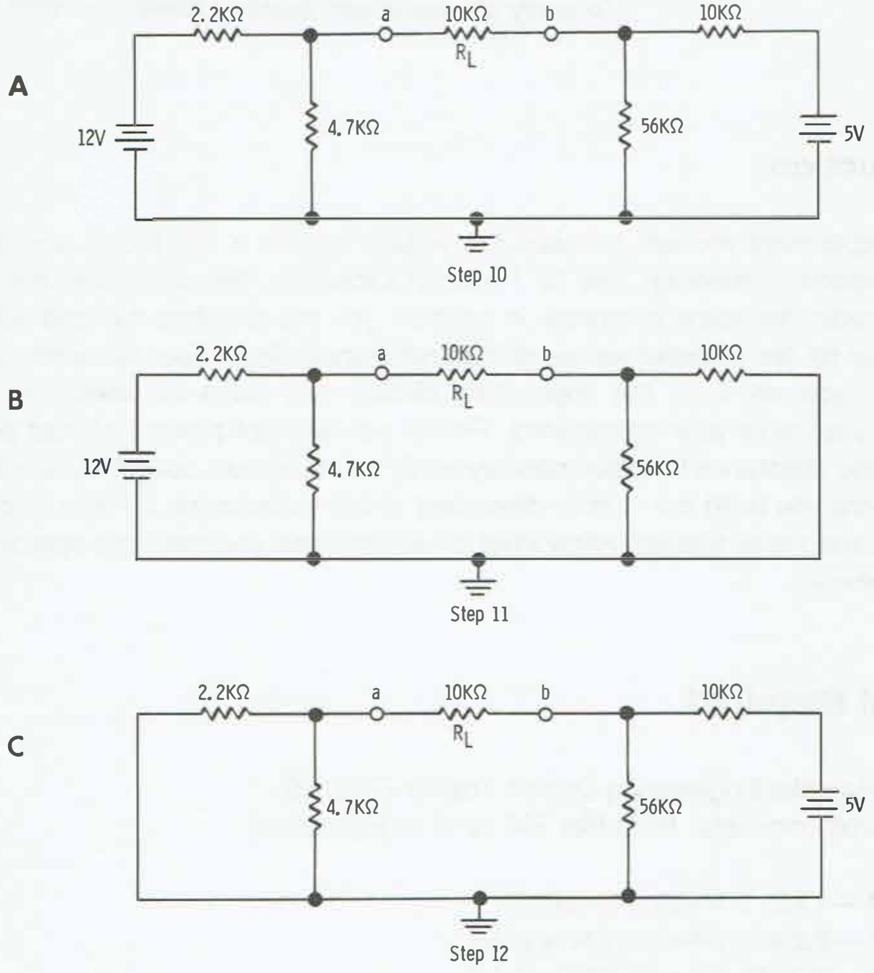


Figure 8-1

Experimental circuits for steps 10-12.

3. Use the values calculated in steps 1 and 2 to predict the voltage across R_L when **both** sources are present. $V_{ab} = V_{R_L} =$ ________V.

4. Using the relationship $P_L = V_L^2/R_L$, calculate the load power for steps 1, 2, and 3.

$P_{L_1} =$ ________ μW (step 1)
$P_{L_2} =$ ________ μW (step 2)
$P_{L_3} =$ ________ μW (step 3)

5. Does $P_{L_1} + P_{L_2} = P_{L_3}$? __________ Why? __________________________
____

6. Calculate the value of R_{TH} for the circuit in Figure 8-1A. R_{TH} = ___________ kΩ.

7. Calculate the value of V_{TH} for the circuit in Figure 8-1A. V_{TH} = _____________ kΩ.

8. Use the Thevenin equivalent circuit to predict the load voltage and load power for the following values of R_L:0, 1kΩ, 2.2kΩ, 3.3kΩ, 4.7kΩ, 6.8kΩ, 10kΩ, 15kΩ, 22kΩ, 33kΩ, and ∞ ohms. Record your calculated values in Table 8-1. For what value of R_L is the load power maximum? _____________ R_L = _____________ kΩ.

R_L (Ω)	V_L (V)	$P_L = V_L^2/R_L$ (μW)
0		
1k		
2.2k		
3.3k		
4.7k		
6.8k		
10k		
15k		
22k		
33k		
∞		

Table 8-1

Calculations for Step 8.

Discussion

In the first part of this experiment, you were asked to analyze the circuit in Figure 8-1A. The analysis is provided at this point so that you can check your calculations prior to building the actual circuits.

Step 1 Refer to Figure 8-2.

Reducing the 5V source to zero you have:

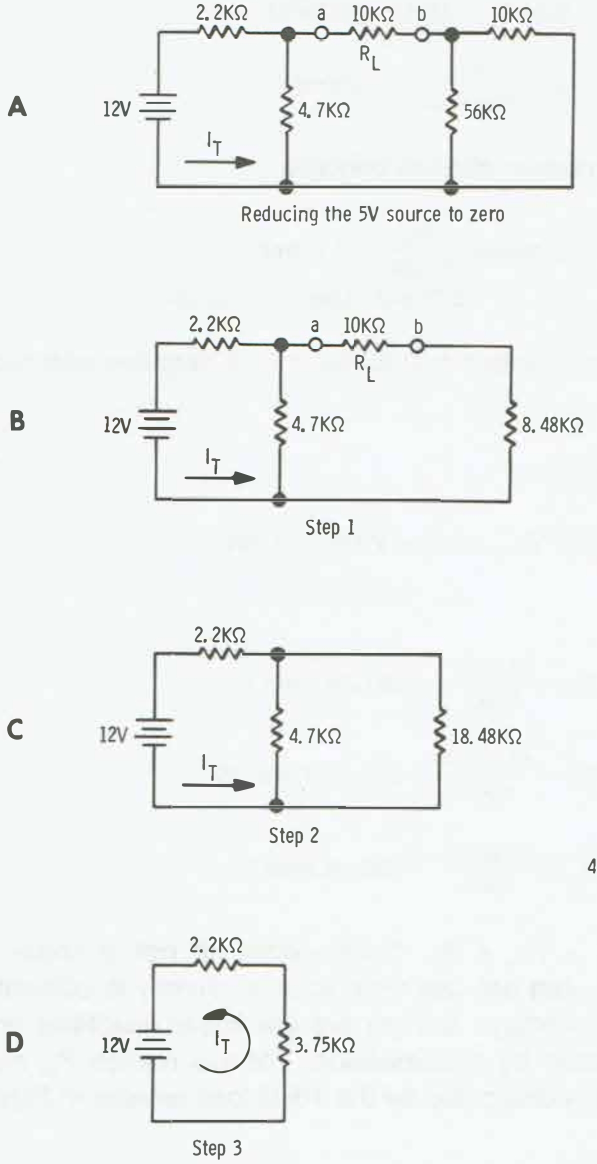
$$\begin{aligned} 56\text{k}\Omega \parallel 10\text{k}\Omega &= 8.48\text{k}\Omega \\ 10\text{k} + 8.48\text{k} &= 18.48\text{k}\Omega \\ 18.48\text{k} \parallel 4.7\text{k} &= 3.75\text{k}\Omega \\ 3.75\text{k} + 2.2\text{k} &= 5.95\text{k}\Omega \end{aligned}$$

$$I_T = \frac{12\text{V}}{5.95\text{k}\Omega} = 2.02\text{mA}$$

Employing the current division principle:

$$I_{RL} = 2.02\text{mA}\frac{(4.7\text{k}\Omega)}{(23.18\text{k}\Omega)} = 0.41\text{mA}$$

$$V_{RL} = V_{ab} = 0.41\text{mA}\ (10\text{k}\Omega) = 4.1\text{V}$$



10K||56K=8.48KΩ

10K+8.48K=18.48KΩ

4.7K||18.48K=3.75KΩ

3.75KΩ + 2.2KΩ = 5.95KΩ

Figure 8-2

Calculating the voltage across R_L due to the 12V source.

Step 2 Refer to Figure 8-3.

$$
\begin{aligned}
&2.2\text{k}\Omega \parallel 4.7\text{k}\Omega = 1.5\text{k}\Omega\\
&10\text{k}\Omega + 1.5\text{k}\Omega = 11.5\text{k}\Omega\\
&11.5\text{k}\Omega \parallel 56\text{k}\Omega = 9.54\text{k}\Omega\\
&9.54\text{k}\Omega + 10\text{k}\Omega = 19.54\text{k}\Omega
\end{aligned}
$$

$$I_T = \frac{5\text{V}}{19.54\text{k}\Omega} = 0.256\text{mA}$$

Employing the current division principle:

$$
\begin{aligned}
I_{RL} &= 0.256\text{mA}\frac{56\text{k}\Omega}{67.5\text{k}\Omega} = 0.212\text{mA}\\
V_{RL} &= V_{ab} = -0.212\text{mA}\,(10\text{k}\Omega) = -2.12\text{V}
\end{aligned}
$$

The minus sign indicates that terminal A is negative with respect to terminal B.

Step 3 By superposition:

$$V_{ab} = V_{RL} = 4.1\text{V} - 2.12\text{V} = 1.98\text{V}$$

Step 4

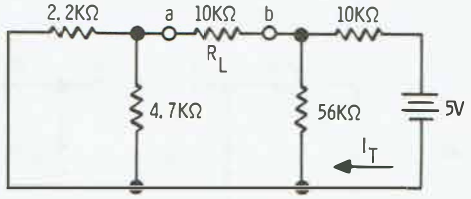
$$P_{L_1} = \frac{(4.1\text{V})^2}{10\text{k}\Omega} = 1681\mu\text{W (step 1)}$$

$$P_{L_2} = \frac{(2.12\text{V})^2}{10\text{k}\Omega} = 449.4\mu\text{W (step 2)}$$

$$P_{L_3} = \frac{(1.98\text{V})^2}{10\text{k}\Omega} = 392\mu\text{W (step 3)}$$

Step 5 Note that $P_{L_1} + P_{L_2} \neq P_{L_3}$. Since power is **not** a linear quantity ($P = I^2R$), you **can not** use superposition directly to calculate power in DC circuits. Voltage and current **are** linear quantities and, thus, can be calculated by superposition. For this reason P_{L_3} represents the actual power dissipated by the 10kΩ load resistor in Figure 8-1A.

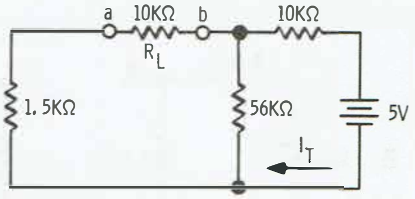
A



Reducing the 12V source to zero

$2.2K \parallel 4.7K = 1.5K\Omega$

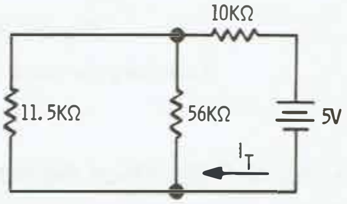
B



Step 1

$10K + 1.5K = 11.5K\Omega$

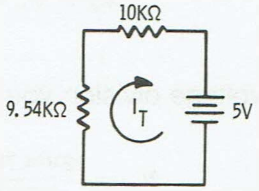
C



Step 2

$11.5K \parallel 56K = 9.54K\Omega$

D



Step 3

$9.54K + 10K = 19.54K\Omega$

Figure 8-3

Calculating the voltage across R_L due to the 5V source.

Step 6 Refer to Figure 8-4A. Reducing each source to zero and removing the 10kΩ load resistor you have:

$$
\begin{aligned}
R_{ab} = R_{TH} &= 2.2\text{k}\Omega \parallel 4.7\text{k}\Omega + 10\text{k}\Omega \parallel 56\text{k}\Omega \\
&= 1.5\text{k}\Omega + 8.48\text{k}\Omega \\
&= 9.98\text{k}\Omega \approx 10\text{k}\Omega
\end{aligned}
$$

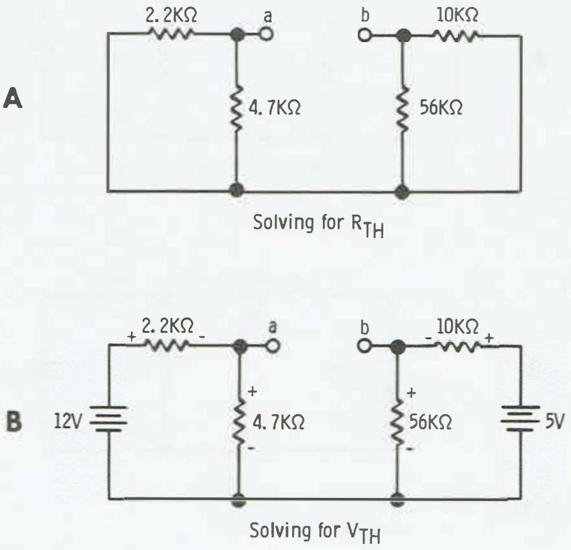


Figure 8-4

Thevenizing the circuit in Figure 8-1 A.

Step 7 Refer to Figure 8-4B. Once the load is removed, the 2.2kΩ and 4.7kΩ resistors are connected in series. Similarly, the 10kΩ and 56kΩ resistors are also connected in series. By inspection, you can see that:

$$V_{ab} = V_{TH} = V_{4.7k} - V_{56k}$$

Using voltage division you have:

$$V_{TH} = \frac{12\text{V}(4.7\text{k}\Omega)}{6.9\text{k}\Omega} - \frac{5\text{V}(56\text{k}\Omega)}{66\text{k}\Omega}$$

$$V_{TH} = 8.17\text{V} - 4.24\text{V} = 3.93\text{V}$$

Figure 8-5 illustrates the Thevenin equivalent circuit.

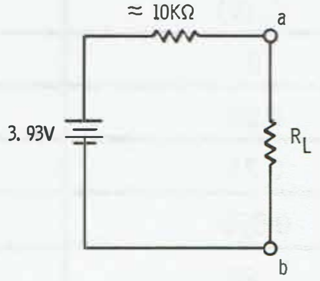


Figure 8-5

Thevenin equivalent for the circuit in Figure 8-4A based on **calculated values**.

Step 8 In Figure 8-5, the voltage across R_L is:

$$V_{R_L} = \frac{V_{TH}\,R_L}{R_{TH} + R_L} = \frac{3.93V(R_L)}{10k\Omega + R_L}$$

(R_{TH} is rounded from 9.98kΩ to 10kΩ for simplicity in the above equation).

By substituting the values of R_L given in steps 8 (0, 1kΩ, 2.2kΩ, 3.3kΩ, 4.7kΩ, 6.8kΩ, 10kΩ, 15kΩ, 22kΩ, 33kΩ, and ∝ ohms) into the above formula, you can calculate the load voltage for each value of load resistance. Once you know the load voltage, you can calculate the load power from: $P_L = V_L^2/R_L$. For example when $R_L = 1k\Omega$ you have:

$$V_{R_L} = \frac{3.93V(1k\Omega)}{11k\Omega} = 0.36V$$

$$P_L = \frac{(0.36V)^2}{1k\Omega} = 130\mu W$$

Similar calculations for the remaining values of R_L yields:

R_L (Ω)	V_L (V)	$P_L = V_L^2/R_L$ (μW)
0	0	0
1k	0.36	130
2.2k	0.71	229
3.3k	0.975	288
4.7k	1.26	336
6.8k	1.59	372
10k	1.97	388
15k	2.36	371
22k	2.70	332
33k	3.02	276
∞	3.93	0

Table 8-2

Calculated values (Step 8).

Note that maximum load power results when $R_L = 10k\Omega$.

Figure 8-6 illustrates the plot of load power versus load resistance **based on the values in Table 8-2**.

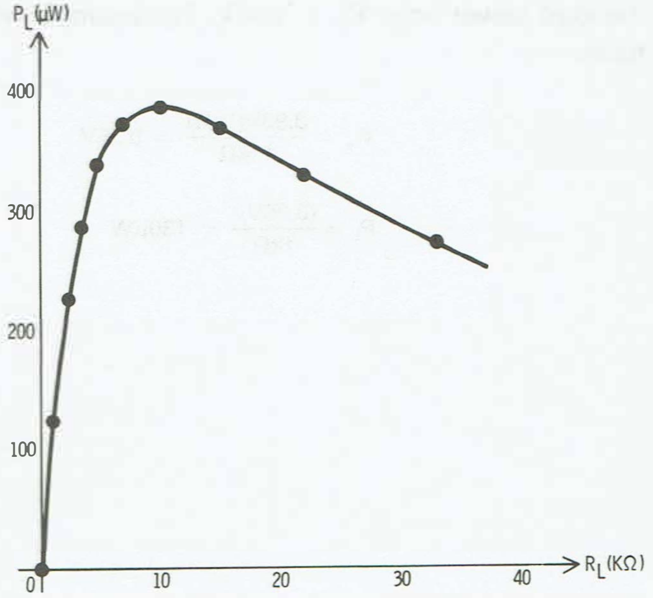


Figure 8-6

Load power versus load resistance based on the **calculated values** in Table 8-2.

At this point you are almost ready to begin building the circuits and recording your experimental (measured) values. Naturally, you are interested in determining the **degree of agreement** (DOA) between your calculated and measured values. You can do this very easily by expressing your measured values as a percentage of your calculated values. For example, suppose a voltage in a circuit is calculated to be 2v, ($V_c = 2v$). Further assume two experimenters report the following measured values:

Experimenter #1, $V_{m1} = 1.87V$

Experimenter #2, $V_{m2} = 2.18V$

You can compute the degree of agreement (DOA) between calculated (V_c) and measured (V_m) values for each experimenter as follows:

$$\#1\ DOA = \frac{1.87}{2} \times 100 = 93.5\%$$

This indicates that the first experimenter reported a measured value that was 93.5% of the calculated value.

$$\#2\ DOA = \frac{2.18}{2} \times 100 = 109\%$$

This indicates that the second experimenter reported a measured value that was 109% of the calculated value.

Rarely will calculated and measured values agree **exactly**! By calculating the degree of agreement, however, you will quickly determine if you are "in the ballpark". We will have more to say about this concept in later units when we begin to design circuits using "real" components. For now, remember that you can calculate the degree of agreement between calculated and measured values from the following formula:

$$DOA = \frac{V_m}{V_c} \times 100 \qquad \text{(Eq. 8-1)}$$

where: V_m = measured value
V_c = calculated value

Procedure (continued)

9. Build the circuit shown in Figure 8-1 A.

10. Measure V_{ab} . V_{ab} = _____V. Calculate the DOA with the value computed in step 3.

$$\text{DOA} = \frac{\rule{2em}{0.4pt}}{1.98} \times 100 = \rule{4em}{0.4pt}\ \%.$$

11. Build the circuit shown in Figure 8-1B. Measure V_{ab}. V_{ab} = _____V. Calculate the DOA with the value computed in step 1.

$$\text{DOA} = \frac{\rule{2em}{0.4pt}}{4.1} \times 100 = \rule{4em}{0.4pt}\ \%.$$

12. Build the circuit shown in Figure 8-1C. Measure V_{ab}. V_{ab} = _____V. Calculate the DOA with the value computed in step 2.

$$\text{DOA} = \frac{\rule{2em}{0.4pt}}{2.12} \times 100 = \rule{4em}{0.4pt}\ \%.$$

13. Build the circuit shown in Figure 8-1A. Measure V_{ab} for the following values of R_L: 0, 1kΩ, 2.2kΩ, 3.3kΩ, 4.7kΩ, 6.8kΩ, 10kΩ, 15kΩ, 22kΩ, 33kΩ, and ∞ ohms. Record your measured values in Table 8-3. Complete the load power column by using the relationship $P_L = V_L^2/R_L$.

R_L (Ω)	V_L (V)	$P_L = V_L^2/R_L$ (μW)
0		0
1k		
2.2k		
3.3k		
4.7k		
6.8k		
10k		
15k		
22k		
33k		
∞		

Table 8-3

Experimental data for step 13.

14. Calculate the DOA between measured and calculated values. Record this information in Table 8-4.

R_L (Ω)	Voltages $V_m/V_c \times 100 =$	%	Powers $P_m/P_c \times 100 =$	%
0	———		———	
1k	$\frac{\quad}{0.36} \times 100 =$		$\frac{\quad}{130} \times 100 =$	
2.2k	$\frac{\quad}{0.71} \times 100 =$		$\frac{\quad}{229} \times 100 =$	
3.3k	$\frac{\quad}{0.975} \times 100 =$		$\frac{\quad}{288} \times 100 =$	
4.7k	$\frac{\quad}{1.26} \times 100 =$		$\frac{\quad}{336} \times 100 =$	
6.8k	$\frac{\quad}{1.59} \times 100 =$		$\frac{\quad}{372} \times 100 =$	
10k	$\frac{\quad}{1.97} \times 100 =$		$\frac{\quad}{386} \times 100 =$	
15k	$\frac{\quad}{2.36} \times 100 =$		$\frac{\quad}{371} \times 100 =$	
22k	$\frac{\quad}{2.70} \times 100 =$		$\frac{\quad}{332} \times 100 =$	
33k	$\frac{\quad}{3.02} \times 100 =$		$\frac{\quad}{276} \times 100 =$	
∞	$\frac{\quad}{3.93} \times 100 =$		———	

Table 8-4

Degrees of agreement between calculated (step 8) and measured (step 13) values.

15. Use your experimental data to plot a curve of load power versus load resistance on the graph in Figure 8-7.

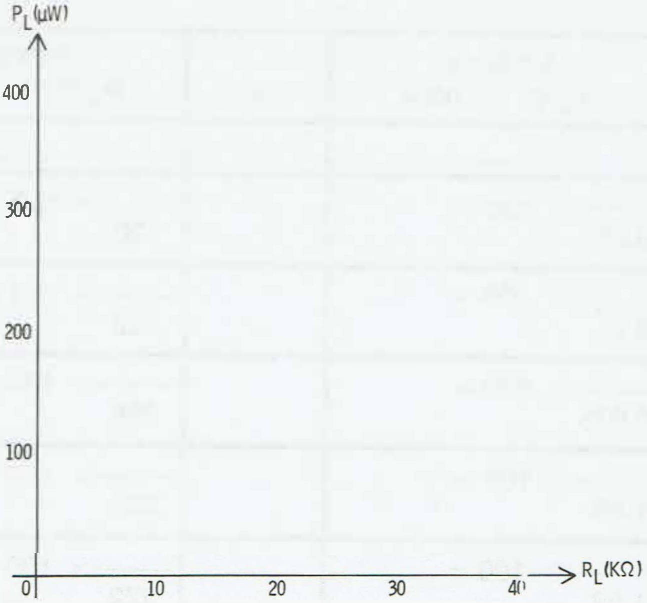


Figure 8-7

Load power versus load resistance based on the **measured values** in Table 8-3.

Discussion

Perhaps your measured and calculated values disagree to a greater extent than you thought they would. If this is the case, welcome to the real world! If you compare the calculated and experimental power curves, you should note the following:

1. Both curves have the same general shape.
2. Both curves peak (indicating maximum load power) when $R_L \approx 10k\Omega$.
3. The calculated power curve predicts a maximum load power of approximately 388μW.
4. Your experimental power curve **probably does not** predict a maximum load power of 388μW (it may, but this is unlikely).

To help you correctly interpret your data, let's begin by considering the effects of the resistor tolerances. Since resistors whose tolerance is ± 5% are used, **we can explain differences between calculated and measured voltages** (and currents) **up to 5%** just due to the tolerances of the resistors used in the experiment. Depending upon the type of volt-ohmmeter you used to make your measurements and how carefully you read the scales, an additional 5% error could have easily been introduced. Typically then, your voltage measurements probably disagreed with your calculated voltages by approximately 10%. It is also very likely that your calculated and measured powers disagreed by a significantly larger percentage. To understand why this is so, recall that power varies with the square of the voltage. For example, assume that you calculate a voltage of 2V across a 10kΩ resistor. The calculated power is:

$$P_c = \frac{(2V)^2}{10k\Omega} = 400\mu W$$

Now assume you measure the voltage across the 10kΩ resistor and read 10% low or 1.8v. The measured power is therefore:

$$P_m = \frac{(1.8V)^2}{10k\Omega} = 324\ \mu W$$

The DOA between the measured and calculated power is:

$$DOA = \frac{324}{400} \times 100 = 81\%$$

As you can see, even though the measured and calculated voltages differed by only 10%, the difference between the respective powers was 19%! Typically then, your power measurements probably disagreed with your calculated powers by approximately 20%.

Up until this point, when you Thevenized a circuit, you did so "on paper". This is not always practical, especially for complex circuits. Fortunately, you can arrive at the values of R_{TH} and V_{TH} experimently. The following method is one way of doing this:

TO MEASURE V_{TH}

1. Physically remove the load from the circuit.
2. Measure the open circuit voltage. This open circuit voltage **is** V_{TH}.

TO MEASURE R_{TH}

1. Remove the load and replace it with a potentiometer (variable resistor).
2. Vary the resistance of the potentiometer until the voltage across it equals one half V_{TH}.
3. Remove the potentiometer from the circuit and measure its resistance. This equals R_{TH}.

Procedure (continued)

16. Using the method outlined in the preceding Discussion section, experimentally determine the values of V_{TH} and R_{TH}. V_{TH} = ______V. R_{TH} = ______kΩ. The 100kΩ potentiometer on the ET-1000 Design Experimenter can be used to determine R_{TH}.

17. Adjust the output voltage of the variable power supply to the value of V_{TH} measured in step 16. Set the 100kΩ potentiometer to the value of R_{TH} measured in step 16, and connect it in series with the supply. This circuit is the Thevenin equivalent circuit. You will use it in the remainder of the experiment.

18. Measure the load voltage in the Thevenin equivalent circuit when R = 0, 1kΩ, 2.2kΩ, 3.3kΩ, 4.7kΩ, 6.8kΩ, 10kΩ, 15kΩ, 22kΩ, 33kΩ, and ∞ ohms. Record the voltages in Table 8-5.

R_L (Ω)	V_L (V)	$P_L = V_L^2/R_L$ (μW)
0		0
1k		
2.2k		
3.3k		
4.7k		
6.8k		
10k		
15k		
22k		
33k		
∞		

Table 8-5

Measured values for step 18.

Discussion

The values recorded in Table 8-5 should agree closely with those in Table 8-3, since the Thevenin equivalent circuit used here was based on measured values of V_{TH} and R_{TH}. The principle source of error in this portion of the experiment is the measured value of R_{TH} in step 16. R_{TH} should have measured close to 10kΩ. If you measured a value of R_{TH} significantly different from 10kΩ, the batteries in your volt-ohmmeter might need to be replaced.

EXPERIMENT 2
MATHEMATICS FOR CIRCUIT DESIGN

Objectives:

To experimentally verify the exponential equation $\nu(t) = (V_1 - V_2)\, e^{-t/\tau} + V_2$.

To experimentally determine the resistance of an inductor.

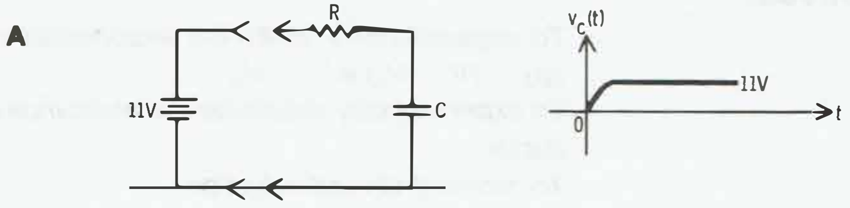
To demonstrate self induction.

Introduction

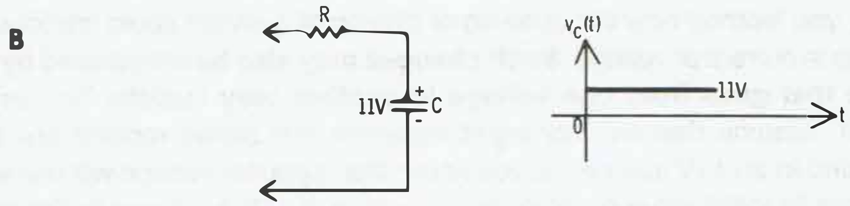
Earlier, you learned how the opening or closing of a switch could introduce abrupt changes in current or voltage. **Such changes may also be introduced by a signal source that goes from one voltage to another very rapidly.** To illustrate the concept, assume that an uncharged capacitor and series resistor are suddenly connected to an 11V source. As you know, the capacitor voltage will rise exponentially from its initial value of 0V to the final value of 11V as shown in Figure 8-8A. If the 11V source is now removed, the capacitor voltage remains at 11V as shown in Figure 8-8B.

Now suppose the RC network is suddenly connected to a "zero volt" source as shown in Figure 8-8C. Clearly, the capacitor voltage will decay from its initial value of 11V to the final value of 0V.

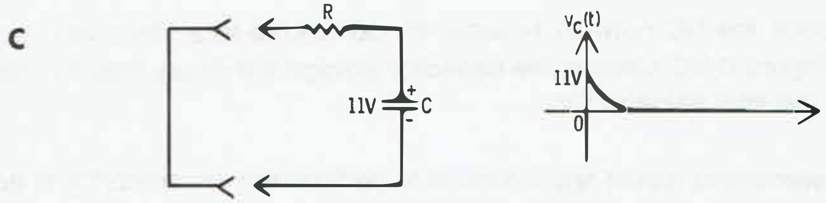
Most experimenters would find it difficult to perform the experiment just described and simultaneously observe the capacitor voltage on an oscilloscope! Fortunately, you can get the same effect by driving the RC network from a square-wave voltage source as shown in Figure 8-8D. In this circuit, when the input voltage is 11V, the capacitor voltage will rise exponentially from 0V to 11V just as it did in Figure 8-8A. Similarly, when the input voltage "switches" to 0V, the capacitor voltage will decay exponentially from 11V to 0V just as it did in Figure 8-8C. Since the square-wave input voltage continuously drives the RC network, a train of periodic rising and decaying voltages appear across the capacitor. Naturally, we have assumed that the input voltage remains at 11V and 0V long enough for the capacitor to completely charge and discharge. In this experiment, you will use the popular 555 timer integrated circuit to generate the square-wave input voltage.



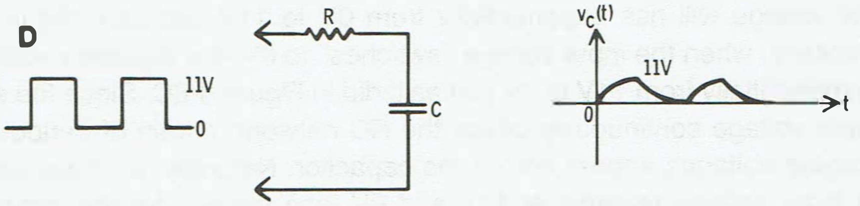
The 11V source produces a rising exponential



Capacitor charged to 11V



A "zero volt" source produces a decaying exponential



A square-wave input produces periodic rising and decaying exponentials.

Figure 8-8

Circuits for Introduction to Experiment 2.

Material Required

Heathkit Engineering Design Trainer ET-1000
Oscilloscope (Heathkit SO-4550 or equivalent)
Volt-ohmmeter (Heathkit SM 1210 or equivalent)
1 — 100Ω, ½-watt, 5% resistor
1 — 470Ω, ½-watt, 5% resistor
1 — 33kΩ, ½-watt, 5% resistor
1 — 470kΩ, ½-watt, 5% resistor
1 — .1 µF, 25v capacitor
1 — .01 µF, 25v capacitor
1 — .001 µF, 25v capacitor
1 — 4.5 H iron-core inductor (#46-3)
1 — Neon Lamp (#412-15)
1 — 555 Timer (#442-53)

Procedure

1. Calculate the time constant of the RC network in Figure 8-9. $\tau =$ ______µs.

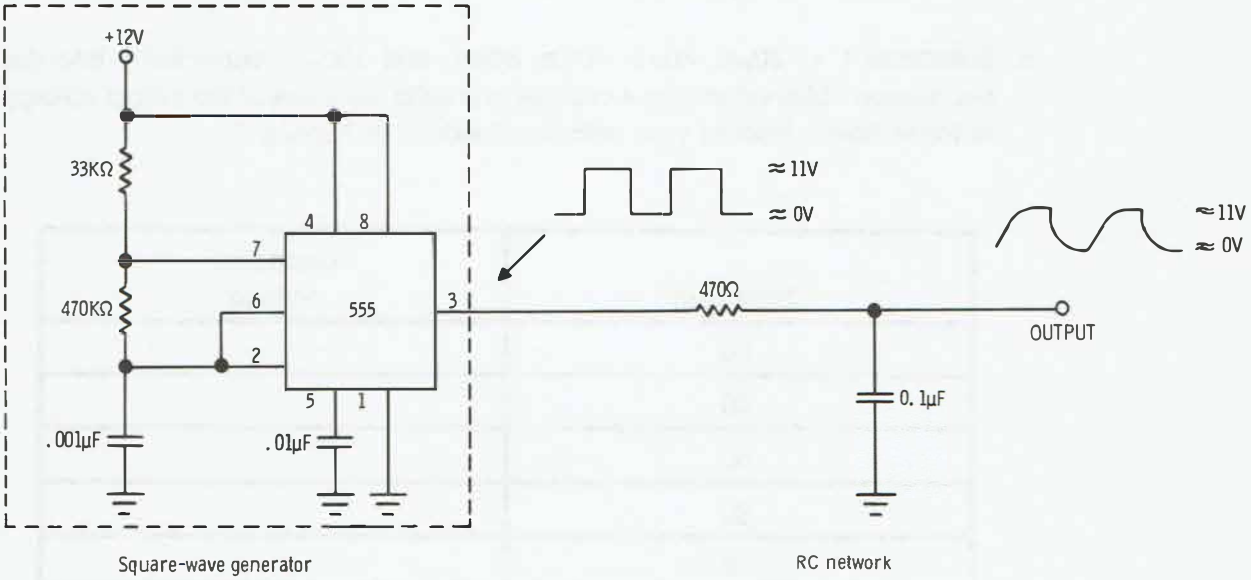


Figure 8-9

Experimental circuit for steps 1 to 11.

2. Substitute the initial voltage (0V), final voltage (11V), and time constant into the exponential equation to obtain an expression for the rising portion of the output voltage $v(t)$. $v(t) =$ ______.

3. Substitute t = 20μs, 40μs, 60μs, 80μs, and 100μs respectively into the expression obtained in step 2 in order to predict the value of the output voltage at these times. Record your calculated values in Table 8-6.

Time (μs)	Capacitor voltage
20	
40	
60	
80	
100	

Table 8-6

Calculated values for the rising exponential.

4. Substitute the initial voltage (11V), final voltage (0V), and time constant into the exponential equation to obtain an expression for the decaying portion of the output voltage $v_2(t)$. $v_2(t) =$ ______.

5. Substitute t = 20μs, 40μs, 60μs, 80μs, and 100μs respectively into the expression obtained in step 4 in order to predict the value of the output voltage at these times. Record your calculated values in Table 8-7.

Time (μs)	Capacitor voltage
20	
40	
60	
80	
100	

Table 8-7

Calculated values for the decaying exponential.

Discussion

Our calculations for steps 1 through 5 are provided at this point so that you can check your calculations prior to building the circuit.

Step 1

$$\tau = RC = 470\Omega\,(0.1\mu F) = 47\mu s$$

Step 2

$$\upsilon_1(t) = (V_1 - V_2)\,e^{-t/\tau} + V_2$$
$$\upsilon_1(t) = 0 - 11\,e^{-t/47\mu s} + 11$$
$$\upsilon_1(t) = 11 - 11\,e^{-t/47\mu s}$$
$$\upsilon_1(t) = 11\,(1 - e^{-t/47\mu s})$$

Step 3 For t = 20μs we have:

$$\upsilon_1(20\mu s) = 11\,(1 - e^{-20\mu s/47\mu s})$$
$$\upsilon_1(20\mu s) = 11\,(1 - e^{-0.426})$$
$$\upsilon_1(20\mu s) = 11\,(1 - 2.718^{-0.426})$$
$$\upsilon_1(20\mu s) = 11\,(1 - 0.653)$$
$$\upsilon_1(20\mu s) = 11\,(0.347) = 3.82V$$

Similar calculations for the remaining times yield:

$$\upsilon_1\,(40\mu s) = 6.3V$$
$$\upsilon_1\,(60\mu s) = 7.94V$$
$$\upsilon_1\,(80\mu s) = 9V$$
$$\upsilon_1\,(100\mu s) = 9.7V$$

Step 4

$$\upsilon_2(t) = (V_1 - V_2)\,e^{-t/\tau} + V_2$$
$$\upsilon_2(t) = (11 - 0)\,e^{-t/47\mu s} + 0$$
$$\upsilon_2(t) = 11\,e^{-t/47\mu s}$$

Step 5 For t = 20μs we have:

$$\upsilon_2(20\mu s) = 11\,e^{-20\mu s/47\mu s}$$
$$\upsilon_2\,(20\mu s) = 11\,e^{-0.426}$$
$$\upsilon_2\,(20\mu s) = 11\,(2.718^{-0.426})$$
$$\upsilon_2(20\mu s) = 11\,(0.653)$$
$$\upsilon_2\,(20\mu s) = 7.18V$$

Similarly calculations for the remaining times yield:

$$\upsilon_2\,(40\mu s) = 4.7V$$
$$\upsilon_2\,(60\mu s) = 3.1V$$
$$\upsilon_2\,(80\mu s) = 2.01V$$
$$\upsilon_2\,(100\mu s) = 1.3V$$

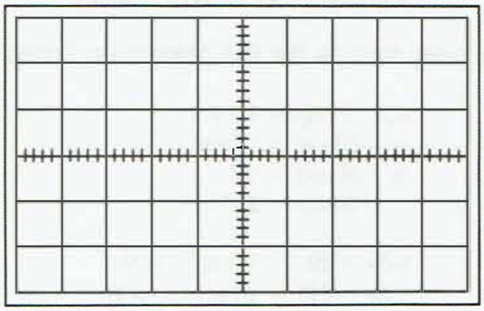
You can do the above calculations with a pocket calculator that has a y^x function.

Procedure (continued)

6. Build the circuit shown in Figure 8-9. Set the vertical sensitivity of the oscilloscope to 5V/division, and the horizontal sensitivity to 0.1 ms/division.

7. Measure the output voltage from the 555 timer circuit (pin 3). You should observe a square wave of approximately 11V peak whose frequency is approximately 1.5 k Hz. If this is not the case recheck your wiring.

8. Measure the voltage across the 0.1μF capacitor (If you have a dual trace oscilloscope, you can do steps 7 and 8 simultaneously). You should observe periodic rising and decaying exponential waves. Adjust the trace so that the rising portion of the output voltage starts at the center of the screen, which corresponds to $t = 0$. Sketch the observed waveform in Figure 8-10.



Volts/division= 5 Time/division =0.1ms

Figure 8-10

Waveform observed in step 8.

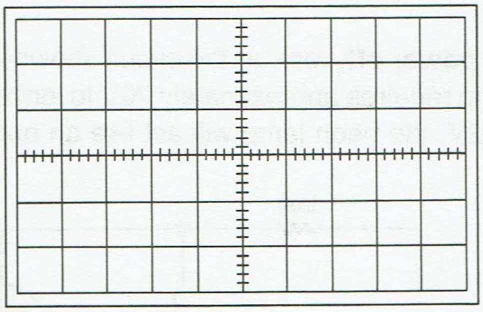
9. Carefully measure the output voltage at $t = 20\mu s$, $40\mu s$, $60\mu s$, $80\mu s$, and $100\mu s$. Since the horizontal sensitivity is set to 0.1mS (100μs) per division, one fifth of a division represents 20μs. Record your measured values in the column labeled v_1 (t) in Table 8-8.

Time (μs)	$v_1(t)$	$v_2(t)$
20		
40		
60		
80		
100		

Table 8-8

Data for steps 9 and 11.

10. Adjust the trace so that the decaying portion of the output voltage starts at the middle of the screen, which corresponds to t = 0. Sketch the observed waveform in Figure 8-11.



Volts/division = 5 Time/division = 0.1ms

Figure 8-11

Waveform observed in step 11.

11. Carefully measure the output voltage at t = 20μs, 40μs, 60μs, 80μs, and 100μs. Record your measured values in the column labeled v_2 (t) in Table 8-8.

Discussion

If you took careful measurements, your calculated (Table 8-5 and Table 8-6) and measured (Table 8-8) values should agree reasonably well. It is especially important that you carefully defined the point corresponding to t = 0 **before** you took data. When you examined the output voltage from the 555 timer (step 7), you probably noted that the square-wave remained at +11V slightly longer than it did at 0V. Also, the square-wave probably did not have a peak value of exactly 11V, and tended to "droop" slightly. These and other factors, such as measurement technique, component tolerances, etc., introduced some errors into your measurements. Nevertheless, your calculated and measured values probably did not differ more than ±15%. If your calculated and measured values differed significantly, try repeating the measurements exercising great care in defining the point on the waveforms corresponding to t = 0 prior to taking data.

Procedure (continued)

12. With the **power off**, wire up the circuit shown in Figure 8-12.Since the neon lamp requires approximately 70V to ignite and the source voltage is only 12V, the neon lamp will act like an open circuit.

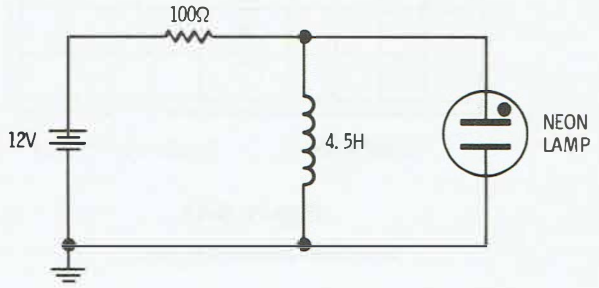


Figure 8-12

Experimental circuit for steps 12 to 14.

13. Turn the power on. Ideally, what should the steady-state inductor voltage equal? V_L (ideal) = ______V. Measure the voltage across the inductor. V_L (meas) = ______V.

14. Disconnect the **ground lead**. Describe what happens.____________
____.

Discussion

Ideally, the voltage across an inductor is 0V when the current through the inductor is constant. Since there is no relative motion between the (constant) magnetic field and inductor, the induced voltage is zero. Real inductors have resistance (as well as inductance) because the wire used to wind the inductor has resistance. Thus, in the steady-state, a real inductor acts like a resistance (equal to the resistance of the wire from which it is wound) rather than a short circuit. In some circuits the resistance of the inductor is negligible, while in other circuits the resistance of the inductor is quite significant. Clearly, in this experiment, the resistance of the inductor was significant. You can estimate the resistance of your inductor by employing the voltage division principle; since, in the steady-state, the 12V source divides between the series 100Ω resistor and the resistance of the inductor. Thus:

$$V_{L(meas)} = \frac{12V \cdot R_L}{100\ \Omega + R_L}$$

When you are finished reading the discussion portion of this experiment, you should employ the equation just provided and your measured value of inductor voltage to calculate the resistance of your inductor.

When you disconnected the ground lead (step 14), you should have observed that the neon lamp lit momentarily. When the ground lead was "broken," the magnetic field surrounding the inductor collapsed rapidly. As a result, the relative motion between the rapidly collapsing magnetic field and inductor induced a large voltage in the inductor. Since the neon lamp is connected in parallel with the inductor, the same voltage was across the neon lamp. Obviously, the induced voltage was large enough to light the lamp. Once the magnetic field was completely collapsed, the induced voltage was zero and the lamp extinguished.

How much energy did the inductor deliver to the lamp? To begin to answer this fascinating question, you can use your experimental values of inductor voltage and resistance to estimate the steady-state inductor current from:

$$I_L = \frac{V_{L\ (meas)}}{R_{L\ (cal)}}$$

Recall that the energy stored in the magnetic field of an inductor is given by:

$$W = \frac{1}{2} L I^2$$

Although the measured values will differ from experiment to experiment, the calculations for energy stored in the inductor are given here using typical values.

The voltage measured across the inductor, V_L, is approximately 8.2V. Using the voltage division principle:

$$8.2V = \frac{12V \cdot R_L}{100\ \Omega + R_L}$$

$$(100\ \Omega + R_L)\ 8.2V = 12\ V \cdot R_L$$

$$R_L = 215.79\ \Omega$$

Therefore, the steady-state inductor current is:

$$I_L = \frac{V_{L\ (meas)}}{R_{L\ (cal)}}$$

$$I_L = \frac{8.2V}{215.79\Omega}$$

$$I_L = 38\ mA$$

Now, using the steady-state current and the value of the inductor, you have:

$$W = \frac{1}{2} L I^2$$

$$W = \frac{1}{2} \cdot 4.5H \cdot (.038)^2$$

$$W = 3.2\ mW$$

Thus, you can calculate the energy stored by your inductor under steady-state conditions. This **is** the energy that was delivered to the lamp when you suddenly opened the circuit by disconnecting the ground lead in step 14.

EXPERIMENT 3
AC CIRCUIT ANALYSIS

Objectives: *To analyze elementary AC circuits by series- parallel techniques, and by Thevenin's theorem.*
To verify the maximum power transfer theorem for the case of a complex source impedance, and a purely resistive load.

Introduction

Complex numbers enable you to use the methods of DC circuit analysis in AC circuits. When you are calculating AC power, you must use rms values of current and voltage. The total power supplied by an AC source to the circuit is given by $P = V_T I_T \cos\theta$. In this formula, the term $\cos\theta$ is called the power factor. Maximum power transfer occurs in AC circuits when the load impedance equals the conjugate of the Thevenin source impedance. In the case of a purely resistive load, maximum power transfer occurs when the load resistance, R_L, equals the absolute value of the Thevenin impedance of the source $\sqrt{R_{TH}^2 + X_{TH}^2}$.

Material Required

Heathkit Engineering Design Trainer ET-1000
Oscilloscope (Heathkit SO-4550 or equivalent)
1 — 1kΩ, ¼-watt, 5% resistor
1 — 2.2kΩ, ¼-watt, 5% resistor
1 — 4.7kΩ, ¼-watt, 5% resistor
1 — 6.8kΩ, ¼-watt, 5% resistor
3 — 10kΩ, ¼-watt, 5% resistor
1 — 15kΩ, ¼-watt, 5% resistor
1 — 22kΩ, ¼-watt, 5% resistor
3 — 33kΩ, ¼-watt, 5% resistor
1 — .01μF, 25V capacitor
1 — 741 Op Amp (442-22)

Procedure

1. Calculate the reactance of the 0.01μF capacitor in Figure 8-13A. $X_c = -j$ ______ohms.

2. Calculate the impedance between points A and B in Figure 8-13A. Note that $Z_{AB} = R_2 \parallel X_c$, $Z_{AB} =$ ______kΩ − j ______kΩ. Convert Z_{AB} from rectangular to polar form. $Z_{AB} =$ ______kΩ ∠ ______.

3. Calculate the total impedance in Figure 8-13A. Note that $Z_T = R_1 + Z_{AB}$. $Z_T =$ ______kΩ − j ______kΩ. Convert Z_T from rectangular to polar form. $Z_{AB} =$ ______kΩ ∠______.

4. Calculate I_T from $I_T = \frac{5V}{Z_T}$. Note that the calculated value of I_T is a peak value. $I_T =$ ______mA ∠ ______.

5. In Figure 8-13A, $V_{R_2} = V_C = V_O = V_{AB}$. Using the value of Z_{AB} obtained in step 2 and the value of I_T from step 4, calculate the peak output voltage from $V_O = I_T Z_{AB}$. $V_O =$ ________ V∠________.

Discussion

The calculations for steps 1 through 5 are provided at this point so that you may check your calculations prior to building the circuit.

Step 1

$$X_c = \frac{-j}{2\pi fC} = \frac{-j}{2\pi(1\text{kHz})(.01 \times 10^{-6})} = -j\,15.9\ \text{k}\Omega$$

Step 2

$$Z_{AB} = 33\text{k}\Omega \parallel -j\,15.9\ \text{k}\Omega$$

$$Z_{AB} = \frac{33\,(-j\,15.9)}{33 - j\,15.9}\ \text{k}\Omega = \frac{-j\,524.7 \quad 33 + j\,15.9}{33 - j\,15.9 \quad 33 + j\,15.9}\ \text{k}\Omega$$

$$Z_{AB} = \frac{-j\,524.7\,(33 + j\,15.9)}{33^2 + 15.9^2}\ \text{k}\Omega$$

$$Z_{AB} = \frac{8\,342.7 - j\,17\,315.1}{1\,341.81}\ \text{k}\Omega = 6.22\ \text{k}\Omega - j\,12.9\ \text{k}\Omega$$

Converting to polar form:

$$Z_{AB} = \sqrt{(6.22)^2 + (12.9)^2}\ \text{k}\Omega - \angle \arctan\frac{12.9}{6.22}$$

$$Z_{AB} = 14.32\ \text{k}\Omega \angle -64.26^\circ$$

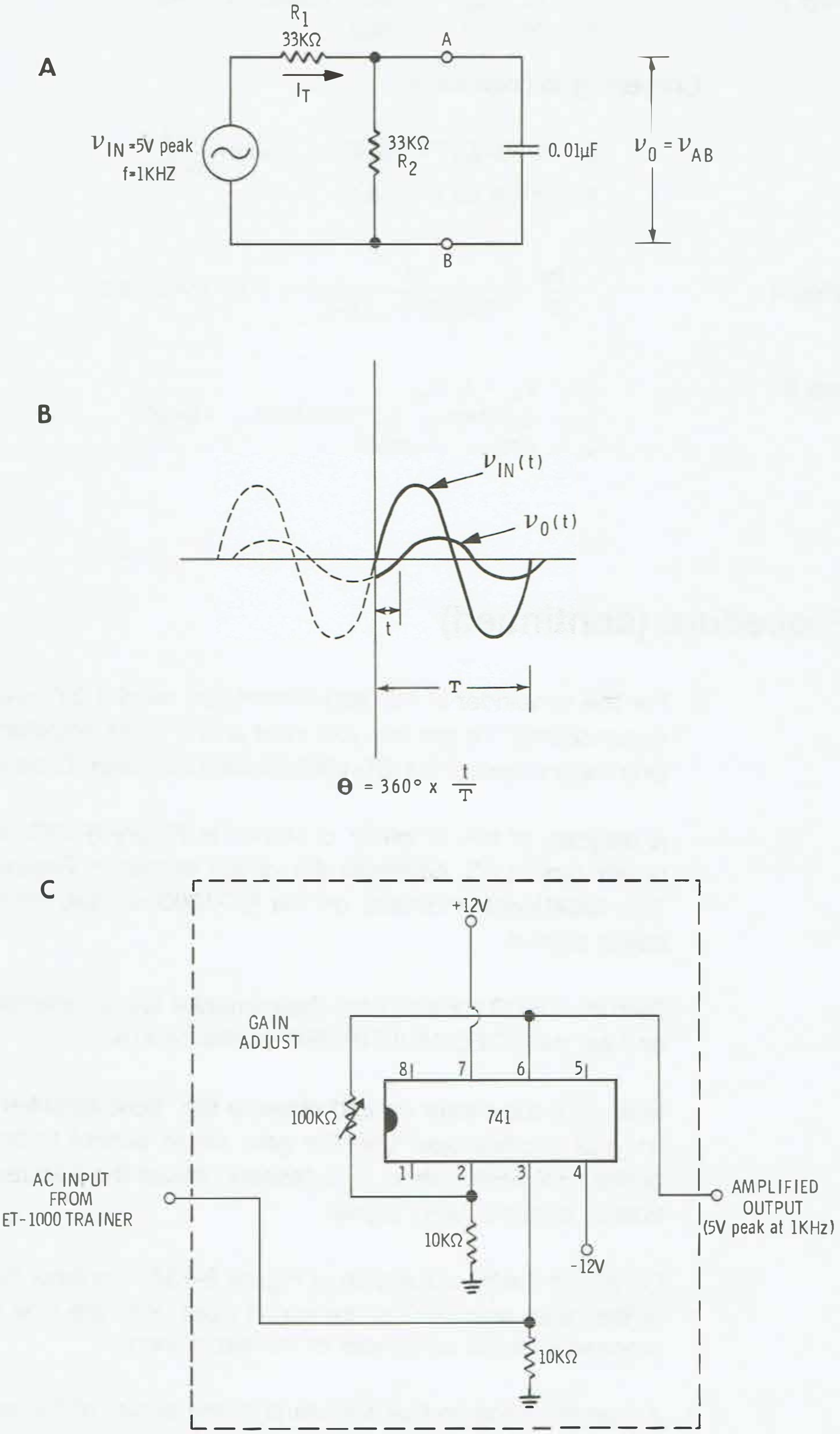


Figure 8-13

Measuring voltage and phase shift.

Step 3

$$Z_T = R_1 + Z_{AB} = 33k\Omega + 6.22k\Omega - j\,12.9k\Omega$$
$$Z_T = 39.22k\Omega - j\,12.9k\Omega$$

Converting to polar form:

$$Z_T = \sqrt{(39.22)^2 + (12.9)^2} - \angle \arctan\frac{12.9}{39.22}$$
$$Z_T = 41.29\ k\Omega \angle -18.2°$$

Step 4

$$\therefore I_T = \frac{5V}{Z_T} = \frac{5V}{41.29k\Omega \angle -18.2°} = 0.121\ mA \angle 18.2°$$

Step 5

$$V_O = V_{AB} = I_T\ Z_{AB}$$
$$V_O = (0.121mA \angle 18.2°)\ (14.32k\Omega \angle -64.26°)$$
$$V_O = 1.73V \angle -46.06°$$

Procedure (continued)

6. For the remainder of this experiment you need a 5V peak, 1kHz signal source. To get this you must add a "post amplifier" to the sine wave output of the ET-1000 Electronic Design Experimenter.

   A diagram of this amplifier is shown in Figure 8-13C. With the power turned off, construct the circuit shown in Figure 8-13C. The 100kΩ potentiometer on the ET-1000 is used for the gain adjust control.

   Turn the FREQ control of the Experimenter fully counterclockwise and set the FREQ MULTIPLIER control to x1K.

   Now, turn the power on and observe the "post amplifier" output on your oscilloscope. Use the gain adjust control to obtain the desired 5V peak output. If necessary, adjust the frequency controls to obtain a 1kHz signal.

   Construct the circuit shown in Figure 8-13A. Connect the output of the "post amplifier" to the circuit input. You are now ready to proceed with the remainder of the experiment.

7. Adjust the trace so that Vin starts at the center of the screen as shown in Figure 8-13 B. If you have a dual-trace oscilloscope, measure *v*o using the second channel. Record the peak value of the output voltage. Vo = ______V.

8. With a dual-trace oscilloscope, you can measure the angle θ by which the output voltage lags the input voltage as shown in Figure 8-13B. Record the values of t, T, and the angle θ. t = ________, T = ______, $\theta = 360° \times \frac{t}{T} =$ ______. If you took careful measurements your measured values should be in close agreement with the values calculated in step 5.

In the second part of the experiment, you will employ Thevenin's theorem to experimentally verify the maximum power transfer theorem for the case of a complex source impedance, and a purely resistive load.

9. In Figure 8-14, the circuit from part one of the experiment is used to drive a resistive load. Thus, the circuit serves as a source for the resistive load, R_L. Calculate the Thevenin impedance of the circuit as seen from load terminals A and B.
   Z_{TH} = ______kΩ − j ______kΩ. Convert Z_{TH} from rectangular to polar form. Z_{TH} = ______kΩ ∠ ______.

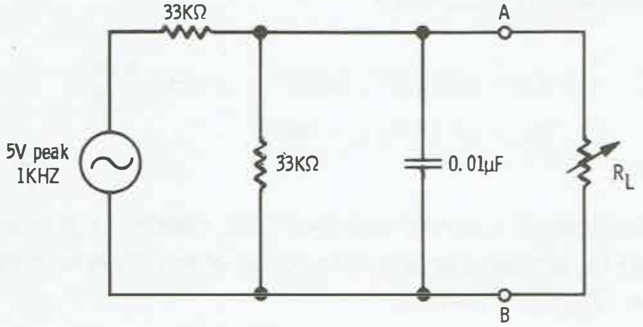


Figure 8-14
Maximum power transfer.

10. Based on the maximum power transfer theorem, what value of R_L should produce maximum power transfer? R_L = ______kΩ.

11. Calculate the Thevenin voltage V_{TH} as seen from load terminals A B. V_{TH} = ______V ∠ ______.

12. Use the Thevenin equivalent circuit to predict the peak voltage across the load resistor when R_L equals the value calculated in step 10. V_{RL} = ______V ∠ ______.

13. Calculate the maximum possible load power. $P_{RL\ (max)}$ = ______.

Discussion

Our calculations for steps 9 through 13 are provided at this point so that you may check your calculations prior to building the circuit.

Step 9 To calculate Z_{TH}, you remove the load and reduce the voltage source to zero as shown in Figure 8-15A. Thus:

$$Z_{TH} = 33\text{k}\Omega \parallel 33\text{k}\Omega \parallel -j\,15.9\text{ k}\Omega$$

$$Z_{TH} = 16.5\text{k}\Omega \parallel -j\,15.9\text{k}\Omega$$

$$Z_{TH} = \frac{16.5\,(-j\,15.9)}{16.5\,-j\,15.9}\text{ k}\Omega$$

$$Z_{TH} = \frac{-j\,262.35}{16.5\,-j\,15.9}\cdot\frac{16.5\,+\,j\,15.9}{16.5\,+j\,15.9}\text{ k}\Omega$$

$$Z_{TH} = \frac{-j\,262.35\,(16.5\,+j\,15.9)}{16.5^2\,+\,15.9^2}\text{ k}\Omega$$

$$Z_{TH} = \frac{4171.36\,-j\,4328.8}{525.06}\text{ k}\Omega$$

$$Z_{TH} = 7.94\text{ k}\Omega\,-j\,8.24\text{k}\Omega$$

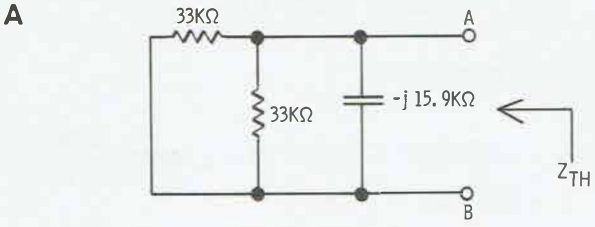
Converting to polar form:

$$Z_{TH} = \sqrt{(7.94)^2 + (8.24)^2}\; - \angle\,\arctan\frac{8.24}{7.94}$$

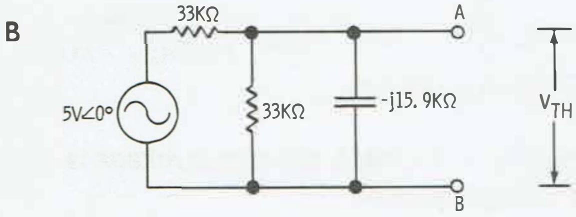
$$Z_{TH} = 11.44\text{ k}\Omega\,\angle\,-\,46.06°$$

Step 10 In the case of a purely resistive load, maximum power transfer occurs when R_L equals the absolute value of the Thevenin impedance. Thus $R_L = |Z_{TH}| = 11.44\text{k}\Omega$.

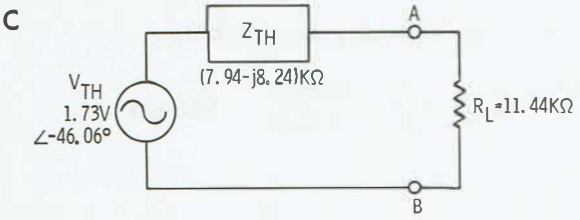
Step 11 To calculate V_{TH}, you remove the load, R_L, and determine the open circuit voltage as shown in Figure 8-15 B. Since the circuit in part one is used to drive the load, V_{TH} equals the value of the voltage calculated in step 5. Thus $V_{TH} = 1.73\text{ V}\,\angle\,-46.06°$.



Calculating Z_{TH}



Calculating V_{TH}



Thevenin equivalent

Figure 8-15

Thevenizing the circuit in Figure 8-14.

Step 12 The Thevenin equivalent circuit is illustrated in Figure 8-15C. We will use the voltage division principle to calculate V_{RL}.

The total impedance Z_T is:

$$Z_T = Z_{TH} + R_L$$
$$Z_T = 7.94\ \text{k}\Omega - j\,8.24\text{k}\Omega + 11.44\text{k}\Omega$$
$$Z_T = 19.38\ \text{k}\Omega - j\,8.24\ \text{k}\Omega$$

Converting to polar form:

$$Z_T = \sqrt{(19.38)^2 + (8.24)^2}\ \text{k}\Omega - \angle \arctan\frac{8.24}{19.38}$$
$$Z_T = 21.06\ \text{k}\Omega \angle -23.03^\circ$$

Applying the voltage division principle:

$$V_{RL} = V_{TH} \cdot \frac{R_L}{Z_T} = 1.73\text{V} \angle -46.06^\circ \frac{11.44\text{k}\Omega}{21.06\text{k}\Omega \angle -23.03^\circ}$$
$$V_{RL} = 0.94\ \text{V} \angle -23.03^\circ$$

Thus, when $R_L = 11.44\text{k}\Omega$, the output voltage is 0.94V peak, and lags the input voltage by 23.03°.

Step 13 Converting the peak voltage calculated in step 12 to an rms value, you obtain:

Thus: $V_{RL} = 0.707\ (0.94) = 0.664V_{rms}$

$$P_{RL(max)} = \frac{V^2}{R_L} = \frac{(0.664)^2}{11.44\text{k}\Omega} = 38.5\ \mu\text{W}$$

Procedure (continued)

14. With the input voltage adjusted to 5V peak at 1kHz, measure the peak voltage between terminals A and B, Figure 8-4, for the following values of R_L: 1k, 2.2, 4.7k, 6.8k, 10k, 11.5k, 15k, 22k, and 33kΩ. You can connect a 6.8kΩ and 4.7kΩ resistor in series to get 11.5kΩ. Record your measured values in Table 8-8.

R_L (Ω)	V_{AB} (peak)	V_{AB} (rms)	P_{RL} (μW)
1k			
2.2k			
4.7k			
6.8k			
10k			
11.5k			
15k			
22k			
33k			

Table 8-9

Data For steps 14 and 15.

15. Calculate the rms voltages and power for each value of load resistance in Table 8-9.

Discussion

The data in Table 8-9 should indicate that maximum power transfer occurs when $R_L \approx 11.5k\Omega$, since this is reasonably close to the calculated value of 11.44kΩ in step 10. The calculations in this experiment are tedious due to the fact that complex numbers must be used to analyze AC circuits. A dual-trace oscilloscope is useful for measuring the difference in phase between two voltages when the phase angle is a significant portion of one cycle. Small phase differences are not easily measured with an oscilloscope. For this reason, you may have experienced some difficulty if you tried to measure the phase angle for those values of R_L where the phase shift was not a significant portion of one cycle.

Table 8-10 contains the data and calculations derived from typical results for this Experiment. As you can see, maximum power transfer occurs when R_L is *approximately* equal to 11.5kΩ.

Remember, component tolerances, instrument loading effects, and other factors usually result in actual voltages that are different from the calculated values. Also, recall from Experiment 1 that such errors are "magnified" when calculating power. For these reasons, your data probably indicates that maximum power transfer actually occurs for some values of R_L between 10kΩ and 15kΩ.

R_L (Ω)	V_{AB} (peak)	V_{AB} (RMS)	P_{RL} (μW)
1k	150mV	106.05mV	11.24μW
2.2k	318mV	224.83mV	23μW
4.7k	570mV	403mV	34.5μW
6.8k	720mV	509.04mV	38.1μW
10k	900mV	636.3mV	40.49μW
11.5k	1000mV	707mV	43.46μW
15k	1100mV	777.7mV	40.32μW
22k	1350mV	954.45mV	41.41μW
33K	1500mV	1060.5mV	34.08μW

Table 8-10

Typical values for steps 14 and 15.

EXPERIMENT 4
CAPACITORS

Objective: *To demonstrate the use of coupling and bypass capacitors.*
To select capacitor values for coupling and bypass circuits.

Introduction

Virtually all capacitor types find applications in coupling and bypass circuits. The function of a coupling capacitor is to pass AC signals and block DC signals. Bypass or decoupling capacitors serve just the opposite function. Consequently, bypass capacitors decouple AC signals and allow DC signals to be present at the circuit output.

The values of both coupling and bypass capacitors are chosen so that their reactances at the lowest frequency to be coupled or bypassed is small compared to the Thevenin resistance as seen from the capacitor terminals. A useful rule of thumb is to select a capacitor whose reactance is $R_{TH}/20$ at the lowest frequency to be coupled or bypassed. Both coupling and bypass capacitors are widely used in transistor circuits, electronic instruments, filters, and many other applications.

Material Required

Heathkit Engineering Design Trainer ET-1000
Oscilloscope (Heathkit SO-4550 or equivalent)
1 — 10kΩ, ¼-watt, 5% resistor
1 — 22kΩ, ¼-watt, 5% resistor
1 — 39kΩ, ¼-watt, 5% resistor
1 — 68kΩ, ¼-watt, 5% resistor
1 — 100kΩ, ¼-watt, 5% resistor
1 — .1 μF, 50V, ceramic capacitor

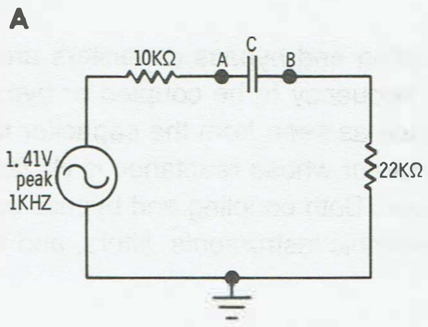
Procedure

1. Calculate the Thevenin resistance between A and B for the coupling circuit in Figure 8-16A. R_{TH} = ______kΩ.

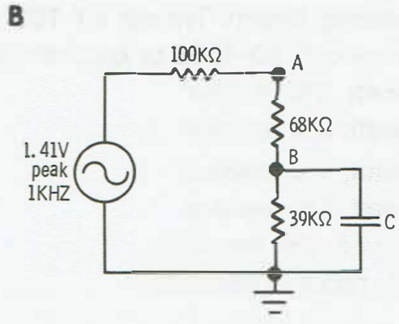
2. Calculate the value of the minimum size capacitor in Figure 8-16A to provide good coupling for signals between 1kHz and 100kHz. C = _______ μF.

3. Calculate the Thevenin resistance between point B and ground for the bypass circuit in Figure 8-16 B. R_{TH} = ________kΩ.

4. Calculate the value of the minimum size capacitor in Figure 8-16 B to provide good bypassing of signals between 1kHz and 100kHz. C = ______μF.



Coupling circuit



Bypass circuit

Figure 8-16

Circuits for Experiment 4.

Discussion

Our calculations for steps 1 through 4 are provided at this point so you can check your calculations prior to building the circuits.

Step 1 To calculate R_{TH}, you remove the capacitor and replace the voltage source by a short circuit. Thus:

$$R_{AB} = R_{TH} = 10k\Omega + 22k\Omega = 32k\Omega$$

Step 2

For good coupling, $|X_c| = \frac{R_{TH}}{20}$ at the lowest frequency to be coupled, which is 1kHz in this case.

Thus:

$$|X_c| = \frac{32k\Omega}{20} = 1.6k\Omega$$

Finally:

$$C = \frac{1}{\omega\,|X_c|} = \frac{1}{(6.28 \times 10^3)\,(1.6k\Omega)} = 0.0995\ \mu F$$

Since a standard value capacitance of $0.1\mu F$ is readily available, we select $C = 0.1\mu F$ for the coupling capacitor.

Step 3 Removing the capacitor and reducing the voltage source to zero in Figure 8-16B we have:

$$R_{TH} = (100k\Omega + 68k\Omega) \parallel 39k\Omega$$

$$R_{TH} = 168k\Omega \parallel 39k\Omega = 31.7k\Omega$$

Step 4

For good bypassing $|X_c| = \frac{R_{TH}}{20}$ at the mimimum frequency of 1kHz.

Thus:

$$|X_c| = \frac{31.7k\Omega}{20} = 1.58\ k\Omega$$

$$C = \frac{1}{\omega|X_c|} = \frac{1}{(6.28 \times 10^3)\,(1.58k\Omega)} = 0.1\mu F$$

Since $0.1\mu F$ is a standard value, we select $C = 0.1\mu F$ for the bypass capacitor.

Procedure (continued)

5. Now that you have determined the capacitors values, draw the waveforms of the voltages that you would expect to see at points A and B for the coupling circuit in Figure 8-16A. Use Figure 8-17 for your sketches. Assume that the input signal is a 1kHz, 1.41V peak sinusoidal waveform.

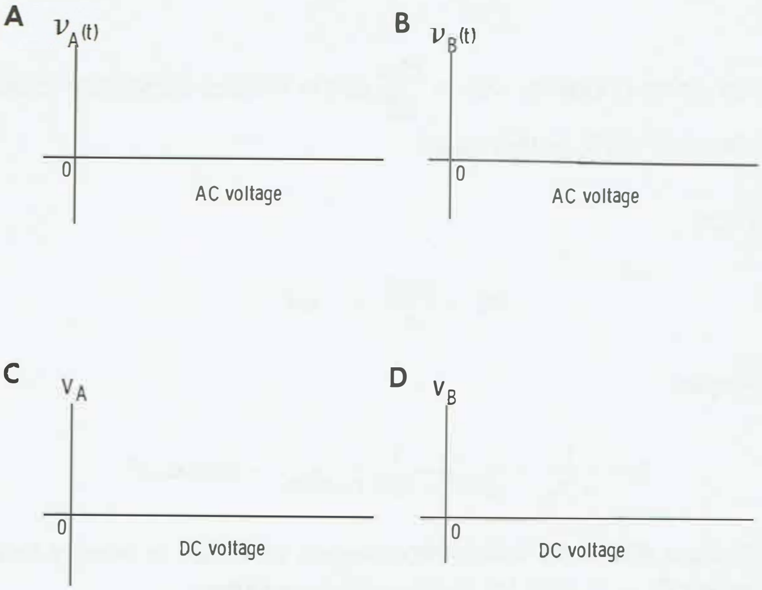


Figure 8-17

Predicted waveforms for steps 5 and 6.

6. Repeat step 5, but assume a 12V DC input voltage.

7. Construct the circuit shown in Figure 8-16A. Measure the peak values at points A and B. V_A = _____V. V_B = _____V.

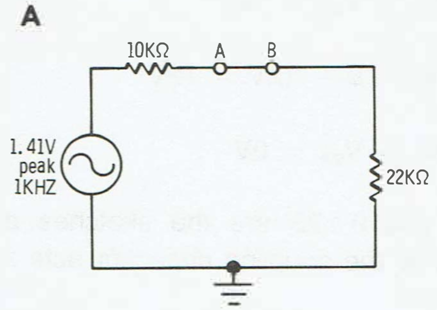
8. Repeat step 7 with the signal source set to minimum frequency. Now increase the frequency of the signal source to maximum. Does the coupling improve as frequency increases?_____

9. Replace the AC source with a 12V DC source. Now measure the voltages at points A and B. V_A = _____V. V_B = _____V.

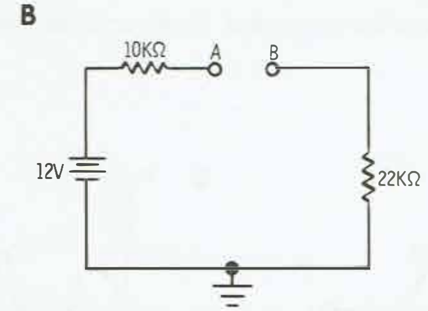
Discussion

To a reasonable approximation, coupling capacitors act like a short circuit for an AC signal whose minimum frequency is equal to or greater than the minimum frequency used to calculate the coupling capacitor's value.

Figure 8-18A and 8-18B illustrate the DC and AC equivalent circuits for the coupling circuit.



Coupling circuit
AC equivalent



Coupling circuit
DC equivalent

Figure 8-18

AC & DC equivalent circuits for steps 5 and 6.

Since the capacitor acts as a short:

Step 5 In figure 8-18A $V_A = V_B = V_{22k}$. Using voltage division:

$$V_A = V_B = \frac{1.41V\ (22k\Omega)}{10k\Omega + 22k\Omega} = 0.97V \text{ peak}$$

Step 6 As you know capacitors act like open circuits for DC signals. Figure 8-18B illustrates the DC equivalent circuit that results once the capacitor is replaced by an open in the coupling circuit.

In Figure 8-18B, the current is zero due to the open circuit. Thus, the voltage across the 10kΩ and 22kΩ resistors is also zero. Consequently:

$$V_A = V_{10k} + 12V$$

$$V_A = 0 + 12V = 12V$$

$$V_B = V_{22k} = 0V$$

Figure 8-19A and 8-19B are the sketches of the predicted AC waveforms. Since the coupling capacitor acts as a short, both voltages are identical.

Figures 8-19C and 8-19D depict the predicted DC voltages. The coupling capacitor acts as an open to DC. In this case, the applied voltage is dropped across the capacitor: Zero volts at point B and +12 volts at point A.

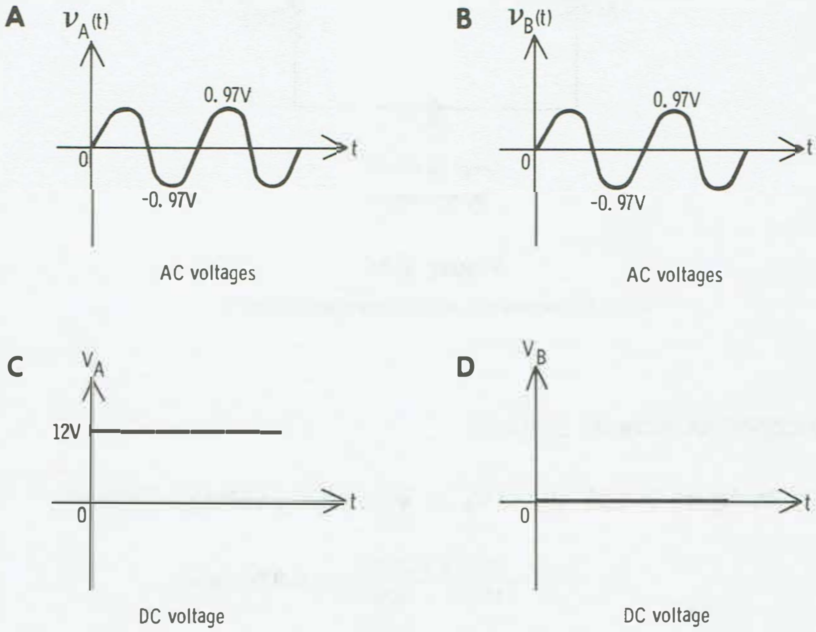


Figure 8-19

Sketches of waveforms for steps 5 and 6.

Step 7 The voltages measured in this step should approximate the voltages calculated in step 5. Bear in mind, however, that the tolerance of the components used in the circuit will effect the actual measurements.

Step 8 Below 1kHz, the coupled signal is attenuated. This happens because the circuit is designed to couple signals from 1kHz to 100kHz. When the frequency of the applied signal exceeds 1kHz, coupling is maximum.

Step 9 Because the coupling capacitor blocks DC, you should have measured 12 volts DC at point A, and 0 volts DC at point B.

Procedure (continued)

10. Draw the voltage waveforms that you should expect to find at points A and B of the bypass circuit shown in Figure 8-16B. Assume an input signal of 1.41V peak at 1kHz. Use Figure 8-20 for your sketches.

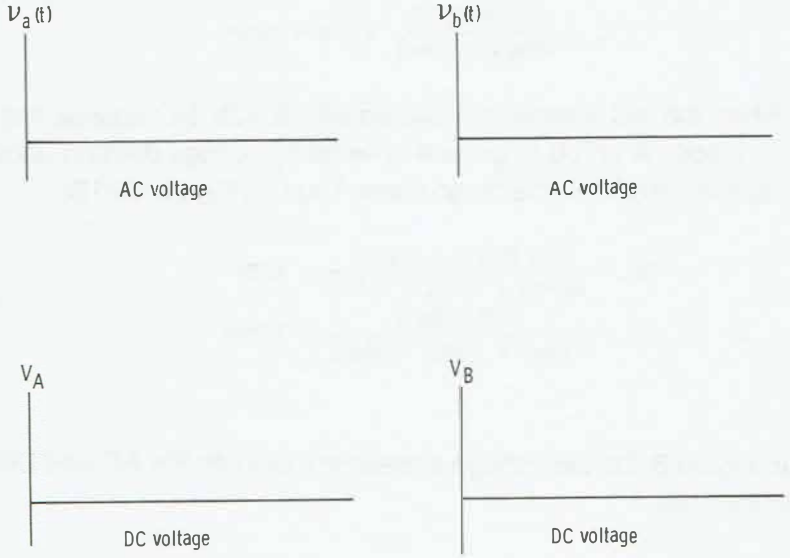


Figure 8-20

Predicted waveforms for steps 10 and 11.

11. Repeat step 10 but assume a 12V DC input.

12. Construct the circuit shown in Figure 8-16B. Measure the peak voltages at points A and B. V_A = _____V. V_B = _____V.

13. Repeat step 12 with the input signal at minimum frequency. Gradually increase the frequency of the input. What effect does the increase in frequency have on the voltages at points A and B?

14. Remove the AC source and replace it with a 12V DC source. Measure the voltages at points A and B. V_A = _____V. V_B = _____V.

Discussion

Step 10 In Figure 8-21A the 39kΩ resistor is shorted by the bypass capacitor. Therefore, $V_B = 0V$.

The voltage at point A equals the voltage across the 68kΩ resistor. Once again, using voltage division:

$$V_A = \frac{1.41V\,(68k\Omega)}{100k\Omega + 68k\Omega} = 0.57V \text{ peak}$$

Step 11 When the AC source is replaced with a 12V DC source, the voltage from point A and B to ground is found by voltage division because the bypass capacitor acts as an open. Thus, in Figure 8-21B:

$$V_A = \frac{12V\,(68k\Omega + 39k\Omega)}{100k\Omega + 68k\Omega + 39k\Omega} = 6.2V$$

$$V_B = \frac{12V\,(39k\Omega)}{100k\Omega + 68k\Omega + 39k\Omega} = 2.26V$$

In Figure 8-22, the voltage waveforms for both the AC and DC circuits are shown.

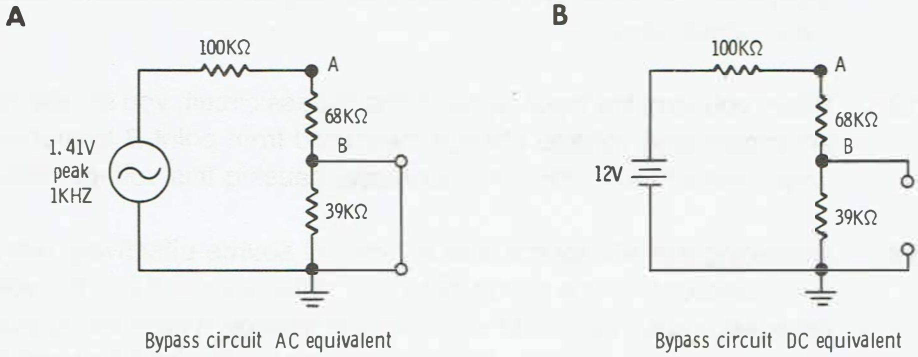


Figure 8-21

AC & DC equivalent bypass circuits.

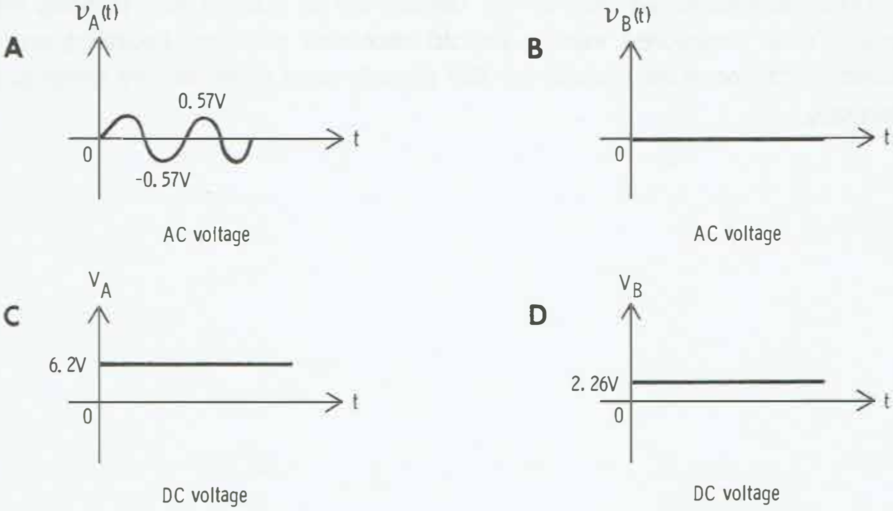


Figure 8-22

Calculated responses for the bypass circuit.

Step 12 The voltage measurements obtained in step 12 should be relatively close to the voltages calculated in steps 10 and 11. Again remember, component and instrument tolerances may cause variations from the ideal calculations.

Step 13 When you vary the input signal to the bypass circuit, you should notice an appreciable voltage change measured from point B to ground. At frequencies below 1kHz, X_c increases, causing this voltage drop.

Step 14 Replacing the AC source with a 12V DC source effectively removes the capacitor from the circuit. Now, the voltage at point B is the voltage dropped across the 39kΩ resistor. The voltage measured at point A is the sum of the voltage drops across the 39kΩ and 68kΩ resistors.

For this experiment, your calculated and measured values should be in reasonably close agreement. Since the capacitor selected for both the coupling and bypass circuits was calculated based on a minimum frequency of 1kHz, increasing the frequency improves the coupling and bypass action. Similarly, for frequencies less than 1kHz, the effectiveness of the capacitors to couple and bypass signals is reduced. Your measured values should convince you that coupling and bypass capacitors act like open circuits for DC signals, and approximate short circuits for AC signals.

EXPERIMENT 5
DIODE APPROXIMATIONS

Objectives: *To experimentally determine the effects of a diodes turn-on voltage, forward resistance, and reverse resistance in a circuit. To develop a model for a type IN4148 diode.*

Introduction

Ideally, a diode should act like a closed switch when it is forward biased, and like an open switch when it is reversed biased. Real diodes have finite values of forward and reverse resistance. Also, real diodes do not conduct appreciably until the forward voltage exceeds the diodes turn-on voltage.

In order to minimize the nonideal properties of real diodes, component values should be chosen so that:

$$V \geqslant 20\ V_T$$

$$R \geqslant 20\ r_F$$

$$R \leqslant 20\ R_R$$

where: V = voltage driving the diode
V_T = diode turn-on voltage
R = resistance in series with the diode
r_F = forward diode resistance
R_R = reverse diode resistance

The turn-on voltage for germanium diodes is about 0.3V. Similarly, silicon diodes typically have a turn-on voltage of approximately 0.7V.

A typical IV curve for a diode is illustrated in Figure 8-23. By experimentally determining the values of V_1, V_2, I_1, and I_2, you can estimate the forward diode resistance as follows:

$$r_F = \frac{V_2 - V_1}{I_2 - I_1}$$

For a given temperature and reverse voltage, the reverse diode resistance is:

$$R_R = \frac{V_R}{I_R}$$

where: V_R = reverse diode voltage
I_R = reverse diode current

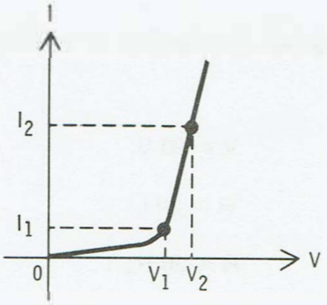


Figure 8-23

Typical diode IV curve.

Material Required

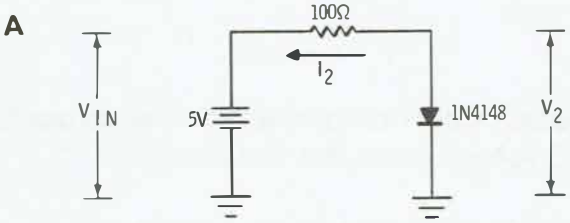
Heathkit Engineering Design Trainer ET-1000
Oscilloscope (Heathkit SO-4550 or equivalent)
Volt-ohmmeter (Heathkit SM-1210 or equivalent)
1 — 100Ω, ¼-watt, 5% resistor
1 — 220Ω, ¼-watt, 5% resistor
1 — 330Ω, ¼-watt, 5% resistor
1 — 470Ω, ¼-watt, 5% resistor
1 — 10MΩ, ¼-watt, 5% resistor
1 — IN4148 silicon diode

Selected specifications for a type 1N4148 diode include:

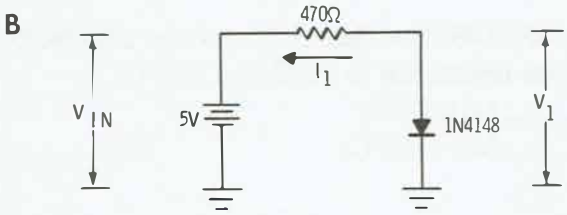
Power dissipation	250 mW
Maximum foward DC current	75mA
Reverse current (25°C) at $V_R = -20V$	25nA maximum
Reverse current (150°C) at $V_R = -20V$	50nA maximum

Procedure

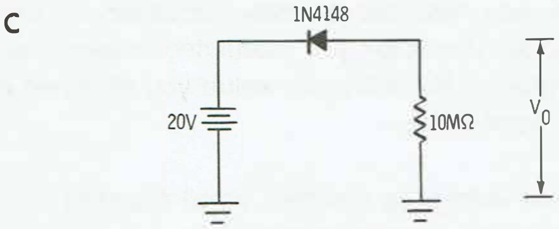
1. Build the circuit shown in Figure 8-24A. Using the oscilloscope, measure V_{IN} and V_2. V_{IN} = _____V. V_2 = _____V.



Steps 1 and 2



Steps 3 to 5



Steps 6 and 7

Figure 8-24

Experimental circuits for steps 1-7.

2. Calculate the current, I_2, from the values measured in step 1.

$$I_2 = \frac{V_{IN} - V_2}{100\Omega} = \underline{\qquad} \text{ mA.}$$

3. Build the circuit shown in Figure 8-24 B. Using the oscilloscope measure V_{IN} and V_1. V_{IN} = ______V V_1 = ______V.

4. Calculate the current I_1 from the values measured in step 3.

$$I_1 = \frac{V_{IN} - V_1}{470\Omega} = \underline{\qquad} \text{ mA}$$

5. Calculate the diodes forward resistance.

$$r_F = \frac{V_2 - V_1}{I_2 - I_1} = \underline{\qquad} \ \Omega$$

6. Build the circuit shown in Figure 8-24C. Use the oscilloscope to measure the DC voltage across the 10 MΩ resistor. V_o = ______V.

7. Heat the diode with a match held approximately one half inch from the diode. As soon as the output voltage starts to increase, extinguish the match. **Exercise caution in this step of the experiment!**

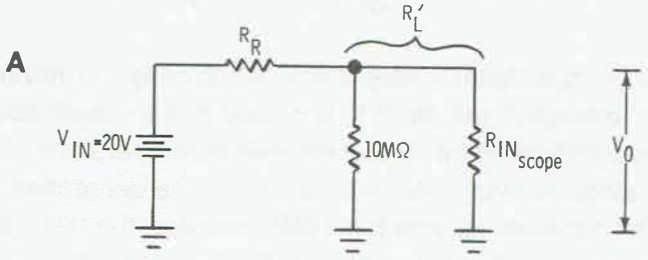
8. From the IN4148 specifications provided previously, estimate the reverse diode resistance at 25°C and 150°C.
   R_R = ______MΩ (25°C)
   R_R = ______MΩ (150°C)

Discussion

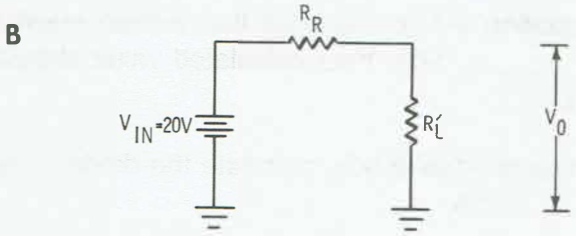
In steps 1 through 4, you recorded the data necessary to estimate the forward resistance of your diode. The value you calculated in step 5 should be less than 10Ω, with 3Ω being typical. Naturally, the value you obtained depends upon the specific properties of your diode.

Once a diode's turn-on voltage is reached, small changes in the voltage across the diode produce relatively large changes in the current flowing through it. For this reason, in order to obtain accurate results, it is essential that you accurately read the voltages you measured in steps 1 and 3. If the diodes forward resistance you obtained in step 5 exceeds 10Ω, repeat steps 1 through 5. Take special care to measure V_2 and V_1 as accurately as possible.

Ideally, the output voltage in Figure 8-24C equals zero, since the diode is reversed biased by the 20V source. The output voltage you measured in step 6 is determined by the reverse resistance of the diode, R_R, and the effective load resistance, R_L', as shown in Figure 8-25. Here, the effective load resistance equals the parallel combination of the 10MΩ resistor and the input resistance of the oscilloscope.



Reversed biased diode



Equivalent circuit

Figure 8-25

Equivalent circuits for steps 6 and 7.

Most oscilloscopes have an input resistance of 1MΩ with a X1 probe, and 10MΩ with a X10 probe. Consequently, if you use a X10 probe in step 6, R_L' will equal 10MΩ || 10MΩ, or 5MΩ. Similarly, with a X1 probe, R_L' will equal 10MΩ || 1MΩ, or 0.909MΩ. In any event, you can see in Figure 8-25B that the input voltage divides between R_R and R_L'. Applying the principle of voltage division, you obtain:

$$V_o = V_{IN} \frac{R_L'}{R_R + R_L'}$$

Solving for R_R yields:

$$R_R = \frac{(V_{IN} - V_o)\, R_L'}{V_o} \quad \text{(Eq. 8-1)}$$

Equation 5-1 can be used to estimate the reverse resistance of the diode from your measured values.

In step 8, the reverse resistance of the diode was calculated from the specifications provided previously. Specifically:

$$R_R = \frac{20V}{25nA} = 800\ M\Omega\ (25°C)$$

$$R_R = \frac{20V}{50\mu A} = 0.4\ M\Omega\ (150°C)$$

The values of R_R calculated in step 8 are "worst case", or **minimum**, values since the maximum expected values of I_R are used in the calculation. As you can see, R_R decreases significantly for large increases in temperature. Consequently, when you heat the diode in step 7, its reverse resistance decreases. Because the input voltage divides between R_R and the 10MΩ resistor, the voltage across the 10MΩ resistor increased significantly when the diode was heated.

Procedure (continued)

9. Use equation 8-1 to calculate the reverse resistance of the diode. R_R = ______ MΩ. Your calculated value should equal or exceed 800MΩ.
10. Assuming an ideal diode, calculate the diode current in Figure 8-26. I_D = _____mA
11. Calculate the diode current in Figure 8-26 taking the diodes turn-on voltage and forward resistance into account. I_D = _____mA
12. Build the circuit shown in Figure 8-26. Use the volt-ohmmeter to measure the diode current. $I_{D\ (meas)}$ = ______ mA.

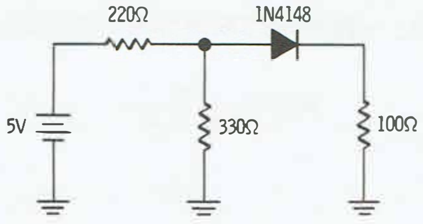


Figure 8-26

Experimental circuit for steps 10-12.

Discussion

Perhaps the easiest way to analyze the circuit in Figure 8-26 is to treat the diode and 100Ω resistor as the load, and then apply Thevenin's theorem. Thus:

Removing the load and replacing the 5V source with a short circuit you can see that:

$$R_{TH} = 330\Omega \parallel 220\Omega = 132\Omega$$

To calculate V_{TH}, the load is removed and the resulting open circuit voltage calculated. Thus:

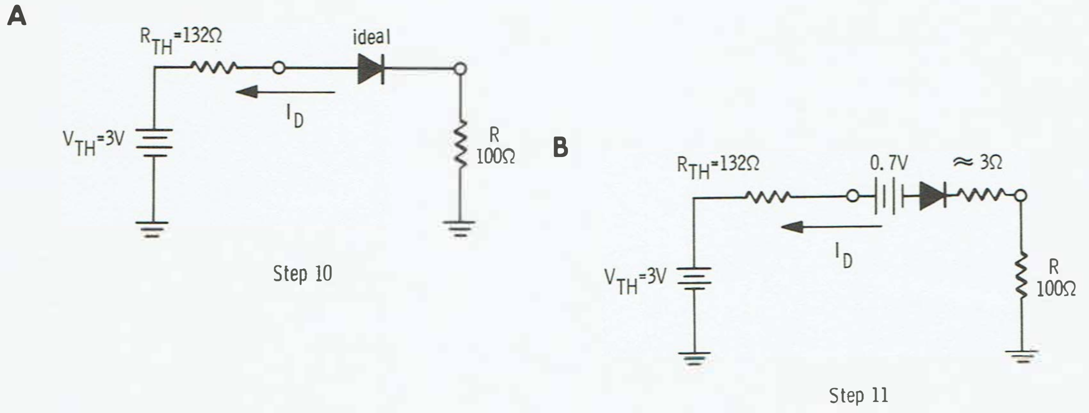
$$V_{TH} = V_{330} = \frac{5V\,(330\Omega)}{220\Omega + 330\Omega} = 3V$$

The Thevenin equivalent circuit for step 10 is shown in Figure 8-27A. Here, the diode current is:

$$I_D = \frac{3V}{132\Omega + 100\Omega} = 12.9\text{ mA}$$

Similarly, Figure 8-27B illustrates the Thevenin equivalent circuit for step 11. Note that the 1N4148 diode has been modeled by the series combination of the diodes turn-on voltage, an ideal diode, and the diodes forward resistance – which is assumed to be 3Ω. The value of r_F you obtained in step 5 may be somewhat larger or smaller than 3Ω. In Figure 8-27B, the diode current is:

$$I_D = \frac{V_{TH} - V_T}{R_{TH} + r_F + R} = \frac{3V - 0.7V}{132\Omega + 3\Omega + 100\Omega} = 9.8\text{ mA}$$



Step 10

Step 11

Figure 8-27

Thevenin equivalent circuits for steps 10 and 11.

The diode current measured in step 12 should agree closely with the value you calculated in step 11.

Based on your data, you should conclude that the turn-on voltage was a significant factor in this experiment. The forward resistance of the diode was not very significant because it was small compared to the equivalent series resistance, 232Ω, in Figure 8-27B.

EXPERIMENT 6
FULL-WAVE RECTIFIERS

Objectives: *To analyze three full-wave rectifier circuits.*
To experimentally verify the calculated values.

Introduction

Full-wave rectifiers form the backbone of modern power supplies. Compared to a half-wave rectifier, full-wave rectifiers offer the following advantages:

1. Larger DC output voltage.
2. Smaller AC output voltage.
3. Simplified transformer construction.

In this experiment you will analyze the three most popular full-wave rectifier circuits. In addition, you will take appropriate measurements to verify your calculated values.

Table 5-2 and Table 5-3 in your text summarize the formulas necessary to analyze the most frequently encountered rectifier circuits.

Material Required

Heathkit Engineering Design Trainer ET-1000
Volt-ohmmeter (Heathkit SM-1210 or equivalent)
Oscilloscope (Heathkit SO-4550 or equivalent)
2 — 1kΩ, ½-watt, 5% resistor
2 — 4.7kΩ, ½-watt, 5% resistor
4 — 1N4002 silicon diodes

Procedure

1. Use the oscilloscope to measure the full secondary transformer voltage on the ET-1000 Trainer. V_{peak} = ________ V.

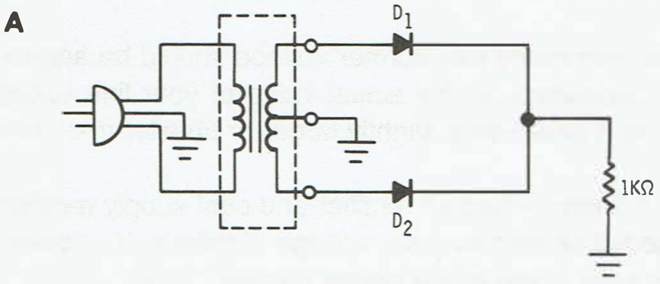
2. Based on the measured value in step 1, predict the peak voltage driving the diodes for the rectifiers in Figure 8-28.

Center-tapped rectifier V_m = ______V
Bridge rectifier V_m = ______V
Dual supply rectifier V_m = ______V

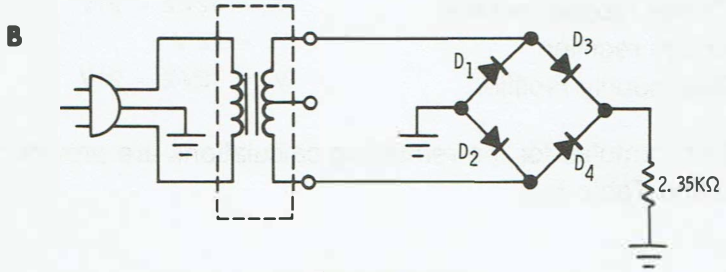
3. Based on the measured value in step 1, predict the peak inverse voltages across the diodes for the rectifiers in Figure 8-28.

Center-tapped rectifier PIV = ______V
Bridge rectifier PIV = ______V
Dual supply rectifier PIV = ______V

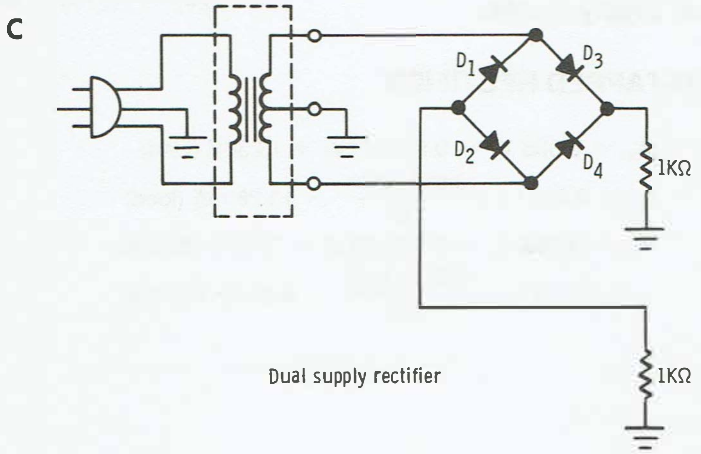
4. For each rectifier in Figure 8-28 calculate the DC load voltage, DC current, DC diode voltages, and DC diode currents. Record your calculated values in Table 8-11 (Page 8-66).



Center-tapped rectifier



Bridge rectifier



Dual supply rectifier

Figure 8-28

Experimental circuits for steps 1-9.

Discussion

Our calculations are provided at this point so that you can check your calculations prior to building the circuits.

Step 1 The full secondary transformer voltage should be approximately 42V peak. Depending on the actual value of your line voltage, you may measure a peak value slightly larger or smaller than 42V.

Step 2 V_m for the center-tapped rectifier and dual supply rectifier equals one half the full secondary peak voltage. Similarly, V_m equals the full secondary peak voltage in a bridge rectifier. Thus:

Center tapped rectifier	$V_m = 42V/2 = 21V$
Bridge rectifier	$V_m = 42\ V$
Dual supply rectifier	$V_m = 42V/2 = 21V$

The formulas for the remaining calculations are provided in Table 5-2 and Table 5-3.

Step 3

Center-tapped rectifier	$PIV = 2V_m = 2(21V) = 42V$
Bridge rectifier	$PIV = V_m = 42V$
Dual supply rectifier	$PIV = 2V_m = 2(21V) = 42V$

Step 4 CENTER-TAPPED RECTIFIER

$$V_{DC} = 0.636\ V_m = 0.636\ (21V) = 13.36V \text{ (load)}$$

$$I_{DC} = 0.636\ I_m = \frac{0.636\ (21V)}{1k\Omega} = 13.36\ mA \text{ (load)}$$

$$V_{DC} = 0.636\ V_m = 0.636\ (21V) = 13.36V \text{ (diodes)}$$

$$I_{DC} = 0.318\ I_m = \frac{0.318\ (21V)}{1k\Omega} = 6.68mA \text{ (diodes)}$$

BRIDGE RECTIFIER

$$V_{DC} = 0.636\ V_m = 0.636\ (42V) = 26.7V\ \text{(load)}$$

$$I_{DC} = 0.636\ I_m = \frac{0.636\ (42V)}{2.35k\Omega} = 11.36\ mA\ \text{(load)}$$

$$V_{DC} = 0.318\ V_m = 0.318\ (42V) = 13.36V\ \text{(diodes)}$$

$$I_{DC} = 0.318\ I_m = \frac{0.318\ (42V)}{2.35k\Omega} = 5.68\ mA\ \text{(diodes)}$$

DUAL SUPPLY RECTIFIER

$$V_{DC} = \pm\ 0.636\ V_m = 0.636\ (21V) = \pm\ 13.36V\ \text{(loads)}$$

$$I_{DC} = 0.636\ I_m = \frac{0.636\ (21V)}{1k\Omega} = 13.36\ mA\ \text{(both loads)}$$

$$V_{DC} = 0.636\ V_m = 0.636\ (21V) = 13.36V\ \text{(diodes)}$$

$$I_{DC} = 0.318\ I_m = \frac{0.318\ (21V)}{1k\Omega} = 6.68\ mA\ \text{(diodes)}$$

Note that since both loads are equal, the peak diode and load currents are equal.

Procedure (continued)

5. Build the circuit shown in Figure 8-28A. Measure the DC load current, and DC diode currents with the volt-ohmmeter connected as a DC ammeter.

   I_{DC} (load) = _______ mA
   I_{DC} (D_1) = _______ mA
   I_{DC} (D_2) = _______ mA

6. Measure the DC load voltage and DC diode voltages with the volt-ohmmeter connected as a DC voltmeter.

   V_{DC} (load) = _______ V
   V_{DC} (D_1) = _______ V
   V_{DC} (D_2) = _______ V

   Record the measured values of load voltage and load current in Table 8-11 (page 8-66). For the diode voltages and currents, record the average of your measured values in Table 8-11. For example, if you measured 13.6V across D_1 and 13.2V across D_2, you would record (13.6V + 13.2V)/2 or 13.4V in Table 8-11 for the measured value.

7. Measure the load and diode voltages with the oscilloscope. Load voltages should be full-wave signals, and diode voltages half-wave signals.

8. Repeat steps 5 through 7 for the circuit in Figure 8-28 B. Connect two 4.7kΩ resistors in parallel to obtain the 2.35kΩ load resistor.

9. Repeat steps 5 through 7 for the circuit in Figure 8-28 C.

Discussion

Our calculated values were based on the assumption that the rectifier diodes were ideal.

For the circuits used in this experiment, the voltages driving the diodes were large compared to the diodes turn-on voltage. Also, the load resistances were large compared to the foward resistance of the diodes. Consequently, the real diodes should have approximated the characteristics of ideal diodes. Thus, your measured and calculated values should have been in close agreement (approximately ± 12%).

Center-tapped rectifier

QUANTITY	CALCULATED	MEASURED
DC load voltage	______	______
DC load current	______	______
DC diode voltages	______	______
DC diode currents	______	______

Bridge rectifier

QUANTITY	CALCULATED	MEASURED
DC load voltage	______	______
DC load current	______	______
DC diode voltages	______	______
DC diode currents	______	______

Dual supply rectifier

QUANTITY	CALCULATED	MEASURED
DC load voltages	______	______
DC load currents	______	______
DC diode voltages	______	______
DC diode current	______	______

Table 8-11

Calculated and measured values for Experiment 6.

EXPERIMENT 7
SMOOTHING FILTERS

Objectives:

To design a choke-input filter.
To investigate the characteristics of a bleeder resistor.
To design a capacitor input filter.
To design a multisection filter.

Introduction

Smoothing filters convert pulsating DC input voltages into relatively pure DC output voltages. In this experiment, you will design both average and peak-type smoothing filters for the full-wave rectifier circuits of Experiment 6.

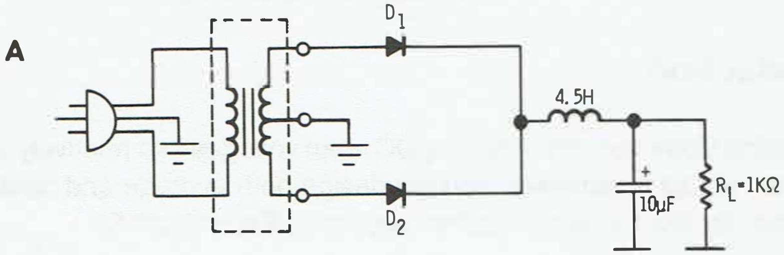
Appropriate analysis and design equations for capacitor-input and choke-input filters are provided in Table 5-4 (page 5-49) and Table 5-5 (page 5-55) respectively.

Material Required

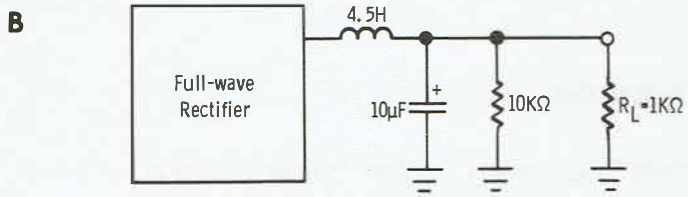
Heathkit Engineering Design Trainer ET-1000
Volt-ohmmeter (Heathkit SM-1210 or equivalent)
Oscilloscope (Heathkit SO-4550 or equivalent)
1 — 1kΩ, ½-watt, 5% resistor
1 — 2.2kΩ, ½-watt, 5% resistor
2 — 4.7kΩ, ½-watt, 5% resistor
1 — 10kΩ, ½-watt, 5% resistor
1 — 10μF, 50 volt electrolytic capacitor
1 — 47μF, 50 volt electrolytic capacitor
4 — 1N4002 silicon diodes
1 — 4.5 H choke

Procedure

1. Figure 8-29A illustrates the rectifier of Figure 8-28A with the addition of a choke-input filter. In Experiment 6, we calculated the DC output voltage of the unfiltered rectifier to be approximately 13.36V. Calculate the minimum inductance required in Figure 8-29A for adequate filtering. L_{min} = ______H.



Choke-input filter



Choke-input filter with bleeder resistor

Figure 8-29

Experimental circuits for steps 1-12.

2. Since a 4.5H choke is included in the parts kit, we select L = 4.5H. Measure the DC resistance of the choke. R_c = ______Ω.

3. Calculate the expected DC output voltage based on the measured value in step 2. V_{DC} = ______V.

4. Calculate the size of the capacitor required so that the percent ripple is 3% or less C = ______μF. Based on this calculation, what standard size capacitor would you recommend for the filter? C = ______μF.

Discussion

Sample calculations for steps 1 through 4 are provided at this point so you can check your calculations prior to building the circuit. Refer to Table 5-5 (page 5-55) for appropriate equations.

Step 1 $L_{min} = \frac{R_L}{1000} = \frac{1k\Omega}{1000} = 1H$

Step 2 The measured DC choke resistance is, approximately 200Ω.

Step 3 The DC choke input voltage is the DC output voltage from the rectifier. Assuming the rectifier output voltage is 13.36V DC, application of the voltage division principle yields:

$$V_{DC} = \frac{13.36v\ (1k\Omega)}{200\Omega + 1k\Omega} = 11.13V$$

Step 4 $C = \frac{830 \times 10^{-9}}{rL} = \frac{830 \times 10^{-9}}{0.03(4.5)} = 6.15\ \mu F$

Since too little ripple is always acceptable, a standard 10μF 50V electrolytic capacitor would be a reasonable choice.

Procedure (continued)

5. Build the circuit shown in Figure 8-29 A. Measure the DC load voltage with the volt-ohmmeter connected as a DC voltmeter. V_{DC} = ______ V.

6. Measure the peak ripple voltage with the oscilloscope. V_r = ______V peak. Multiply your measured value by 0.707 to obtain the approximate rms ripple voltage. V_r ______V_{rms}.

7. Calculate the percent ripple from the measured values in steps 5 and 6. r = ______%.

8. The voltage measured in step 5 represents the full-load voltage. To obtain the no-load voltage, remove the 1kΩ load resistor and measure the DC voltage across the capacitor. V_{NL} = _____V.

9. Calculate the percent regulation of the power supply from the measured values in steps 5 and 8.

$$\% \text{ regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

$$\% \text{ regulation} = \underline{\qquad} \times 100 = \underline{\qquad}\,\%$$

10. Reconnect the 1kΩ load resistor. Place a 10kΩ "bleeder resistor" in parallel with the load as shown in Figure 8-29B. Measure the full-load DC output voltage. V_{FL} = _____V.

11. Remove the 1kΩ load resistor and measure the no-load DC output voltage. V_{NL} = _____V.

12. Calculate the percent regulation of the power supply from the measured values in steps 10 and 11.

$$\% \text{ regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

$$\% \text{ regulation} = \underline{\qquad} \times 100 = \underline{\qquad}\,\%$$

Discussion

Based on the calculated values, you should have measured a DC output voltage of approximately 11.13V with less than 3% ripple. Depending upon the actual line voltage and minor component variations, your measured values may vary slightly from the calculated values. In any event, you should have obtained results that were consistent with Experiment 6.

When the load is removed from the choke-input filter, the capacitor charges to the peak output voltage of the rectifier. Consequently, the no-load voltage you measured in step 8 should have been equal to V_m, or approximately 21V. Since the filter acts like a peak-type filter under no-load and an average-type filter under full-load, the percent voltage regulation is quite large, as demonstrated in step 9.

To improve the voltage regulation, a "bleeder resistor" can be placed in parallel with the load. The value of the bleeder resistor is chosen so that the parallel combination of the bleeder and load resistors approximately equal the value of the load resistor. A useful rule of thumb is to select a bleeder resistor whose resistance is 10 times the load resistance.

A bleeder resistor improves the voltage regulation by making the difference between the no-load and full-load voltages smaller. For example, assuming $R_C = 200\Omega$, $R_B = 10k\Omega$, $R_L = 1k\Omega$, and the DC input voltage to the filter to be 13.36V, we have:

$$V_{FL} = \frac{13.36V\,(10k\Omega \parallel 1k\Omega)}{200\Omega + (10k\Omega \parallel 1k\Omega)} = 10.95V$$

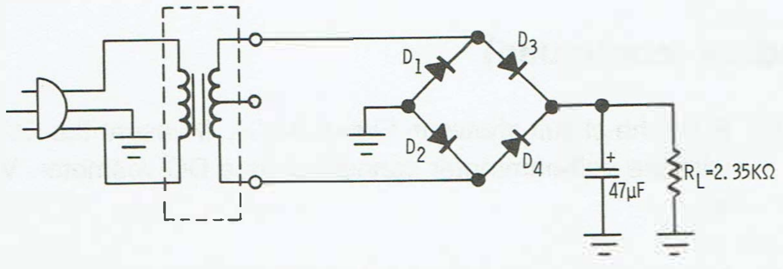
$$V_{NL} = \frac{13.36V\,(10k\Omega)}{200\Omega + 10k\Omega} = 13.1V$$

Consequently, the voltages measured in steps 10 and 11 should have been approximately equal to 10.95V and 13.1V respectively.

Procedure (continued)

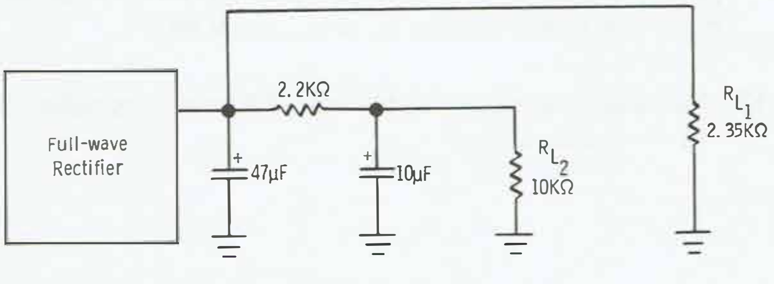
13. Figure 8-30A illustrates the rectifier of Figure 8-28B with the addition of a capacitor-input filter. In Experiment 6, the peak secondary voltage was approximately 44V. Calculate the minimum size capacitor required in Figure 8-30A so that the percent ripple is 3% or less. C_{min} = ______µF. Based on this calculation, what standard size capacitor would you recommend? C = ______µF.

A



Capacitor input filter

B



A multi-section filter supplying two loads

Figure 8-30

Experimental circuits for steps 13-21.

14. Calculate the expected DC output voltage, assuming C = 47μF. Take the turn-on voltages of the diodes into account when you are making this calculation. V_{DC} = ______V.

Discussion

Sample calculations for steps 13 and 14 follow.

Step 13

$$C_{min} = \frac{2.4 \times 10^{-3}}{r\ R_L} = \frac{2.4 \times 10^{-3}}{0.03\ (2.35)(10^3)} = 34\ \mu F$$

A 47μF, 50V electrolytic capacitor is a reasonable choice.

Step 14

$$V_{DC} = V_m \left[1 - \frac{4.17 \times 10^{-3}}{R_L C} \right]$$

The effect of the diode's turn-on voltage is to reduce V_m by approximately 1.4V. Thus:

$$V_{DC} = (44\ V - 1.4V) \left[1 - \frac{4.17 \times 10^{-3}}{2.35(10^3)(47)(10^{-6})} \right]$$

$$V_{DC} = 42.6V\ (0.962) = 41V.$$

Procedure (continued)

15. Build the circuit shown in Figure 8-30A. Measure the DC load voltage with the volt-ohmmeter connected as a DC voltmeter. V_{DC} = ______ V.

16. Measure the peak ripple voltage with the oscilloscope. V_r = ______V peak. Multiply your measured value by 0.707 to obtain the approximate rms ripple voltage. V_r = ______V rms.

17. Calculate the percent ripple from the measured values in steps 15 and 16. r = ______%.

18. In Figure 8-30 B, an RC section has been added to provide additional filtering for a second load, R_{L_2}. Calculate the DC voltage across R_{L_2}. V_{DC} = _____V.

19. Calculate the smoothing factor for the RC section. α = _____.

Discussion

Based on the calculations done previously, the voltage across the 2.35kΩ load resistor in Figure 3-30A should be approximately 41V with 3% or less ripple. Note in Figure 3-30B that the output voltage from the capacitor-input filter is the input voltage to the RC section. Thus:

Step 18

$$V_{DC} = \frac{41V\ (10k\Omega)}{2.2k\Omega + 10k\Omega} = 33.6V$$

Step 19

$$\alpha = 754\ RC - 1$$

$$\alpha = 754\ (2.2)(10^3)(10)(10^{6}) - 1 = 15.5$$

Therefore, the ripple voltage across R_{L_2} should be at least 15.5 times smaller than the ripple voltage across R_{L_1}.

Procedure (continued)

20. Add the second stage of filtering as shown in Figure 8-30B. Measure the DC voltage across R_{L_2} with the volt-ohmmeter. V_{DC} = _____V.

21. Measure the peak ripple voltage across R_{L_2} with the oscilloscope. V_r = _____V peak. Multiply your measured value by 0.707 to obtain the approximate rms ripple voltage. V_r = _____V_{rms}.

Discussion

Multisection filters are often used when a power supply must simultaneously provide two or more load voltages. The principle advantage of this approach is economy, since sufficient filtering is provided for each load at minimum cost.

The DC voltage measured in step 20 should be close to the calculated value of 33.6V. You can obtain the experimental smoothing factor by dividing the ripple voltage measured in step 16 by the ripple voltage measured in step 21. Naturally, the experimental smoothing factor should equal or exceed the calculated value of 15.5.

EXPERIMENT 8
CLIPPERS

Objectives:

To investigate the characteristics of shunt-type clippers.
To observe the effects of load, and potentiometer resistance, on the operation of the clipper.

Introduction

Clippers "clip off" voltages above or below a reference level established by the circuit designer. Biased clippers employ a DC voltage source to obtain either positive or negative reference levels. Frequently, biased clippers will also employ a potentiometer so that adjustable clipping levels may be obtained.

The various nonideal effects due to source resistance, load resistance, and the properties of real diodes must be taken into account when you design clipper circuits. Specifically, you should select component values so that the following inequalities are satisfied:

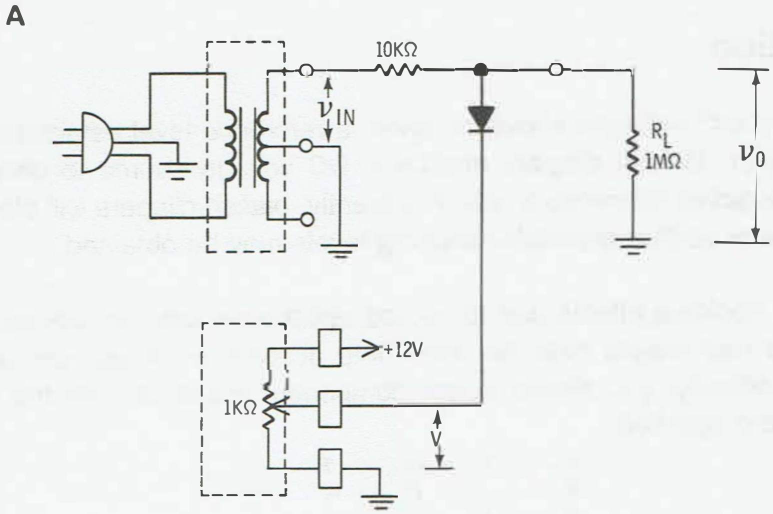
$V_{TH} >> V_T$ $\quad$ $R_{TH} << R$
$R >> r_F$ $\quad$ $R_L >> R$
$R << R_R$ $\quad$ $R_P << R$

Material Required

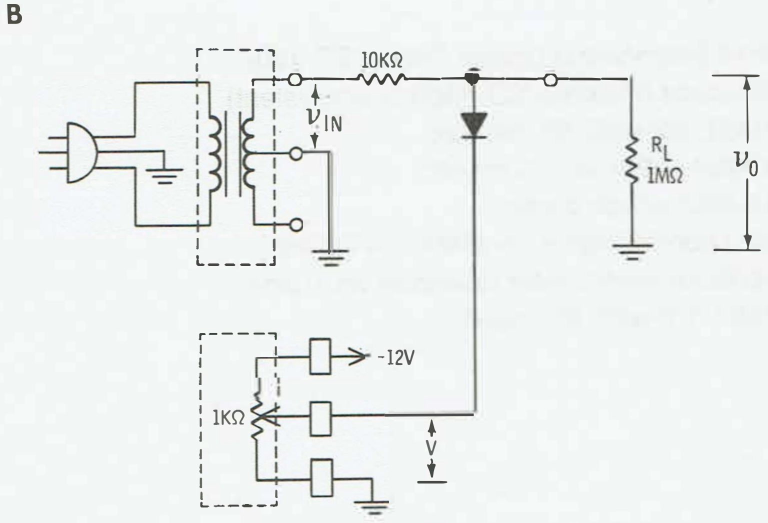
Heathkit Engineering Design Trainer ET-1000
Oscilloscope (Heathkit SO-4550 or equivalent)
2—10kΩ, 1/2-watt, 5% resistor
1—100kΩ, 1/2-watt, 5% resistor
1—1N4002 silicon diode
1—1kΩ potentiometer (available on Trainer)
1—100kΩ potentiometer (available on Trainer)
1—1MΩ, 1/2-watt, 5% resistor

Procedure

1. Build the circuit shown in Figure 8-31A. This circuit is similar to the positive peak type clipper in Figure 6-9A. In this case, we are using half the transformer secondary voltage as the input. Note also that the 1kΩ potentiometer on the ET-1000 Trainer is being used so you can obtain adjustable clipping levels.



Positive peak-type



Positive base-type

Figure 8-31

Experimental shunt clipper circuits.

2. Adjust the potentiometer wiper so that V = 12V.

3. Measure and record the peak value of the input voltage, ν_{in}. V_m = ________ V_{peak}.

4. Examine the output voltage. You should obtain an output voltage whose positive peak is clipped at approximately 12V. Carefully measure and record the level at which clipping occurs. Also note the value of $-V_m$. Positive clipping level = _____V. $-V_m$ = _____V.

5. Slowly vary the potentiometer wiper. How does this affect the output voltage? ____
____

6. Readjust the potentiometer wiper so that V = 12V.

7. Reverse the direction of the diode. In this case, the circuit is similar to the negative base type clipper in Figure 6-9B.

8. Examine the output voltage. You should obtain a waveform similar to the one shown in Figure 6-9B. Measure and record the values of V_m and the clipping level. V_m = ______V. Clipping level = ______V.

9. Vary the potentiometer wiper to verify that the clipping level is adjustable.

10. Reverse the direction of the diode. Also connect the upper end of the potentiometer to −12V as shown in Figure 8-31B. The circuit should now be similar to the positive base-type clipper in Figure 6-9C. Verify the operation of the circuit by varying the potentiometer wiper, and noting the affect on the output waveform.

11. Reverse the direction of the diode. You should now have a circuit similar to the negative - peak type clipper in Figure 6-9D. Once again, verify the operation of the circuit by varying the potentiometer wiper and noting the affect on the output voltage.

Discussion

In this first part of the experiment, the clippers should have closely approximated the characteristics of ideal clippers, since component values were employed that satisfied the inequalities stated previously. Specifically:

1. The effective transformer secondary resistance is negligible compared to the 10kΩ value of R.

2. The peak transformer secondary voltage, approximately 21V, is large compared to the diodes turn-on voltage.

3. The 10kΩ value of R is certainly large compared to the diodes forward resistance, and small compared to the diodes reverse resistance.

4. The 1MΩ load resistor is large compared to the 10kΩ value of R.

5. The 1kΩ potentiometer is small compared to the 10kΩ value of R. In this case, $R_p = R/10$. A value of $R_p = R/100$ would be even more effective in masking out the non-ideal effects of R_p. However, as you saw in steps 1-11, satisfactory results were obtained with $R_p = 1k\Omega$.

In the remainder of the experiment you will investigate what happens if R_L is too small or R_p is too large.

Procedure (continued)

12. Build the positive peak-type clipper in Figure 8-31A. Adjust the potentiometer wiper so that V = 12V.

13. Change the value of R_L to 100kΩ. Make a neat sketch of the output voltage, indicating appropriate voltage levels.

14. Repeat step 13 for R_L = 10kΩ.

15. Reverse the direction of the diode. Sketch the output voltage, indicating appropriate voltage levels for R_L = 100kΩ, and R_L = 10kΩ.

Discussion

A simplified schematic for the circuit in steps 12-14 is illustrated in Figure 8-32A. By considering the diode and DC voltage source as the "load," you obtain the Thevenin equivalent circuit in Figure 8-32B. Here:

$$V_{TH} = \frac{V_m \cdot R_L}{R + R_L} \text{ and } R_{TH} = R \parallel R_L$$

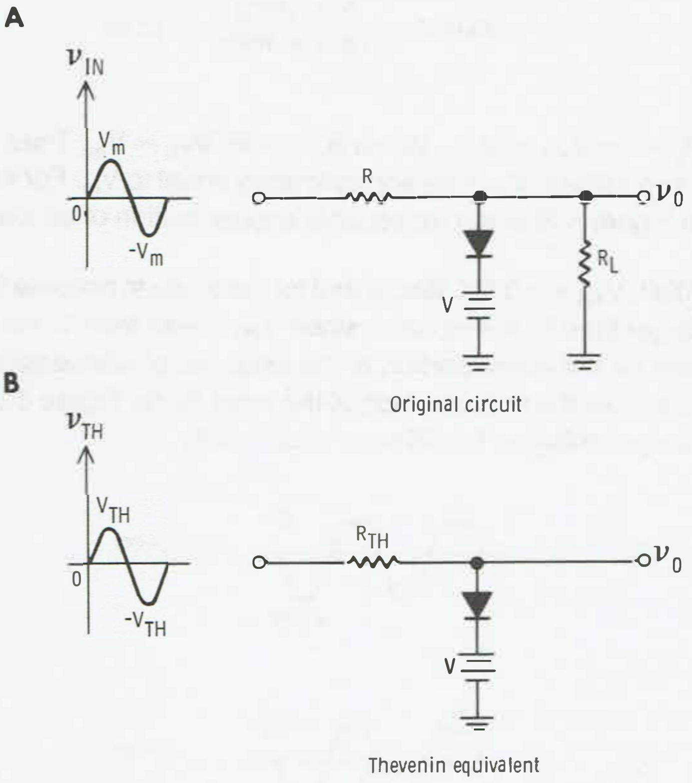


Figure 8-32

Simplified schematic for steps 12-14.

The operation of the Thevenin equivalent circuit in Figure 8-32B is identical to the operation of the positive peak-type clipper in Figure 6-9. Thus:

1. When ν_{TH} exceeds V, the diode is forward biased, and $\nu_o = 12V$.
2. When ν_{TH} is less than V, the diode is reversed biased. Consequently, ν_o equals the instantaneous value of ν_{TH}.

Clearly, the closer V_{TH} is to V_m the more ideal will be the operation of the original circuit in Figure 8-32A. Lets calculate the value of V_{TH} for different values of R_L.

$$R_L = 1M\Omega \quad V_{TH} = \frac{21V\,(1M\Omega)}{10k\Omega + 1M\Omega} = 20.8V$$

$$R_L = 100k\Omega \quad V_{TH} = \frac{21V\,(100k\Omega)}{10k\Omega + 100k\Omega} = 19V$$

$$R_L = 10k\Omega \quad V_{TH} = \frac{21V\,(10k\Omega)}{10k\Omega + 10k\Omega} = 10.5V$$

Idealy, when $R_L = \infty$ $V_{TH} = V_m$. When $R_L >> R$, $V_{TH} \approx V_m$. Thus, when R_L was equal to 1MΩ, and 100kΩ, V_{TH} was approximately equal to V_m. For these values of R_L the circuit in Figure 8-32A is a reasonable approximation of an ideal clipper.

When $R_L = 10k\Omega$, $V_{TH} = 10.5V$. Recall that for the diode to become forward biased, ν_{TH} must be larger than V. In this case, since V_{TH} is less than V, the diode remains reversed biased for the entire portion of the input cycle! Consequently, the output voltage equals ν_{TH} for the entire portion of the input cycle. Figure 8-33 summarizes the expected output voltages for different values of R_L.

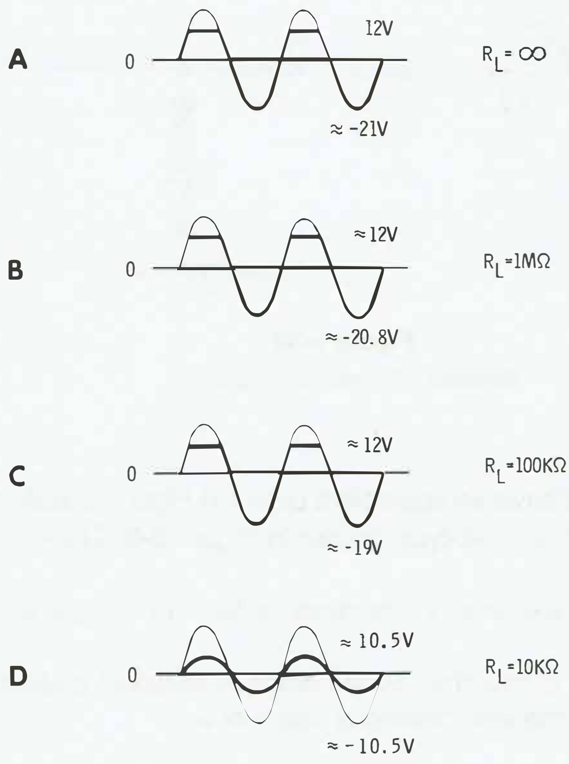


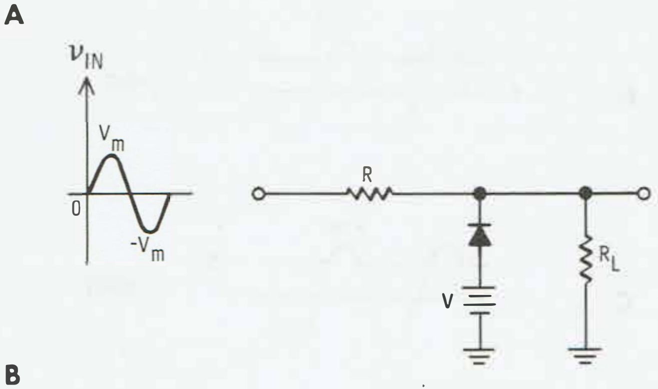
Figure 8-33

Expected output voltages for the positive peak-type clipper.

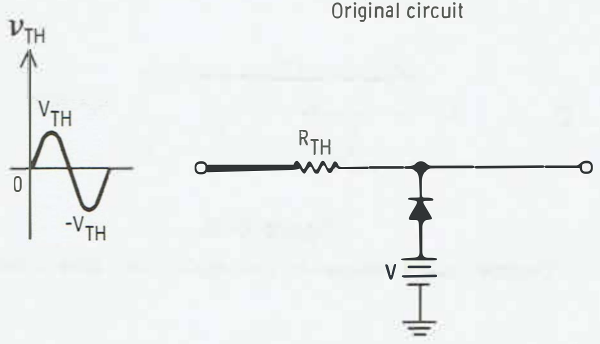
A simplified schematic for the circuit in step 15 is illustrated in Figure 8-34. Once again, we will focus our attention on the Thevenin equivalent circuit.

The Thevenin equivalent circuit in Figure 8-34B is an example of a negative base type clipper. The operation of the circuit may be described as follows:

1. Initially, the diode is forward biased by the DC voltage source, V. Thus, $\nu_o = V$.
2. On the positive half cycle of the input, ν_{TH} and V are connected series opposing. Thus, when ν_{TH} exceeds V, the diode is reversed biased. Therefore, ν_o equals the instantaneous value of ν_{TH}.
3. The diode is forward biased throughout the entire negative half cycle of the input, since ν_{TH} and V are connected series aiding.



Original circuit



Thevenin equivalent

Figure 8-34

Simplified schematic for step 15.

When $R_L = \infty$, the output voltage would appear as shown in Figure 8-35A. The values of V_{TH} calculated previously still apply. Consequently, the output waveforms for $R_L = 1M\Omega$, $100k\Omega$, and $10k\Omega$ appear as shown in Figure 8-35B, C, and D respectively.

Since v_{TH} never exceeds V when $R_L = 10k\Omega$, the diode in Figure 8-34 remains forward biased throughout the entire portion of the input cycle.

This is why the output voltage in Figure 8-35 D is a constant 12V.

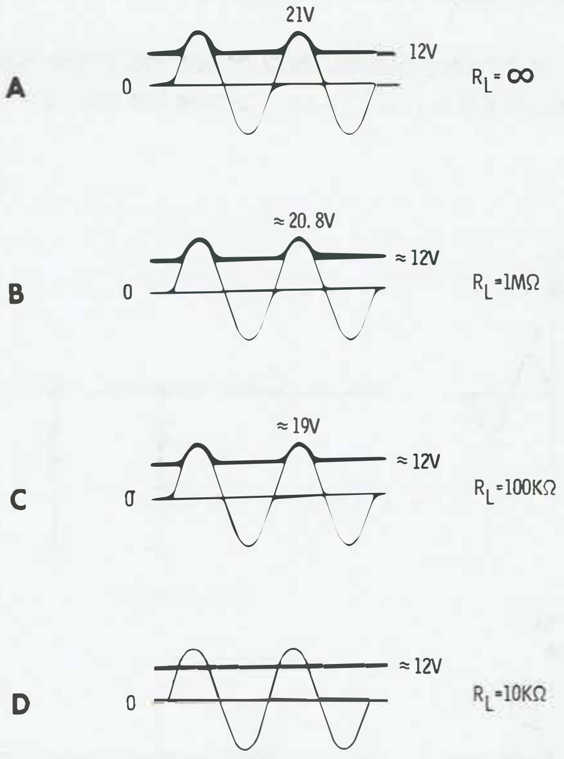


Figure 8-35

Expected output voltages for the negative base type clipper.

Procedure (continued)

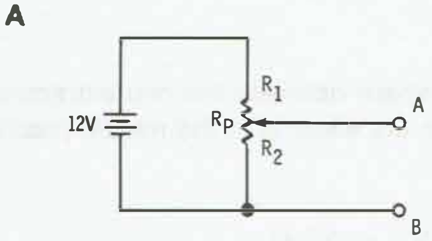
16. Build the positive peak-type clipper shown in Figure 8-31A using the 100kΩ potentiometer on the ET-1000 Trainer for R_p.

17. Slowly vary the potentiometer wiper and observe the output waveform. Are the results the same as those obtained when the 1kΩ potentiometer was employed?

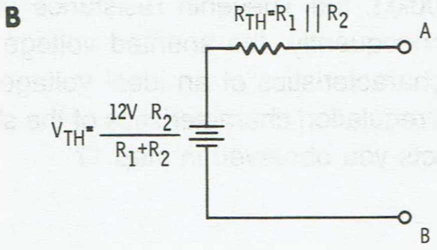
Discussion

When $R_p = 1k\Omega$, in steps 1-15, the clipping levels were well defined. When $R_p = 100k\Omega$ in steps 16 and 17, you observed clipping levels that were "bowed" as you varied the potentiometer wiper.

Figure 8-36 illustrates the 12V source shunted by the potentiometer. As you vary the position of the wiper, the output voltage, V_{AB}, can be adjusted between 0V and 12V. The Thevenin equivalent circuit for any position of the wiper is illustrated in Figure 8-36B. Lets examine what happens for three positions of the wiper.



Circuit



Thevenin equivalent

Figure 8-36

Potentiometer voltage divider.

1. **WIPER AT TOP.**

   In this case, $R_1 = 0$ and $R_2 = R_p$. Thus, $R_{TH} = 0 \parallel R_p = 0$. Clearly, $V_{TH} = 12V$.

2. **WIPER AT BOTTOM.**

   Here, $R_1 = R_p$ and $R_2 = 0$. Thus, $R_{TH} = R_p \parallel 0 = 0$. Obviously, $V_{TH} = 0V$.

3. **WIPER IN MIDDLE.**

   Since the wiper is in the middle $R_1 = R_2 = \frac{R_p}{2}$.

Thus $R_{TH} = \frac{R_P}{2} \parallel \frac{R_P}{2} = \frac{R_P}{4}$.

The Thevenin voltage is $\frac{V}{2}$ or 6V.

When the wiper is at either extreme, $R_{TH} = 0$. Thus, the shunted voltage source acts very much like an ideal voltage source. In these cases, the clipping levels are well defined.

When the wiper is somewhere between the two extremes, $R_{TH} = R_1 \parallel R_2$. The "worst case" occurs when the wiper is in the middle position. Here, $R_{TH} = R_p/4$. Thus:

When $R_p = 1k\Omega$, $R_{TH} = 250\Omega$.
When $R_p = 100k\Omega$, $R_{TH} = 25k\Omega$.

Note that when $R_p = 100k\Omega$, the Thevenin resistance of the shunted voltage source is quite large. Consequently, the shunted voltage source does not approximate very well the characteristics of an ideal voltage source. In this case, the relatively poor voltage regulation characteristics of the shunted voltage source produces the bowing effects you observed in step 17.

EXPERIMENT 9
CLAMPERS AND PEAK DETECTORS

Objectives: *Investigate the characteristics of clampers and peak detectors. To verify the operation of a peak-to-peak detector.*

Introduction

Clampers are used to fix the positive or negative peak of a signal waveform at a desired DC level. The inequalities that must be satisfied when you are designing clipper circuits also apply to clamper circuits.

Peak detectors provide a DC output voltage ideally equal to the positive or negative peak of the input voltage. By cascading a clamper and peak detector, you can construct a peak-to-peak detector. This circuit provides a DC output voltage equal to twice the positive or negative peak value of the input voltage.

In both clampers and peak detectors, the RC time constant must be large enough to ensure good clamping or peak detection. However, if the time constant is too large, the circuit will not respond quickly to amplitude changes in the input voltage. As a guide, the capacitor in a clamper or peak detector is chosen so that $C = \frac{50\ T_{max}}{R}$

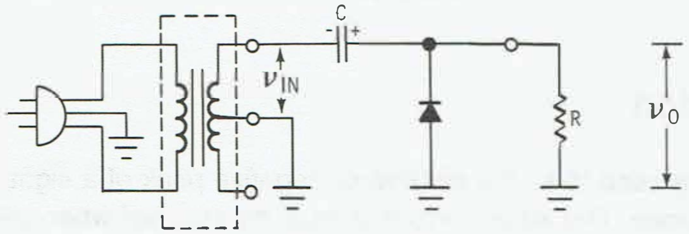
Material Required

Heathkit Engineering Design Trainer ET-1000
Oscilloscope (Heathkit SO-4550 or equivalent)
1 — 10kΩ, ½-watt, 5% resistor
1 — 100kΩ, ½-watt, 5% resistor
1 — 1MΩ, ½-watt, 5% resistor
2 — 1N4002 Silicon Diodes
2 — 10μF, 50V electrolytic capacitors

Procedure

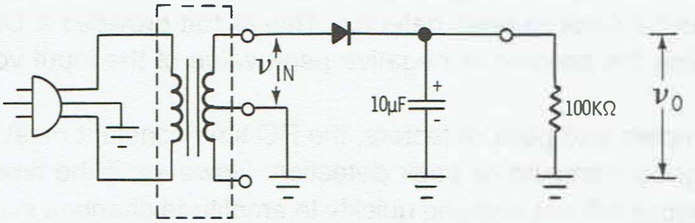
1. Assume R = 100kΩ for the positive clamper in Figure 8-37A. The input to the clamper is approximately 21V peak at 60Hz. Calculate the required value of C. C = ______μF.

A



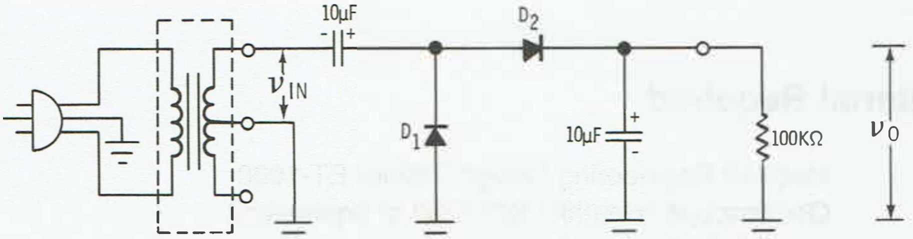
Positive clamper

B



Positive peak detector

C



Positive peak-to-peak detector

Figure 8-37

Experimental circuits.

Discussion

The calculations for step 1 follows:

Since f = 60Hz, T = 1/60Hz or 16.67 ms. Thus:

$$C = \frac{50\ (16.67\ \text{ms})}{100\ \text{k}\Omega} \approx 8.34\ \mu\text{F}$$

For the remainder of the experiment, you will use a 10μF, electrolytic capacitor for C. Since electrolytic capacitors are polarized, it is essential that you observe the correct polarity when you place the capacitor in a given circuit.

Procedure (continued)

2. Build the circuit shown in Figure 8-37A. Here, R = 100kΩ and C = 10μF. Note that the input voltage is half the transformer secondary voltage.

3. Measure the input and output voltages. Record the peak value of the input voltage and the positive and negative peak values of the output voltage. V_m = _____V. V_o (positive peak) = _____V.
V_o (negative peak) = _____V.

4. Shut off the ET-1000 Trainer. Reverse the direction of the diode and the capacitor. Turn the Trainer back on and repeat steps 3 and 4.
V_m = _____V. V_o (positive peak) = _____V.
V_o (negative peak) = _____V.

Discussion

The value of capacitance, C, was chosen to make the time constant large compared to the period of the signal waveform. For this reason, you should have observed close to ideal clamping action in steps 3 and 4.

Ideally, the negative peak of the positively-clamped output, step 3, and the positive peak of the negatively-clamped output, step 4, should be 0V. Due to the diodes turn on voltage, the negative peak of the output voltage in step 3 was clamped slightly below 0V. Similarly, the positive peak of the output voltage in step 4 was clamped slightly above 0V.

Procedure (continued)

5. Build the circuit shown in Figure 8-37B. Connect all grounds to one point and then connect that point to the 30 VAC output center tap with a single wire. Measure and record the value of the DC output voltage. With the oscilloscope AC coupled, measure and record the peak-to-peak value of the output ripple voltage.

   V_{DC} = _____ V, V_r = _____ mV_{pp}.

6. Change R to 1MΩ. Repeat the measurements made in step 5.

   V_{DC} = _____ V, V_r = _____ mV_{pp}.

7. Change R to 10kΩ. Repeat the measurements made in step 5.

   V_{DC} = _____ V, V_r = _____ mV_{pp}.

8. With R = 1MΩ, 100kΩ and 10kΩ respectively, perform the following experiment. While observing the output waveform, alternately break and make the ground wire going to the transformer center tap. For each resistance value, indicate whether the ability of the circuit to respond to sudden changes in the amplitude of the input signal is excellent, fair, or poor.

   R = 1MΩ, Response is _____
   R = 100kΩ, Response is _____
   R = 10kΩ, Response is _____

Discussion

In clampers and peak detectors, increasing the time constant improves the "quality" of the clamping on peak detection. Thus, in steps 5-7, you saw that the DC output voltage with R = 1MΩ contained very little ripple, and was almost equal to V_m. Similarly, when R = 10kΩ, the ripple content of the output voltage increased significantly and the DC level decreased slightly. When R = 100kΩ, the results were between the previous extremes.

Unfortunately, as the time constant increases, the ability of a clamper or peak detector to respond to sudden changes in the amplitude of the input voltage decreases. This fact was demonstrated in step 8, where you saw that large values of resistance resulted in a "sluggish" response.

Hence, the value of C selected for a particular clamper, or peak detector represents a compromise between the overall quality of clamping or peak detection and the ability of the circuit to respond quickly to sudden amplitude changes in the input waveform.

Procedure (continued)

9. Build the circuit shown in Figure 8-37C. Note the waveforms appearing across D_1 and the output.
10. Turn off the power to the ET-1000 Trainer. Reverse the direction of both capacitors and diodes. Then turn the power back on and note the waveforms across D_1 and the output.

Discussion

The circuit in Figure 8-37C is simply the circuit in Figure 8-37A, the positive clamper, and Figure 8-37B, the positive peak detector, connected in cascade. Consequently, the result is a positive peak-to-peak detector. Thus, the output voltage observed in step 9 should have been a DC voltage essentially equal to $2V_m$.

By reversing the direction of the components in step 10, you obtained a negative clamper cascaded with a negative peak detector. For this reason, the resulting circuit was a negative peak-to-peak detector. In this case, the DC output voltage should have been equal to approximately $-2V_m$.

If you obtained a DC output voltage of 0V in either step 9 or 10, you probably had one of the diodes reversed. If this was the case, repeat the appropriate step, being careful to place the components in the circuit in the correct direction.

EXPERIMENT 10
MEASUREMENT CIRCUITS

Objectives: *To construct and verify the operation of an elementary diode checker. To convert a DC voltmeter to a peak-type AC voltmeter.*

Introduction

There are numerous applications for passive circuits in basic instruments and measurement systems. In this experiment you will construct the elementary diode checker discussed in Unit 7. This circuit is especially useful for quickly checking an assortment of diodes in order to isolate defective units.

Peak-type AC voltmeters are useful for measuring both sinusoidal and nonsinusoidal waveforms. You can easily convert a DC voltmeter to a peak-type AC voltmeter by using a peak detector ahead of the DC voltmeter. In this experiment, you will also design a peak-type AC voltmeter, and experimentally verify its operation.

Material Required

Heathkit Engineering Design Trainer ET-1000
Oscilloscope (Heathkit SO-4550 or equivalent)
Volt-ohmmeter
1—680Ω, 1/2-watt, 5% resistor
1—2.2kΩ, 1/2-watt, 5% resistor
1—.039μF capacitor
1—10μF electrolytic capacitor
1—100μF electrolytic capacitor
1—1N4002 Silicon Diode
2—LEDs (light emitting diodes), red

Procedures

1. Most LED's have a forward voltage drop of approximately 1.6V. Assuming that the secondary transformer voltage from the center-tap of either end in Figure 8-38 is $15V_{rms}$, calculate the value of R required to limit the peak diode current to 10mA. R = ______.

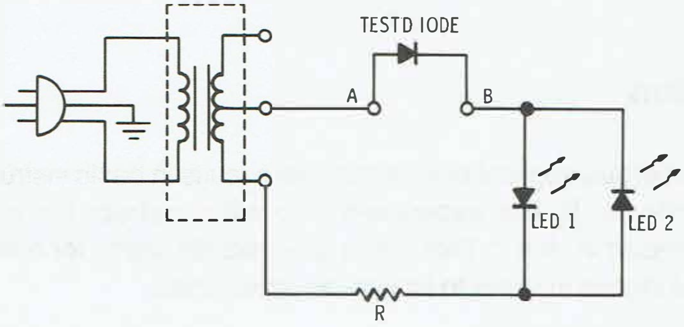


Figure 8-38

Experimental diode checker.

Discussion

The calculation for step 1 follows:

$$1.414\ (15V_{rms}) = 21.21V_{peak}$$

Typically, the test diode has a forward voltage drop of 0.7V. Since the LED is assumed to have a forward voltage drop of 1.6V, we have:

$$R = \frac{V_m - (0.7 + 1.6)}{I} = \frac{21.21V - 2.3V}{10mA} = 1.89k\Omega$$

Selecting the next larger standard value resistor, we chose R = 2.2kΩ.

Procedure (continued)

2. Build the circuit shown in Figure 8-38. Be careful to observe the correct polarity for the diodes. The cathode side of each LED is the long lead. Insert a 1N4002 diode for the test diode. Which LED lights?______ ____________________.

3. To simulate an open diode, remove the test diode. Which LED's are lit?__________________.

4. To simulate a shorted diode, connect a clip lead from A to B. In this case, which LED's are lit?__________________ .

Discussion

As we explained in Unit 7, the operation of the diode checker should agree with the following table:

CONDITION	LED STATUS
Good diode	LED 1 lit
Open diode	Neither LED lit
Shorted diode	Both LED's lit

Table 8-12

Data for steps 2 to 4.

If you have several diodes in your work area, use the diode checker to verify that none of the diodes are opened or shorted.

Procedure (continued)

5. Examine the circuit in Figure 8-39. Here, note that the 680Ω resistor and the 1kΩ potentiometer simply serve as a voltage divider for the secondary transformer voltage.

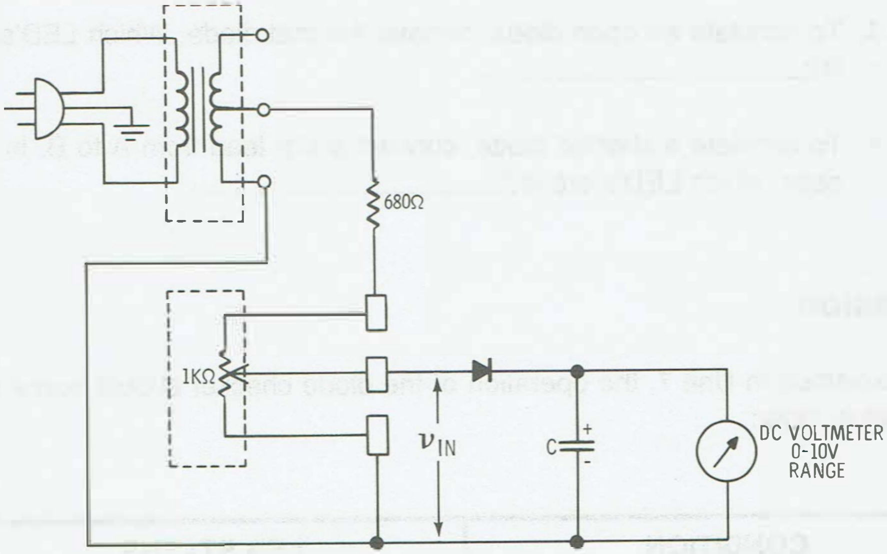


Figure 8-39

Experimental peak type AC voltmeter.

6. Examine the front of your volt-ohmmeter. Record the DC ohms per volt rating of your instrument. Ohms/volt rating = ______kΩ/v.

7. Calculate the input resistance of your volt-ohmmeter when it is used as a DC voltmeter on the 0-10V DC range. R_{IN} = ______kΩ.

8. Using the value of R_{IN} obtained in step 7, calculate the value of C required in Figure 8-39 to obtain good peak detection. C = ______ μF.

Discussion

Most volt-ohmmeters have an ohms per volt rating of either 1kΩ/V or 20kΩ/V when they are used as a DC voltmeter. Consequently, you probably recorded one of these values in step 6. Digital voltmeters, however, have considerably higher ohms per volt ratings.

The input resistance of a DC voltmeter equals the ohms per volt rating times the value of V_{IN} required for full-scale deflection. Thus:

For 1kΩ/V, $R_{IN} = 1\text{k}\Omega/\text{V}\ (10\text{V}) = 10\text{k}\Omega$

For 20kΩ/V, $R_{IN} = 20\text{k}\Omega/\text{V}\ (10\text{V}) = 200\text{k}\Omega$

Recall that the value of capacitance, C, in a peak detector is calculated from $C = \frac{50\ T}{R}$.

Since f = 60Hz, T equals 1/60Hz or 16.67 mS. Thus, if $R_{IN} = 10\text{k}\Omega$:

$$C = \frac{50\ (16.67\ \text{mS})}{10\text{k}\Omega} = 83.4\ \mu\text{F}$$

Similarly, if $R_{IN} = 200\text{k}\Omega$:

$$C = \frac{50\ (16.67\ \text{mS})}{200\ \text{k}\Omega} = 4.17\ \mu\text{F}$$

If your volt-ohmmeter has a DC ohms/volt rating of 1kΩ/V, you will use a 100μF capacitor for the peak-type AC voltmeter in Figure 8-39. Similarly, a 10μF capacitor will be employed if the ohms per volt rating of your volt-ohmmeter is 20kΩ/V.

When you continue on with this experiment, you should use the .039μF capacitor if you have a digital voltmeter. Although your calculated value may be different, this capacitor should give you satisfactory results.

Procedure (continued)

9. Build the circuit shown in Figure 8-39. Here, C = 100µF for an ohms/volt rating of 1kΩ/V, and 10µF for an ohms/volt rating of 20kΩ/V.

10. Monitor ν_{IN} with an oscilloscope. Adjust the potentiometer to obtain the following peak values of ν_{IN}: 10V, 9V, 8V, 7V, 6V, 5V, 2V, and 1V. In each case, record the reading of the DC voltmeter in Table 8-13.

ν_{in}	ν_o (v)
10V peak	
9V peak	
8V peak	
7V peak	
6V peak	
5V peak	
4V peak	
3V peak	
2V peak	
1V peak	

Table 8-13

Data for step 10.

Discussion

Ideally, ν_o in Table 8-13 should equal the peak value of ν_{IN}. Your data should indicate that ν_o approximately equals the peak value of ν_{IN} when ν_{IN} is large compared to the diode's turn on voltage. If the peak value of ν_{IN} is not large compared to (approximately) 0.7V, the reading of the peak-type AC voltmeter will not be very accurate. Consequently, peak-type AC voltmeters are most accurate for relatively large values of ν_{IN}.

Procedure (continued)

11. Adjust the potentiometer so that ν_{IN} = 10V peak. While observing the reading of the DC voltmeter, disconnect the lead going to the transformer center tap. How long does it take for the reading of the DC voltmeter to decrease to 0V? t = ______s.

Discussion

When ν_{IN} = 10V peak, the capacitor charges to approximately 10V. When you removed the input signal in step 11, the capacitor discharges through the resistance of the DC voltmeter. As you know, it takes approximately 5 time constants for the capacitor to completely discharge. Thus:

For $C = 10\mu F$ and $R_{IN} = 200\ k\Omega$,

$\tau = 5\ (10\mu F)\ (200\ k\Omega) = 10s$

For $C = 100\ \mu F$ and $R_{IN} = 10k\Omega$,

$\tau = 5\ (100\ \mu F)\ (10k\Omega) = 5s$

Consequently, depending upon the values of C and R_{IN} for your circuit, you should have measured approximately 5 or 10s in step 11.

Clearly, the response of your peak-type AC voltmeter to sudden amplitude changes in ν_{IN} was somewhat “sluggish”. To improve the ability of the circuit to quickly respond to such changes, you would have to decrease the R_{IN} C time constant. You could do this by reducing the value of C. If C is too small, however, the capacitor voltage would contain an excessive amount of ripple. As usual, a practical design is a compromise between such often “contradictory” requirements.

APPENDIX A

RESISTOR COLOR CODE CHART

COLOR CODE CHART
FIXED RESISTORS

HEATHKIT
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COLOR CODE F

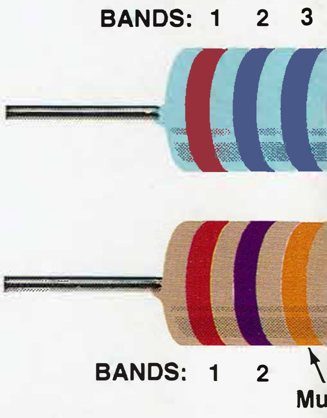
First Band	Second Band	Multiplier Band	Resistance Ω
Brown	Black	Black	10
		Brown	100
		Red	1000
		Orange	10000
		Yellow	0.1 Meg.
		Green	1.0 Meg.
		Blue	10.0 Meg.
	Brown	Black	11
		Brown	110
		Red	1100
		Orange	11000
		Yellow	0.11 Meg.
		Green	1.1 Meg.
		Blue	11.0 Meg.
	Red	Black	12
		Brown	120
		Red	1200
		Orange	12000
		Yellow	0.12 Meg.
		Green	1.2 Meg.
		Blue	12.0 Meg.
	Orange	Black	13
		Brown	130
		Red	1300
		Orange	13000
		Yellow	0.13 Meg.
		Green	1.3 Meg.
		Blue	13.0 Meg.
	Green	Black	15
		Brown	150
		Red	1500
		Orange	15000
		Yellow	0.15 Meg.
		Green	1.5 Meg.
		Blue	15.0 Meg.
	Blue	Black	16
		Brown	160
		Red	1600
		Orange	16000
		Yellow	0.16 Meg.
		Green	1.6 Meg.
		Blue	16.0 Meg.
	Gray	Black	18
		Brown	180
		Red	1800
		Orange	18000
		Yellow	0.18 Meg.
		Green	1.8 Meg.
		Blue	18.0 Meg.

First Band	Second Band	Multiplier Band	Resistance Ω
Red	Black	Black	20
		Brown	200
		Red	2000
		Orange	20000
		Yellow	0.20 Meg.
		Green	2.0 Meg.
		Blue	20.0 Meg.
	Red	Black	22
		Brown	220
		Red	2200
		Orange	22000
		Yellow	0.22 Meg.
		Green	2.2 Meg.
		Blue	22.0 Meg.
	Yellow	Black	24
		Brown	240
		Red	2400
		Orange	24000
		Yellow	0.24 Meg.
		Green	2.4 Meg.
	Violet	Gold	2.7
		Black	27
		Brown	270
		Red	2700
		Orange	27000
		Yellow	0.27 Meg.
		Green	2.7 Meg.

Standard C

5-BAND RESISTORS
(±1%)

4-BAND RESISTORS
(±10%
±5%)



Band 1 1st Digit	
Color	Digit
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

Band 2 2nd Digit	
Color	Digit
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

Band 3 3rd D
Color
Black
Brown
Red
Orange
Yellow
Green
Blue
Violet
Gray
White

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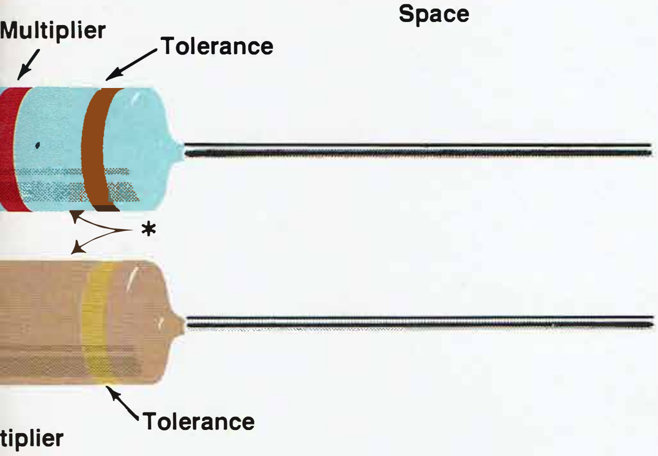
597-1715

R RESISTORS

First Band	Second Band	Multiplier Band	Resistance Ω
Orange	Black	Gold	3.0
		Black	30
		Brown	300
		Red	3000
		Orange	30000
		Yellow	0.30 Meg.
		Green	3.0 Meg.
	Orange	Gold	3.3
		Black	33
		Brown	330
		Red	3300
		Orange	33000
		Yellow	0.33 Meg.
		Green	3.3 Meg.
	Blue	Gold	3.6
		Black	36
		Brown	360
		Red	3600
		Orange	36000
		Yellow	0.36 Meg.
		Green	3.6 Meg.
	White	Gold	3.9
		Black	39
		Brown	390
		Red	3900
		Orange	39000
		Yellow	0.39 Meg.
		Green	3.9 Meg.
Yellow	Orange	Gold	4.3
		Black	43
		Brown	430
		Red	4300
		Orange	43000
		Yellow	0.43 Meg.
		Green	4.3 Meg.
	Violet	Gold	4.7
		Black	47
		Brown	470
		Red	4700
		Orange	47000
		Yellow	0.47 Meg.
		Green	4.7 Meg.
Green	Brown	Gold	5.1
		Black	51
		Brown	510
		Red	5100
		Orange	51000
		Yellow	0.51 Meg.
		Green	5.1 Meg.
	Blue	Gold	5.6
		Black	56
		Brown	560
		Red	5600
		Orange	56000
		Yellow	0.56 Meg.
		Green	5.6 Meg.
Blue	Red	Gold	6.2
		Black	62
		Brown	620
		Red	6200
		Orange	62000
		Yellow	0.62 Meg.
		Green	6.2 Meg.
	Gray	Gold	6.8
		Black	68
		Brown	680
		Red	6800
		Orange	68000
		Yellow	0.68 Meg.
		Green	6.8 Meg.
Violet	Green	Gold	7.5
		Black	75
		Brown	750
		Red	7500
		Orange	75000
		Yellow	0.75 Meg.
		Green	7.5 Meg.
Gray	Red	Gold	8.2
		Black	82
		Brown	820
		Red	8200
		Orange	82000
		Yellow	0.82 Meg.
		Green	8.2 Meg.
White	Brown	Gold	9.1
		Black	91
		Brown	910
		Red	9100
		Orange	91000
		Yellow	0.91 Meg.
		Green	9.1 Meg.

lor Code

* Note Wide Space



(if used) git
Digit
0
1
2
3
4
5
6
7
8
9

Multiplier	
Color	Multiplier
Black	1
Brown	10
Red	100
Orange	1,000
Yellow	10,000
Green	100,000
Blue	1,000,000
Silver	0.01
Gold	0.1

Resistance Tolerance	
Color	Tolerance
Silver	±10%
Gold	± 5%
Brown	± 1%

🟫	Brown	110
	Red	1100
	Orange	11000
	Yellow	0.11 Meg.
	Green	1.1 Meg.
	Blue	11.0 Meg.
🟥	Black	12
	Brown	120
	Red	1200
	Orange	12000
	Yellow	0.12 Meg.
	Green	1.2 Meg.
	Blue	12.0 Meg.
🟧	Black	13
	Brown	130
	Red	1300
	Orange	13000
	Yellow	0.13 Meg.
	Green	1.3 Meg.
	Blue	13.0 Meg.
🟩	Black	15
	Brown	150
	Red	1500
	Orange	15000
	Yellow	0.15 Meg.
	Green	1.5 Meg.
	Blue	15.0 Meg.
🟦	Black	16
	Brown	160
	Red	1600
	Orange	16000
	Yellow	0.16 Meg.
	Green	1.6 Meg.
	Blue	16.0 Meg.
▨	Black	18
	Brown	180
	Red	1800
	Orange	18000
	Yellow	0.18 Meg.
	Green	1.8 Meg.
	Blue	18.0 Meg.

Red	Red	Brown	220
		Red	2200
		Orange	22000
		Yellow	0.22 Meg.
		Green	2.2 Meg.
		Blue	22.0 Meg.
	Yellow	Black	24
		Brown	240
		Red	2400
		Orange	24000
		Yellow	0.24 Meg.
		Green	2.4 Meg.
	Violet	Gold	2.7
		Black	27
		Brown	270
		Red	2700
		Orange	27000
		Yellow	0.27 Meg.
		Green	2.7 Meg.
Orange	Orange	Black	33
		Brown	330
		Red	3300
		Orange	33000
		Yellow	0.33 Meg.
		Green	3.3 Meg.
	Blue	Gold	3.6
		Black	36
		Brown	360
		Red	3600
		Orange	36000
		Yellow	0.36 Meg.
		Green	3.6 Meg.
	White	Gold	3.9
		Black	39
		Brown	390
		Red	3900
		Orange	39000
		Yellow	0.39 Meg.
		Green	3.9 Meg.

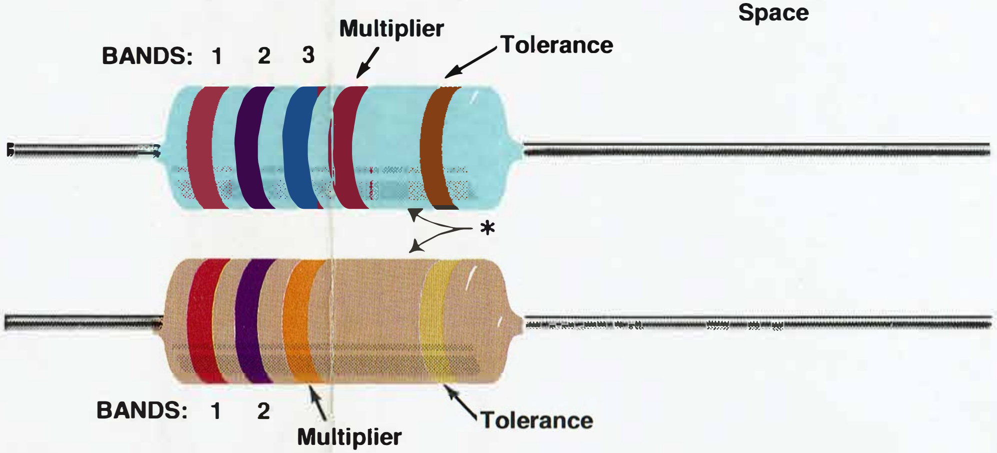
🟨	Violet
Green	Brown
	Blue
Blue	Red
	Gray
Violet	Green
Gray	Red
White	Brown

Standard Color Code

* Note Wide
Space

5-BAND RESISTORS
(±1%)

4-BAND RESISTORS
(±10%
±5%)



Band 1
1st Digit

Color	Digit
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

Band 2
2nd Digit

Color	Digit
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

Band 3 (if used)
3rd Digit

Color	Digit
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

Multiplier

Color	Multiplier
Black	1
Brown	10
Red	100
Orange	1,000
Yellow	10,000
Green	100,000
Blue	1,000,000
Silver	0.01
Gold	0.1

Resistance Tolerance

Color	Tolerance
Silver	±10%
Gold	± 5%
Brown	± 1%

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Appendix B

CIRCUITS, EQUATIONS, AND PARAMETERS

INTRODUCTION

This appendix is a compendium of the basic circuits, parameters, and equations found throughout this text. Beside each major heading is a number, in parenthesis, that indicates the unit from which the material was obtained. For instance:

COUPLING AND BYPASS CIRCUITS (4&6)

indicates that the discussions on these circuits can be found in Unit 4 and Unit 6.

CONTENTS

CLAMPERS B-4

CLIPPERS B-6

COUPLING AND BYPASS CIRCUITS B-9

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RECTIFIER CIRCUITS B-12

THE RC CIRCUIT — PART 1 . . . . B-20

THE RC CIRCUIT — PART 2 . . . . B-23

VOLTAGE MULTIPLIERS B-27

CLAMPERS (6)

When you are designing clamper circuits, choose component values so that:

$V_{TH} >> V_T$	$R >> R_{TH}$
$R >> r_F$	$R_L >> R$
$R << R_R$	$R_P \leqslant \frac{R}{100}$

The value of the shunt resistor, R, should be chosen so that:

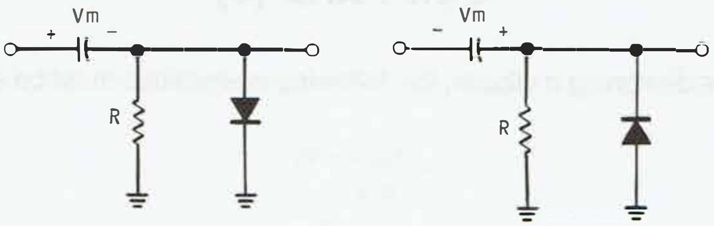
$$R = \sqrt{r_F R_R}$$

In addition, the capacitor should be chosen so that:

$$C = \frac{50T}{R}$$

Where:

C is the capacitance in Farads (F).
T is the period of the input voltage in seconds.
R is the shunt resistance in ohms.



NEGATIVE CLAMPER POSITIVE CLAMPER

Figure B-1

Positive and negative clampers with shunt resistor.

A positive and a negative clamper are shown in Figure B-1. In Figure B-2, positive and negative biased clampers are illustrated. Meanwhile, Figure B-3 depicts an adjustable positive clamper.

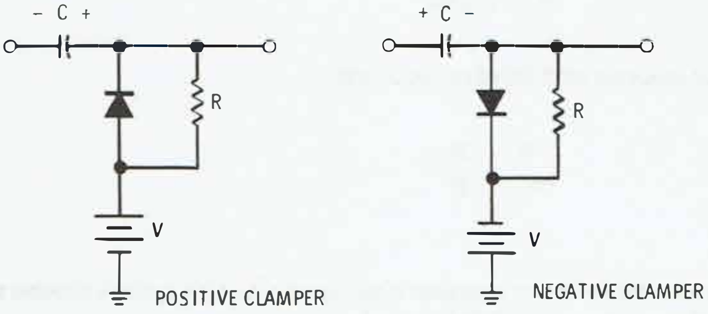


Figure B-2

Biased clamping circuits.

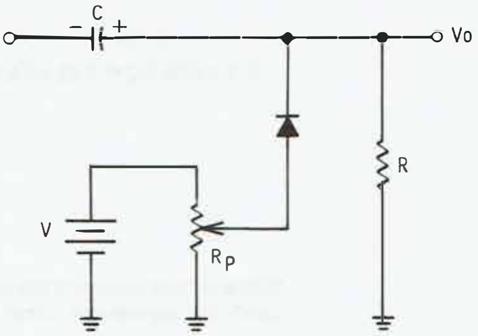


Figure B-3

An adjustable positive clamper.

CLIPPERS (6)

When you are designing a clipper, the following inequalities must be satisfied:

$$V_{TH} >> V_T$$
$$R >> r_F$$
$$R << R_R$$

Where:

V_T is the diode turn-on voltage.
r_F is the diode forward resistance.
R_R is the diode reverse resistance.
V_{TH} is the peak value of the Thevenin voltage driving the circuit.
R is the resistance of the clipping circuit.

The optimum value for R is:

$$R = \sqrt{r_F R_R}$$

To minimize the effects of source and load resistance:

$$R >> R_{TH}$$
$$^*R_L >> R$$

---

*Since R_L may represent the resistance of the next stage connected to the clipper output, it may not be possible to satisfy this requirement. Other inequalities, however, should be observed.

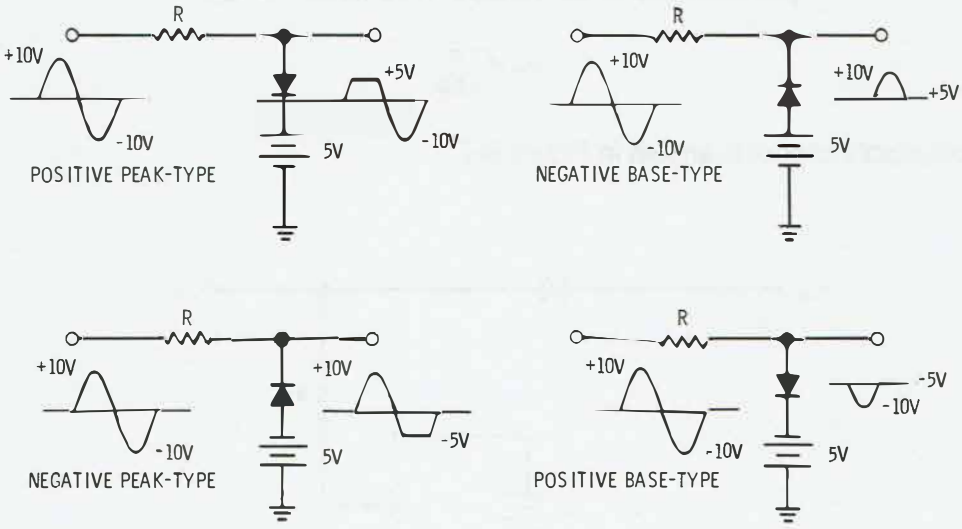


Figure B-4

Biased shunt-type clippers.

Figure B-4 shows four biased shunt-type clippers. Figure B-5 illustrates four biased series-type clippers, and Figure B-6 represents a dual-type clipper.

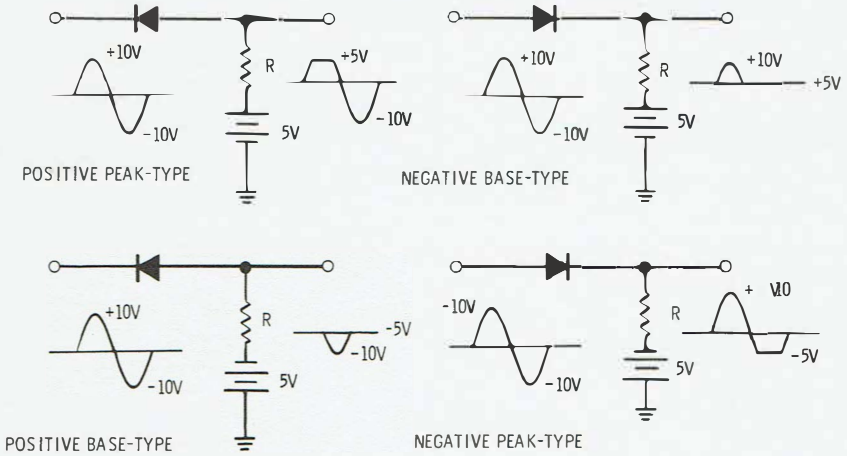


Figure B-5

Biased series-type clippers.

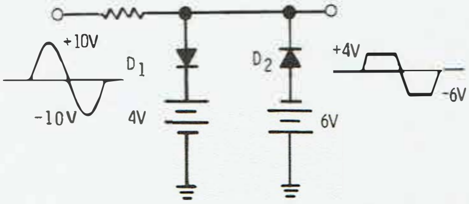


Figure B-6

A dual-type clipper.

If you want an adjustable clipper, select a potentiometer so that:

$$R_P \leq \frac{R}{100}$$

An adjustable clipper is shown in Figure B-7.

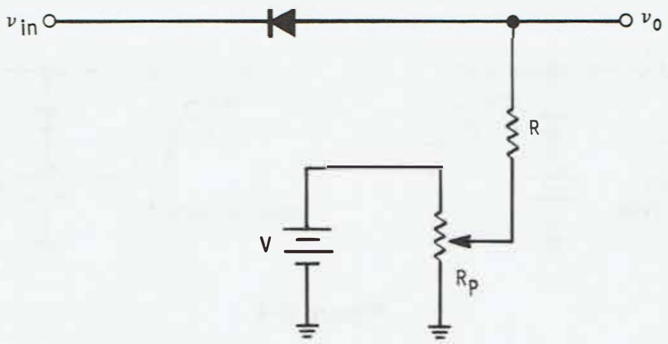


Figure B-7

A positive series-type adjustable clipper.

COUPLING AND BYPASS CIRCUITS (4&6)

The value of the capacitor used in a coupling or bypass circuit is determined as:

$$|X_C| = \frac{R_{TH}}{20}$$

Where:

X_C is the capacitive reactance at the lowest frequency to be coupled.
R_{TH} is the Thevenin resistance as seen from the capacitor.

Figure B-8A shows a capacitor used in a coupling configuration, while Figure B-8B illustrates the use of a capacitor in a bypass configuration.

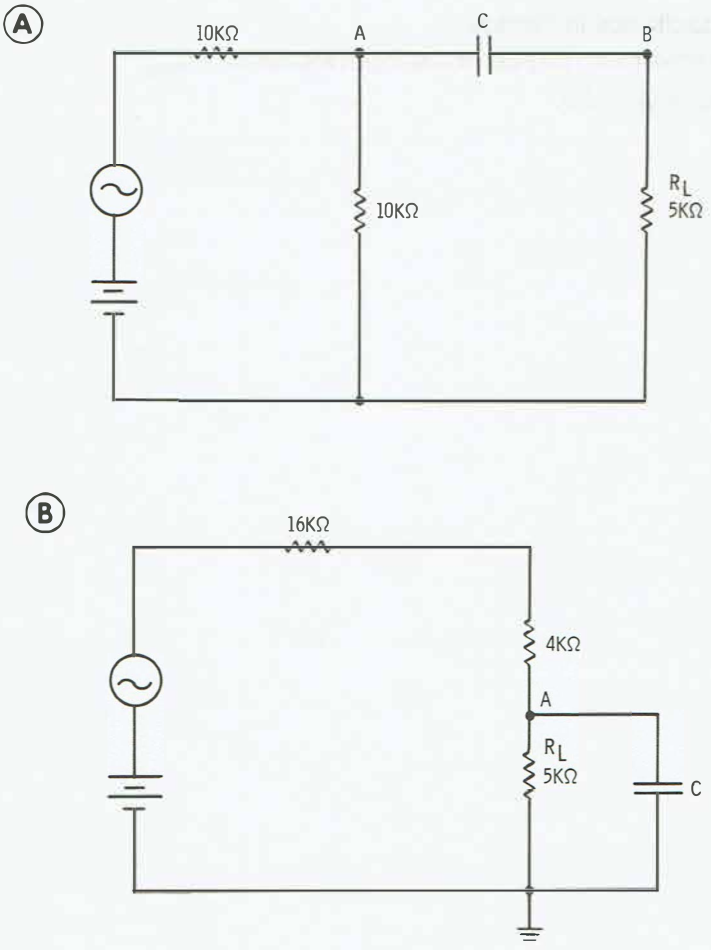


Figure B-8

Coupling and bypass circuits.

PEAK DETECTOR (6)

When you are designing peak detectors, select component values so that:

$$V_M >> V_T$$
$$R >> r_F$$
$$R << R_R$$
$$R >> R_{TH}$$

The capacitor in the peak detector circuit is selected so that:

$$C = \frac{50T_{MAX}}{R}$$

Where:

C is the capacitance in Farads.
T_{MAX} is the maximum period of the input signal or $1/f_{MIN}$.
R is equal to $R_{TH} \times 20$.

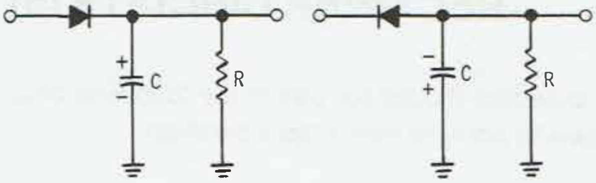
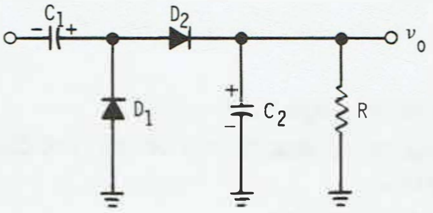


Figure B-9

Peak detector circuits.

Positive and negative peak detectors are shown in Figure B-9.

You can construct a peak-to-peak detector by cascading a clamper and a peak detector. Two such detectors are shown in Figure B-10.



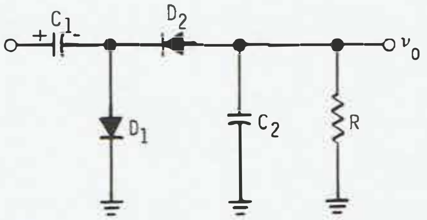


Figure B-10

Peak-to-peak detector circuits.

RECTIFIER CIRCUITS (5)

When you are selecting diodes for use in the following circuits, use Table B-1 for component values to achieve near ideal operation.

Parameter	Negligible When	Appropriate Component Value
V_T	$V >> V_T$	$V \geqslant 20\ V_T$
r_F	$R >> r_F$	$R \geqslant 20\ r_F$
R_R	$R << R_R$	$R \leqslant 20\ R_R$

Table B-1

In this table:

V_T is the diodes turn-on voltage.
V is the source voltage or voltage that is driving the diode.
R_L is the load resistance.
r_F is the diode forward resistance.
R_R is the diode reverse resistance.

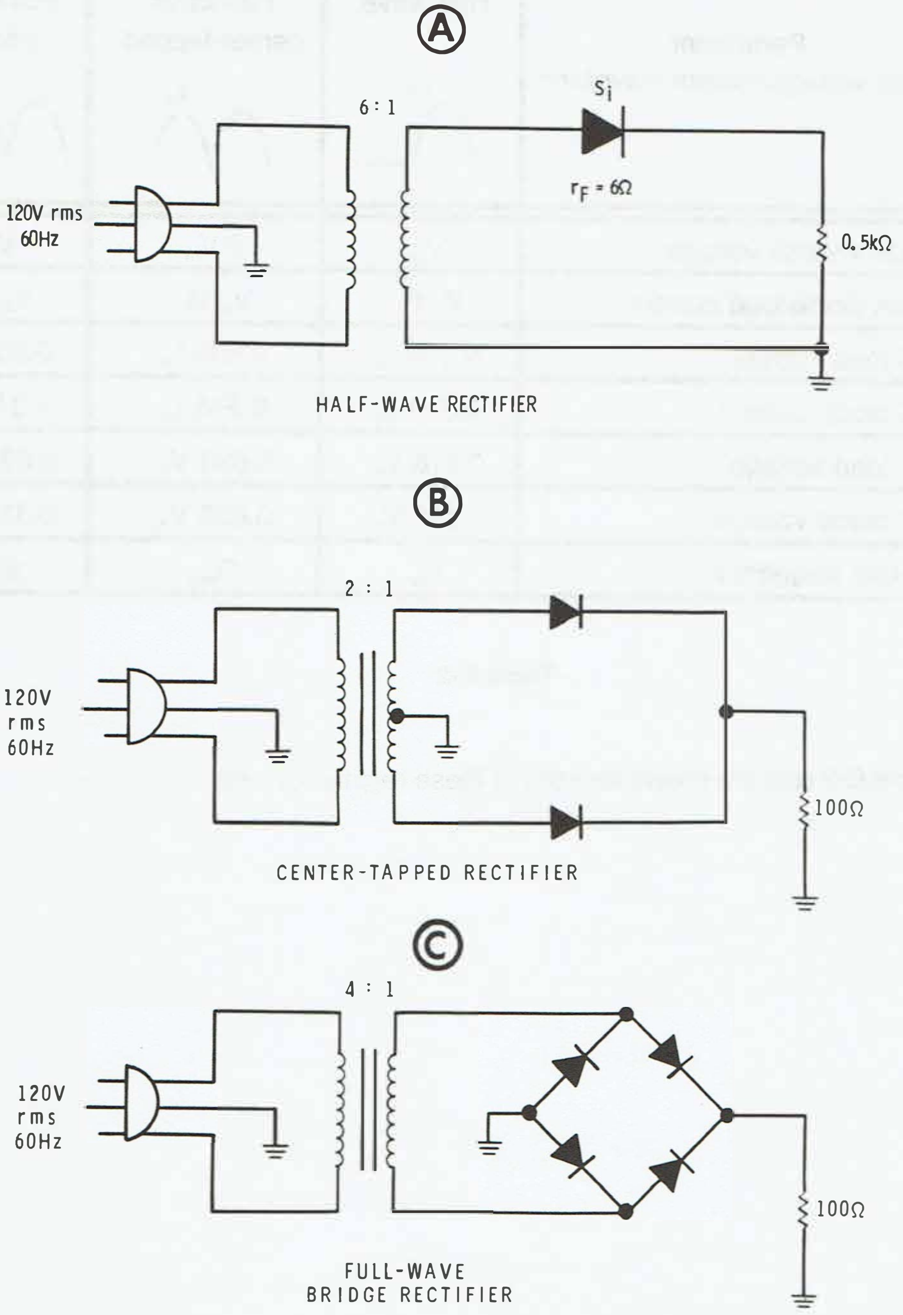


Figure B-11
Rectifier circuits.

Three common rectifier circuits are shown in Figure B-11. Figure B-11A is a half-wave rectifier, Figure B-11B is a full-wave rectifier, and Figure B-11C is a full-wave bridge rectifier.

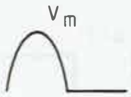
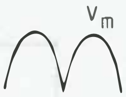
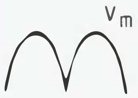
Parameter Load voltage/current waveform	Half wave 	Full-wave center-tapped 	Full-wave bridge 
Peak inverse voltage	V_m	$2\ V_m$	V_m
Peak diode/load current	V_m/R_L	V_m/R_L	V_m/R_L
DC load current	$0.318\ I_m$	$0.636\ I_m$	$0.636\ I_m$
DC diode current	$0.318\ I_m$	$0.318\ I_m$	$0.318\ I_m$
DC load voltage	$0.318\ V_m$	$0.636\ V_m$	$0.636\ V_m$
DC diode voltage	$0.318\ V_m$	$0.636\ V_m$	$0.318\ V_m$
Output frequency	f_{line}	$2f_{line}$	$2f_{line}$

Table B-2

Table B-2 lists the characteristics of these rectifier circuits.

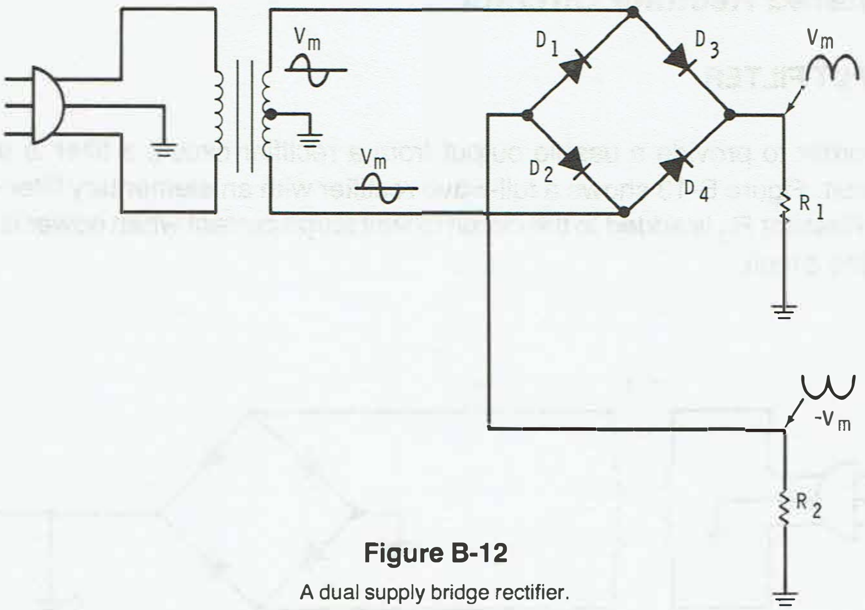


Figure B-12

A dual supply bridge rectifier.

Figure B-12 shows a variation of the full-wave bridge rectifier: a dual supply bridge rectifier. Parameters for this circuit are given in Table B-3.

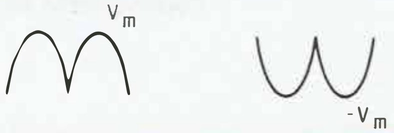
Parameter Load voltage/current waveforms.	
Peak inverse voltage	$2V_m$
Peak diode/load current	$\frac{V_m}{R_1}$ (D_3 & D_4) $\frac{V_m}{R_2}$ (D_1 & D_2)
DC load currents	$0.636\ I_m$
DC diode current	$0.318\ I_m$
DC load voltages	$+0.636\ V_m - 0.636\ V_m$
DC diode voltages	$0.636\ V_m$
Output frequency	$2\ f_{line}$

Table B-3

Filtered Rectifier Circuits

INPUT FILTER

In order to provide a usable output from a rectifier circuit, a filter is added to the circuit. Figure B-13 shows a full-wave rectifier with an elementary filter — capacitor C. Resistor R_S is added to the circuit to limit surge current when power is first applied to the circuit.

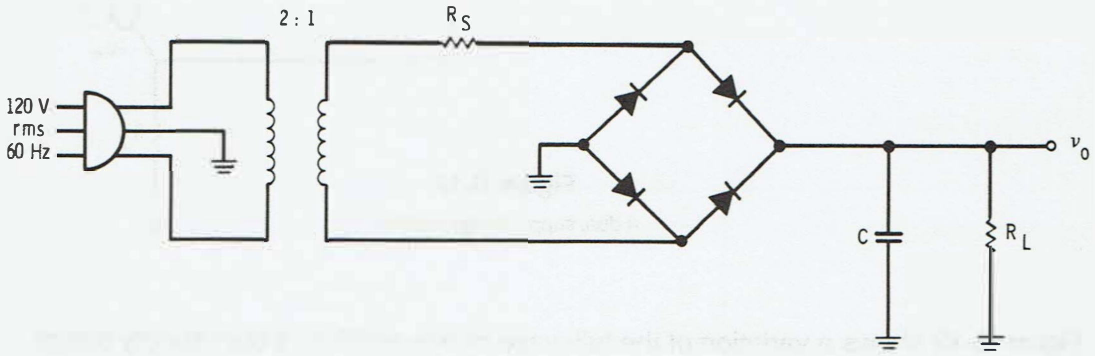


Figure B-13

Full-wave rectifier with filter and surge resistor.

Parameter	Half-Wave Rectifier	Full-Wave Rectifier
DC output voltage	$V_{DC} = V_m \left[1 - \frac{8.34 \times 10^{-3}}{R_L C}\right]$	$V_{DC} = V_m \left[1 - \frac{4.17 \times 10^{-3}}{R_L C}\right]$
RMS ripple voltage	$V_r = \frac{4.8 \times 10^{-3} V_{DC}}{R_L C}$	$V_r = \frac{2.4 \times 10^{-3} V_{DC}}{R_L C}$
Ripple factor	$r = \frac{4.8 \times 10^{-3}}{R_L C}$	$r = \frac{2.4 \times 10^{-3}}{R_L C}$
Minimum capacitance	$C_{min} = \frac{4.8 \times 10^{-3}}{r R_L}$	$C_{min} = \frac{2.4 \times 10^{-3}}{r R_L}$

Table B-4

Capacitor-input filter analysis and design equations for a 60 Hz line frequency.

Table B-4 contains the formulas for designing capacitor input filters.

To calculate the value of the surge limiting resistor, use the formula:

$$R_S = \frac{V_M}{I_S}$$

Where:

I_S is the maximum desired limit for surge current.
V_M is the peak voltage input to the rectifier circuit.
R_S is the value of the surge resistor.

CHOKE-INPUT FILTER

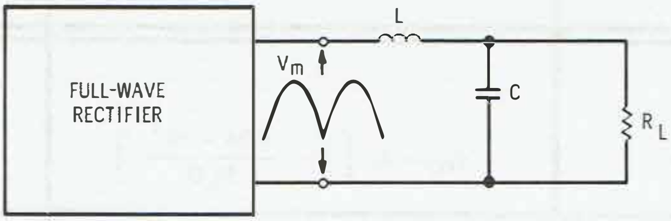


Figure B-14

Choke-input filter.

The choke-input filter is shown in Figure B-14. Parameters and formulas for choke-input filters are given in Table B-5.

Parameter	Full-wave Rectifier
DC output voltage (ideal)	$V_{DC} = 0.636\ V_m$
Actual DC output voltage	$V_{DC} = \dfrac{0.636\ V_m\ (R_L)}{R_C + R_L}$ or $V_{DC} = 0.636\ V_m - I_{DC}\ R_C$
Ripple factor	$r = \dfrac{830 \times 10^{-9}}{LC}$
Minimum inductance	$L_{min} = \dfrac{R_L}{1000}$
Required capacitance	$C = \dfrac{830 \times 10^{-9}}{rL}$
Smoothing factor (ideal)	$\alpha = 5.68 \times 10^5\ LC - 1$

Table B-5

Choke-input filter analysis and design equations for a 60 Hz line frequency full-wave rectifier.

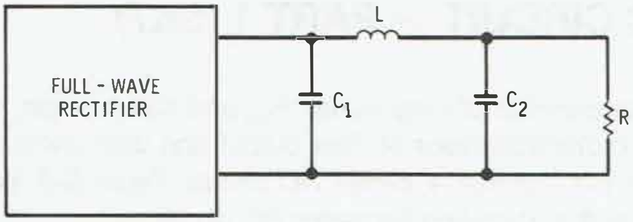


Figure B-15

Cascaded input filter.

CASCADED INPUT FILTER

It is possible to further reduce the ripple voltage from a rectifier circuit by using cascaded filter sections, as shown in Figure B-15. For this type of filter, the following inequalities must be satisfied:

$$|X_{C_2}| << R_L$$
$$|X_L| >> |X_{C_2}|$$
$$|X_{C_1}| << |X_L|$$

π-TYPE FILTER

A resistor may be used instead of a choke in a filter. Figure B-16 depicts a π-type filter. For this filter, the following inequalities must be met:

$$|X_{C_2}| << R_L$$
$$R >> |X_{C_2}|$$
$$|X_{C_1}| << R$$

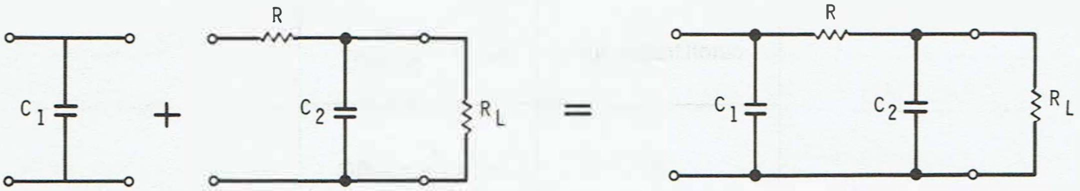


Figure B-16

Resistor π-type filter.

THE RC CIRCUIT — PART 1 (6&7)

Voltage gain in an RC circuit consists of magnitude, $|A_V|$, and phase angle, θ. You must determine the desired characteristics of your output and then choose your component values. Figure B-17 shows a series RC circuit. Table B-6 lists the parameters and formulas used in analyzing the series RC circuit.

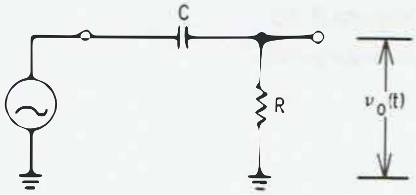


Figure B-17

Series RC circuit.

High-Pass Filter

The circuit in Figure B-17 is a high-pass filter. The formulas in Table B-6 can be used to determine the parameters for this filter.

Parameter	Formula
Voltage gain	$\lvert A_v \rvert = \dfrac{1}{\sqrt{1 + \dfrac{1}{n^2}}}$
Phase angle	$\theta = \arctan \dfrac{1}{n}$
cutoff frequency	$f_{co} = \dfrac{1}{2\pi RC}$
n	$n = \dfrac{f}{f_{co}} = \omega RC$

Table B-6

Characteristics of the series RC circuit when $V_O = V_R$.

Phase Shifter

An RC circuit can be used to shift the phase of a signal. Figure B-18 depicts a phase shifter as well as the appropriate voltage and impedance diagrams for this circuit. The amount of phase shift produced by this circuit can be up to 90° and will vary with component values. Again, use Table B-6 to determine the appropriate values for components.

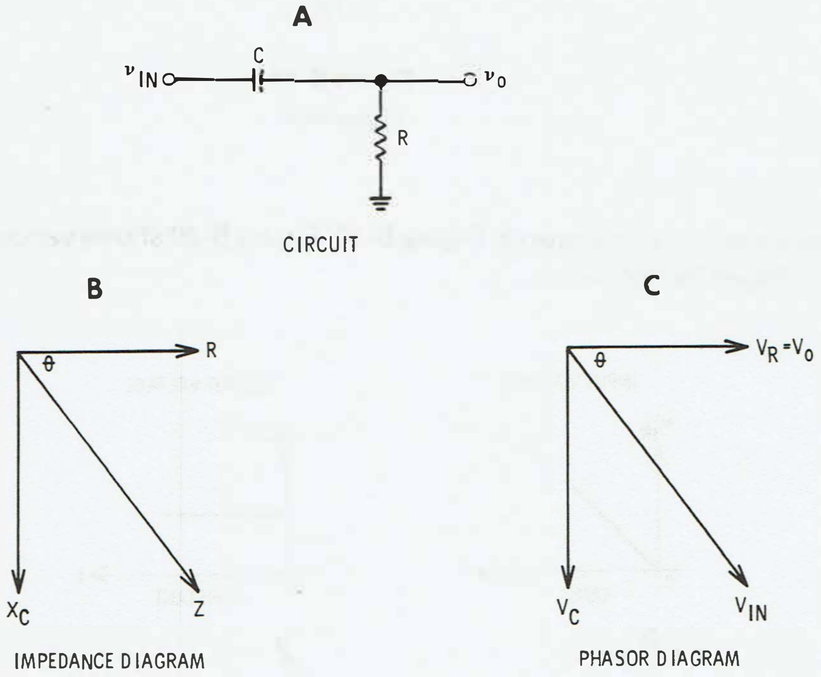


Figure B-18

A 0 to 90° phase shifter. Note that V_R leads V_{IN} by the angle θ.

Differentiator

When the time constant of an RC circuit is small compared to the period of the input signal, the RC circuit acts as a differentiator.

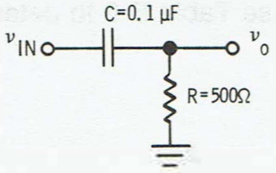


Figure B-19

Differentiator.

A differentiator circuit is shown in Figure B-19. Figure B-20 shows various input and output voltages for this circuit.

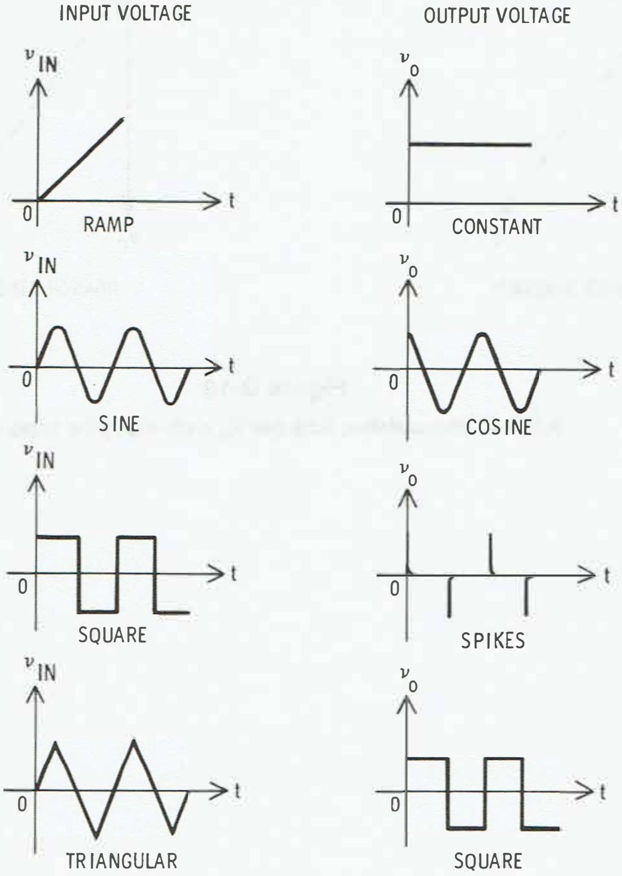


Figure B-20

Differentiator input and output waveforms.

THE RC CIRCUIT PART 2 (6&7)

The output from an RC circuit can be taken across the capacitor as shown in Figure B-21. When this is done, the equations in Table B-7 are used to determine circuit characteristics.

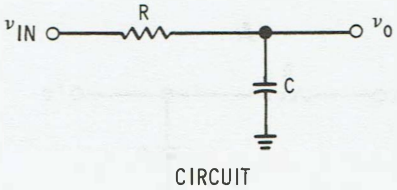


Figure B-21

RC circuit with output across the capacitor.

Low-Pass Filter

The circuit shown in Figure B-21 can be used as a low-pass filter. The formulas necessary to determine circuit parameters are given in Table B-7.

Parameter	Formula
Voltage gain	$\lvert A_v \rvert = \frac{1}{\sqrt{1 + n^2}}$
Phase angle	$-\alpha = \arctan n$
cutoff frequency	$f_{co} = \frac{1}{2\pi RC}$
n	$n = \frac{f}{f_{co}} = \omega RC$

Table B-7

Characteristics of the series RC circuit when $V_O = V_C$.

Phase Shifter

This RC configuration can also be used as a 0 to 90° phase shift circuit as shown in Figure B-22. Use the formulas given in Table B-7 to determine component values and circuit parameters.

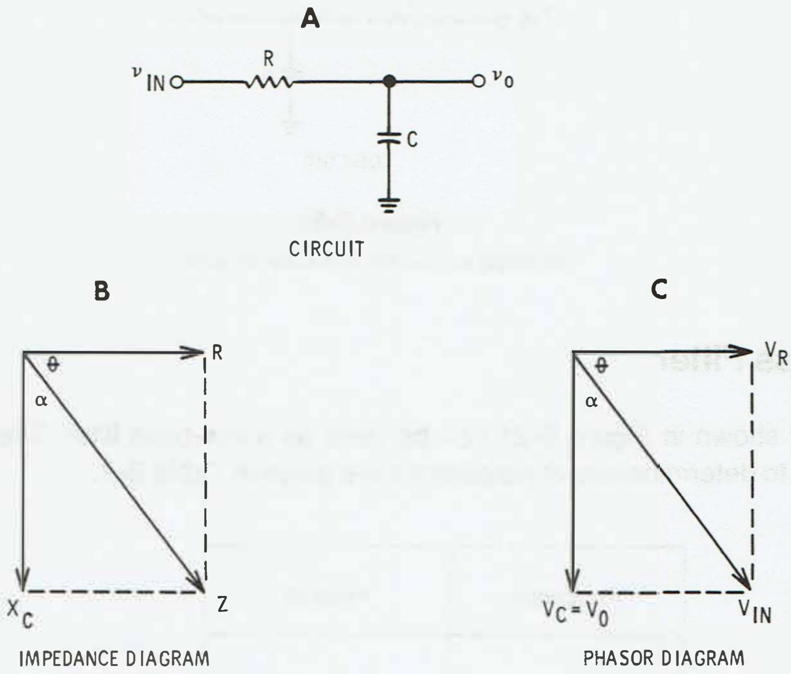


Figure B-22

A 0 to 90° phase shifter. Note that V_C lags V_{IN} by the angle α.

Integrator

If the time constant for an RC circuit is relatively large compared to the period of the input signal, the circuit acts as an integrator. Typically, the components are chosen so that:

$$RC = 20T$$

An integrator is shown in Figure B-23. Input and output voltage waveforms are also shown in this figure.

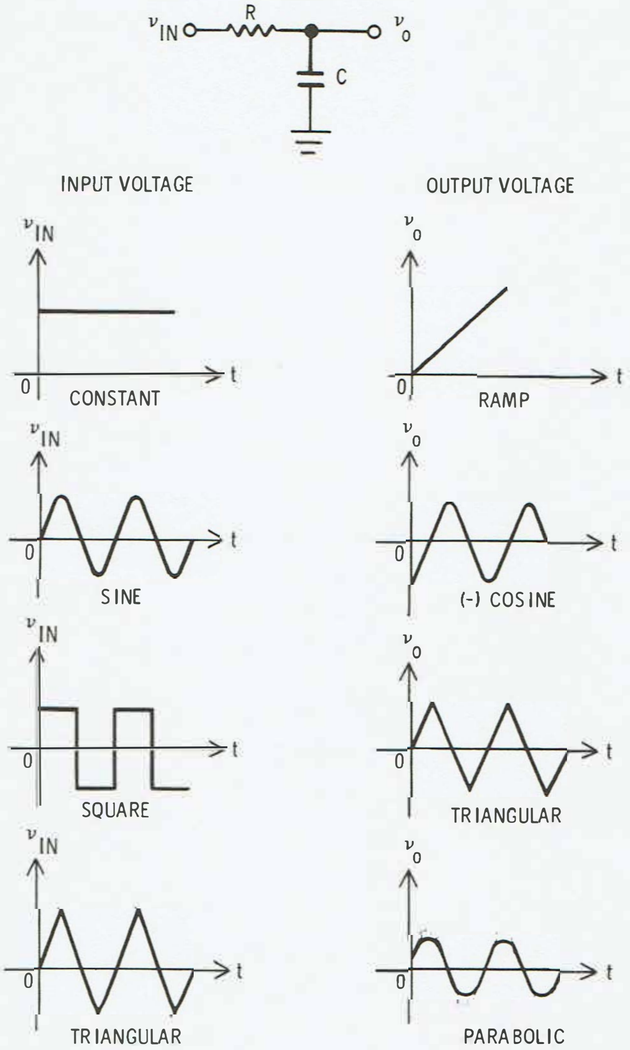


Figure B-23

Integrator input and output waveforms.

Note:

For all RC circuits discussed, a value of R should be chosen so that:

$$R >> R_{TH}$$
$$R_L >> R$$

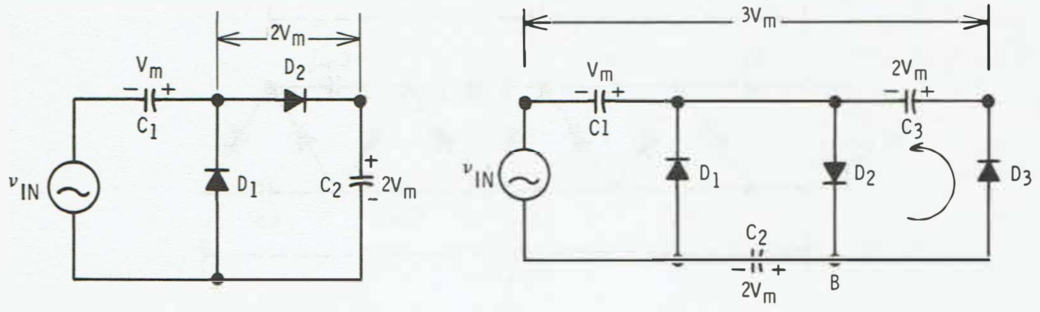
Where:

R_{TH} is the resistance of the source driving circuit.

VOLTAGE MULTIPLIERS (6)

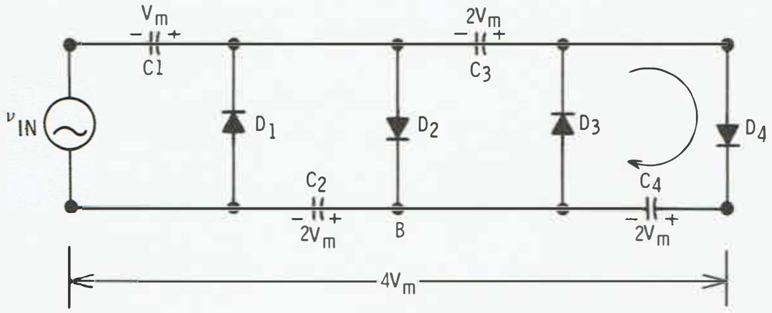
A voltage multiplier provides an output such that:

$$V_O = nV_M$$



VOLTAGE DOUBLER

VOLTAGE TRIPLER



VOLTAGE QUADRUPLER

Figure B-24

Voltage multipliers.

A peak detector is a voltage multiplier with n = 1. A peak-to-peak detector is a voltage multiplier with n = 2. To construct a voltage multiplier, cascade the appropriate number of peak detector circuits. For component parameters, see "Peak Detectors." An n-section voltage multiplier is illustrated in Figure B-25.

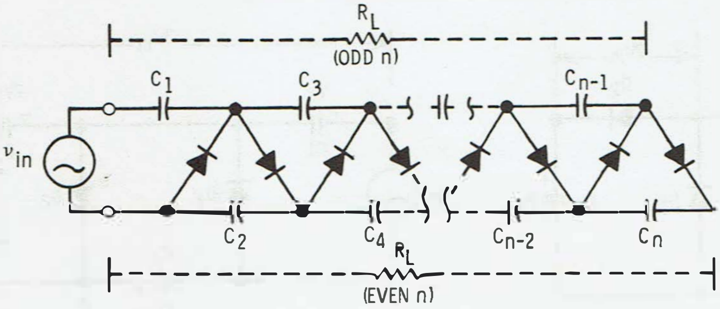


Figure B-25

An n-section voltage multiplier.

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